

precipitation or temperature data in the newspaper, data from the Internet about heights of buildings and other structures), presented in charts, tables, and graphs (including broken-line graphs)

## References

- Compliments *Maximum Raisin Bran Flakes Cereal Box*. (2007). Mississauga, ON: Sobey's.
- Hickman, C.P, Roberts, L.S., & Hickman, F.M. (1982). *Biology of Animals*. Toronto: The C.V. Mosby Company.
- Kellogg's *Two Scoops Raisin Bran Cereal Box*. London, ON: Kellogg Canada.
- Master Choice *40% Raisin Bran Flakes Cereal Box*. Toronto: The Great Atlantic & Pacific Company of Canada.
- Ministry of Education and Training. (1998). *The Ontario Curriculum, Grades 1–8: Mathematics*. Toronto: Queen's Printer.
- President's Choice "Extra Raisin" *Raisin Bran Cereal Box*. Toronto, ON: Loblaws Inc. ▲

## ▲ THINKING OUTSIDE THE RECTANGULAR PRISM: FOSTERING PROBLEM-BASED MATHEMATICS LEARNING

DAN JARVIS

E-MAIL: [dhjarvis@sympatco.ca](mailto:dhjarvis@sympatco.ca)

WEB: [www.nipissingu.ca/faculty/danj](http://www.nipissingu.ca/faculty/danj)



Dan Jarvis is an Assistant Professor of mathematics education in the Faculty of Education, Nipissing University, North Bay, Ontario.

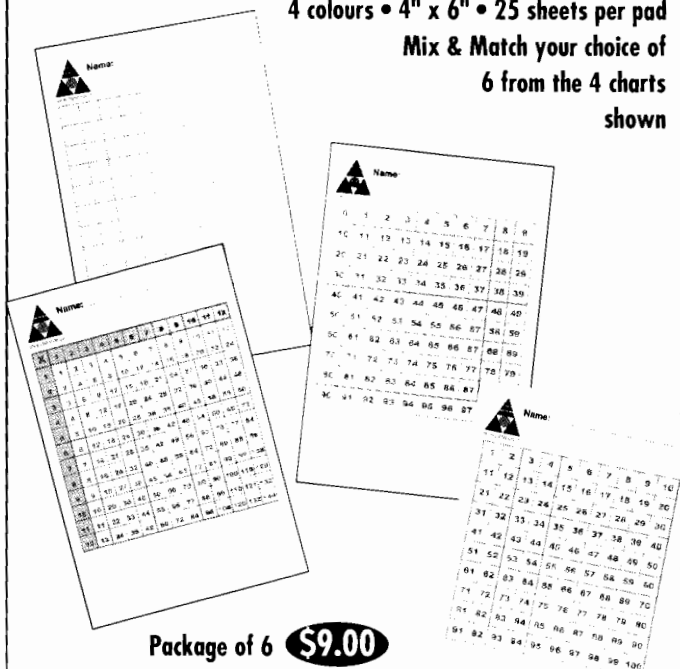
### Problem-Based Learning

"How would you describe your own mathematics education history? What was math class usually like for those 15+ years of your life?" It is this type of question that I like to pose to new teacher candidates each fall as they begin their Bachelor of Education program. For many of them, myself included, similar images, routines, and teacher/student behaviours are recalled, whether success was experienced in school mathematics or not. For example, a more traditional approach to mathematics learning often featured the teaching of a new skill (or skill set), followed by isolated practice questions, and then several application problems. But life's not quite like that, is it? Do we not face complex situations from dawn to dusk that involve people, places, and problematic things? And within these chaotic contexts, do we not continually attempt to look for patterns, draw upon prior knowledge and intuition, make suggestions, try out ideas, negotiate with others, learn from our mistakes, and document our progress? Sometimes, we even solve the problem! However, it is arguably the messy process that is the most meaningful in terms of individual growth, social learning, and the ability to tackle tomorrow's problem with equal—or even increased—determination.

Problem-Based Learning (PBL) is certainly not a new concept, but it is receiving an increasing amount of attention in reform-oriented mathematics resources and from reform-oriented mathematics organizations. For example, the *Ontario Curriculum: Mathematics, Revised* (Ontario Ministry of Education, 2005a, 2005b. 2007)

## OAME ELEMENTARY STICKY CHART PADS

Great for Elementary or Secondary Student Use  
4 colours • 4" x 6" • 25 sheets per pad  
Mix & Match your choice of  
6 from the 4 charts  
shown



Package of 6 **\$9.00**  
plus shipping and handling.

