Nipissing University

Department of Computer Science & Mathematics

MATH 1036 - Calculus I

Instructor: A. Karassev

Time: 3 hours **Final Examination** **SAMPLE EXAM**

CLOSED BOOK. NO NOTES.
NO PROGRAMMABLE OR GRAPHIC CALCULATORS ARE ALLOWED.

IMPORTANT: Show your work! No marks will be given for stating an answer without providing a detailed solution to the question.

Number of marks question is worth is shown in square brackets following the question. The total of marks is 100.

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1. [**5 marks**] Find the range of the function $f(x) = \sqrt{x^2 + 2x + 5}$.

2. [**20 marks (each item (a), (b), (c), (d) is worth 5 marks)**]
Evaluate each limit.

(a) $\lim\limits_{x \to -\infty} \dfrac{\sqrt{2x^2+1}}{3x-5}$ (b) $\lim\limits_{x \to -3} \dfrac{x^2+7x+12}{x^2+5x+6}$ (c) $\lim\limits_{x \to 1} \dfrac{\ln x}{x-1}$

(d) $\lim\limits_{x \to 0} x^2(\cos(1/x) + 1/x)$

3. [**5 marks**] If $f(x) = x^{100} + \dfrac{1}{x}$, find $f^{(100)}(x)$ (i.e. the 100th derivative of f).

4. [**5 marks**] Prove that equation $x^3 + x - 1 = 0$ has exactly one real root.

5. [**5 marks**] Find the constant c that makes the following function continuous on $(-\infty, \infty)$.

$$f(x) = \begin{cases} cx^2 - 1 & \text{if } x \leq 2 \\ x - c & \text{if } x > 2 \end{cases}$$

6. [**5 marks**] Use the definition of derivative to find the derivative of the function $f(x) = \sqrt{x}$.

7. [**5 marks**] Find the x-coordinates of the points on the curve $y = \dfrac{x\tan x + 2\sin x}{\tan x}$ at which the tangent line is horizontal.

8. [**5 marks**] A spherical balloon is being inflated such that its volume is increasing at a constant rate of 2 m^3/h. How fast is the surface area of the balloon increasing when the radius of the balloon is 2 meters? (The volume of the ball that has radius r is $\frac{4}{3}\pi r^3$; the area of its surface is $4\pi r^2$.)

9. [**5 marks**] Use linear approximation to estimate $\sqrt[3]{7}$.

10. [**15 marks (each item (a), (b), (c) is worth 5 marks)**]
Sketch the graph of each of the following functions. Provide detailed solution.

$$\text{(a) } y = |2\sin x - 1| \qquad \text{(b) } f(x) = \frac{2x^2}{x^2-1} \qquad \text{(c) } g(x) = xe^x$$

11. [**5 marks**] A farmer has 2400 ft of fencing and wants to fence off a rectangular field that borders a straight river. He needs no fence along the river. What are the dimensions of the field that has the largest area?

12. [**5 marks**] Find the derivative of the function $f(x) = \left(\dfrac{\sqrt{x-1}}{x^2\sqrt{x+1}}\right)^3 \cdot \dfrac{2x+1}{x+1}$

13. [**5 marks**] Find an equation of the tangent line to the curve $x^3 + xy^2 + 3 = y^3$ at the point $(1, 2)$.

14. [**10 marks (each item (a), (b) is worth 5 marks)**] Let $g(x) = x^5 + x^3 + x$.

(a) Show that g is one-to-one on $(-\infty, \infty)$. What is the domain of g^{-1}? Explain your answer.

(b) Find $(g^{-1})'(3)$.

The End