**Order by mail, fax or online:**

OAME, 70 Chestnut Crt., London, ON N6K 4J5  
Phone/fax (519) 471-6324 or Order online [www.oame.on.ca](http://www.oame.on.ca)  
Do not send cash or cheque. You will be invoiced with shipping and handling costs added.

advocates Problem Solving as one of its seven core mathematical processes<sup>1</sup>, and the National Council of Teachers of Mathematics (NCTM) strongly argues for a problem-solving focus in its *Principles and Standards for School Mathematics* document:

*Solving problems is not only a goal of learning mathematics but also a major means of doing so. Problem solving is an integral part of all mathematics learning, and so it should not be an isolated part of the mathematics program. Problem solving in mathematics should involve all the five content areas... Good problems will integrate multiple topics and will involve significant mathematics. (2000, p. 52)*

Many math teachers are familiar with the work of the late John van de Walle, particularly in terms of his helpful three-part lesson (i.e., *before* [getting ready], *during* [students at work], and *after* [class discussion]) (Van de Walle & Folk, 2008, p. 42). Van de Walle describes a problem-based approach:

*...students solve problems, not to apply mathematics but to learn new mathematics. When students engage in well-chosen problem-based tasks and focus on the solution methods, what results is new understanding of the mathematics embedded in the task. When students are actively looking for relationships, analyzing patterns, finding out which methods work and which don't, justifying results, or evaluating and challenging the thoughts of others, they are necessarily and optimally engaging in reflective thought about the ideas involved... It is a process that requires faith in children—a belief that all children can create meaningful ideas about mathematics. (pp. 37–38)*

In describing her “10 big math ideas,” author Marilyn Burns (2004) notes that “confusion is part of the process” and that we, as teachers, should be continually attempting to “encourage different ways of thinking” (p. 19). In discussing learning contexts that are highly engaging and satisfying for students, Seymour Papert introduced the notion of “hard fun,” a term that he adopted after talking with a young student who had been using his Logo computer software program. Papert further notes:

---

<sup>1</sup> Within the Ontario Curriculum: Mathematics, Revised (2005–07), the following seven core mathematical processes are highlighted: problem solving; reasoning and proving; reflecting; selecting tools and computational strategies; connecting; representing; and communicating. All seven of these can be practised regularly in a classroom where PBL is being used.

*I have no doubt that this kid called the work fun because it was hard rather than in spite of being hard. Once I was alerted to the concept of ‘hard fun’ I began listening for it and heard it over and over. It is expressed in many different ways, all of which all boil down to the conclusion that everyone likes hard, challenging things to do. But they have to be the right things matched to the individual and to the culture of the times. (n.d., paragraph 2)*

Canadian author Marian Small (2008), yet another strong advocate for problem-based mathematics learning, notes that “some of the ways teaching through problem solving differs from a traditional approach include: an increased level of mathematical dialogue between students; the teacher’s role as guide or coach more than as a presenter; and, the teacher’s more judicious use of intervention” (p. 38).

As a teacher who has worked with elementary and secondary school students, and who now has the wonderful opportunity to work with prospective teachers, I have collected many mental images of different forms of PBL through the observation of colleagues, through practice teaching supervision, and through my own experimentation with a more problem-based approach to learning. What follows, therefore, is a distilled version of all of that information—those multiple perspectives—in terms of practical implications of what this type of teaching might include. With grateful acknowledgement of Van de Walle’s “three-part lesson” and of the “MATCH Template” that was further developed by the authors of the Targeted Implementation and Planning Supports (TIPS) resource (Consortium of Ontario School Boards, 2005), I intend to reframe the PBL process using four headings—*Explain, Explore, Express, and Extend*—and to show how the seven Ontario mathematical processes<sup>2</sup> are naturally incorporated within this teaching method. I was tempted to use the pun-laced title, “Problem-Based Learning with E’s,” but those educators who have been brave enough to attempt even a small move toward PBL know that there is nothing particularly simple or “easy” about this type of pedagogy. On the contrary, it’s hard work—but it’s fun.

---

<sup>2</sup> I would like to thank both (anonymous) reviewers of this article, and particularly the one who suggested that I more thoroughly elaborate on the mathematical processes throughout this paper.

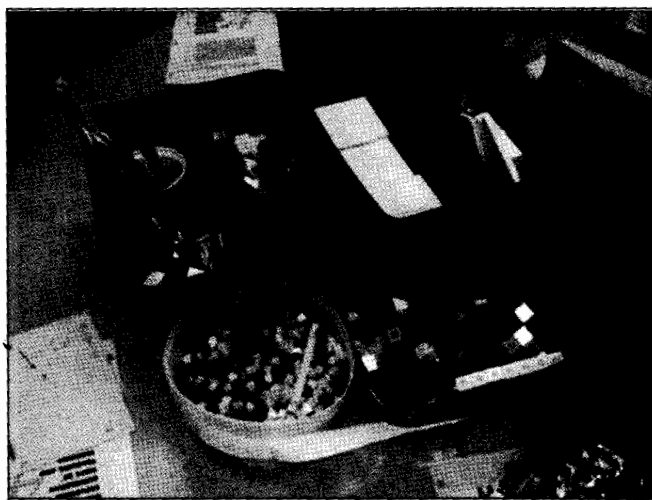
## Explain the Problem

The teacher should begin the class by asking one or two questions that pertain to “real-life” situations and that also relate to the new problem of the day (e.g., for young learners, “How many of you have buttons on your clothing today? Which colours? etc., prior to a graphing exercise; or for older learners, “How many of you have travelled to Toronto? Approximately how long did that take?” etc., leading up to a problem involving travel budgets). These introductory questions (which may also tie into previous learning in the class) serve to engage the group and provide a bridge to the problem (e.g., “Similarly, today’s problem will involve...”). By asking such questions, the teacher is also promoting the mathematical process of **connecting**. The teacher should then present the problem in at least two different ways so that all students will have a chance to absorb the information. For example, the problem should be read aloud to the class, but should also be presented in writing (e.g., in a student handout, on flip-chart paper, or perhaps using a *PowerPoint* slideshow or other presentation software). For the English Language Learner or the ESL or LD student, it would also be appropriate to provide a concrete or pictorial explanation of the problem, but perhaps individually, as it is preferable that most students model the problem on their own.

At this point, it is very important that the teacher now literally stop and ask students to rephrase the problem in their own words, without reading it from the screen or a handout; e.g., “What are we being asked to do in this problem?” and, “What are some important things that we need to consider?” This step accomplishes several objectives. First, it allows the teacher to better understand what it is that the average student has understood, or not understood, about the problem. Second, having the problem restated by a peer in “student-ese” often helps other students “clue in” to the problem. Third, having students mentally summarize the stated problem helps them to develop the mathematical process of **reflecting**. Once the important information has been publicly volunteered and/or solicited, the teacher asks the class if there are any other questions, or if further clarification is needed with respect to the problem. If this crucial step is skipped, the usual result is that the teacher will have to spend 10–20 valuable minutes answering the same clarification question(s) with all groups in the room.

## Explore the Problem

Students are seated in pairs, triads, or small groups with no more than six to a table (as large numbers preclude meaningful dialogue and make the distribution of intellectual work more difficult). At the tables (or grouped desks), there should be manipulatives; possibly technology (e.g., calculators, depending on the problem); and a large sheet of blank flip-chart paper (or grid paper) with markers. Access to manipulatives can be dealt with in several ways. Some teachers have them organized in clearly labelled bins at the periphery of the classroom (on shelves or in cupboards). Others place the desired manipulative(s) on each group’s table in advance of the lesson. In my own workshops with teacher candidates, I have moved toward a slightly different approach—that of providing many different manipulatives from which the teacher candidates themselves can choose. These “Thinking Kits” have gone through several stages of development. I initially used various plastic bins held together with duct tape (The Red Green Special—see *Figure 1*), with each bin containing a particular type of manipulative (e.g., linking cubes, two-sided counters, pattern blocks). I then moved to a set of much larger plastic bins (see *Figure 2*) containing manipulatives separated into individual zip-locked bags. (The double-zip design is best, as it is much easier to open and close and seems to last longer.)



*Figure 1.* First version of the manipulative “Thinking Kits”

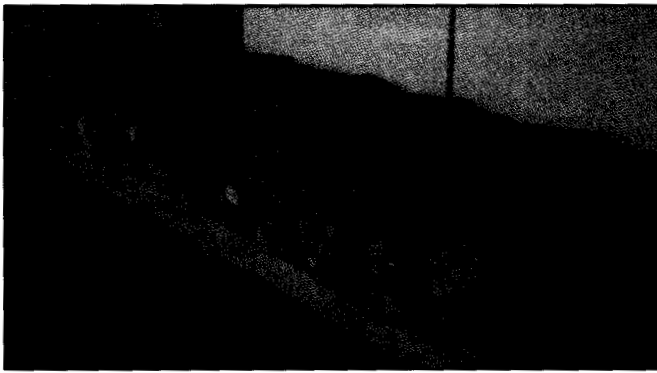


Figure 2. Second version of the manipulative "Thinking Kits"

One may argue, and rightfully so, that using kits filled with manipulatives is not a new idea in mathematics teaching. That being said, the main difference here is that by making these kits always available over the course of the year, and by not prescribing which manipulatives should be used for any given problem, teacher candidates become familiar with the different manipulatives (see *Figure 3*) and develop a sense of what to use when, and why, without being told. In terms of the mathematical processes involved, by following this method, the teacher candidates are **representing** (modelling) the problems and **selecting tools and strategies** to solve the problems. This method also fosters richer discussion, as the teacher candidates will often use a number of approaches and/or materials to tackle the given problem. In the third version of the kits, I removed some manipulatives such as clocks, Tangrams, and Pentominoes—items that are not as useful for solving a variety of problems, but that can be brought out separately during the year to teach particular topics, or for specific types of mathematical explorations.



Figure 3. Common manipulatives found within the kits

Our own *Ontario Curriculum: Mathematics* documents (Ontario Ministry of Education, 2005a, 2005b) note the following regarding the selection and use of manipulatives or concrete learning tools:

*Students should be encouraged to select and use concrete learning tools to make models of mathematical ideas. Students need to understand that making their own models is a powerful means of building understanding and explaining their thinking to others. Using manipulatives to construct representations helps students to see patterns and relationships; make connections between the concrete and the abstract; test, revise, and confirm their reasoning; remember how they solved a problem; and, communicate their reasoning to others. (pp. 14–15)*

Although the types of manipulatives found within a kit may vary, the same approach to PBL can and should be taken, at least occasionally, with secondary school students as well. For example, my calculus students were just as enthusiastic about using cylinders, water, and timers when studying rates of change as my own preschool children were in using linking cubes to study patterns.

During the *explore* stage, teachers are encouraged to first perform a quick rotation through the room, stopping at each group only long enough to ensure that students are on task, and to provide clarification where needed. Following this first circuit, the teacher is now free to make several slower rotations throughout the room for the remainder of the period, during which time he or she is able to collect valuable assessment information. The ability to ask good questions is developed during this time, as are the skills of listening and scaffolding. Anecdotal information is often collected from one or two groups during a problem-based lesson, with the teacher planning to eventually assess all students over subsequent lessons. This is also an opportune time to encourage and assess students as they demonstrate the mathematical processes<sup>3</sup>.

### Express the Problem/Solution

The *expression* (or consolidation) stage is, unfortunately, the one that is most often poorly done or entirely ignored, yet it is arguably the most important part of the PBL approach. There are many ways to orchestrate this sharing of student work. The method that I have witnessed and like the most—and have hence adopted—is as follows: the teacher first asks for one

<sup>3</sup> For an excellent set of questions that encourages development of the mathematical processes, the reader may wish to consult the *Targeted Implementation and Planning Supports for Revised Mathematics (TIPS4RM) Process Package*, available online at [www.edu.gov.on.ca/eng/studentsuccess/lms/files/tips4rm/TIPS4RMPProcesses.pdf](http://www.edu.gov.on.ca/eng/studentsuccess/lms/files/tips4rm/TIPS4RMPProcesses.pdf).

group (or pair) representative to stand and express the group's solution attempt. This solution is then posted at the bottom of the left-hand side of the board. The teacher then asks if any other groups have solved the problem in a similar way. If so, representatives from those groups come forward and pin or tape their solutions directly above the first group's sheet. Now the teacher asks for another way to approach the problem, followed by a student presentation. These students pin their solutions to the wall in a second column, adding similar group strategies to this column vertically. When complete, all group solutions appear on the front board in a sort of visual histogram format (i.e., a vertical bar graph of solution types). Note that students can choose to use actual manipulatives and/or technology to present their solutions to the larger group; often the group (or partner) recorder will use an illustration to show how the group or pair used the manipulatives, making it somewhat easier to present this thinking to the class. In this situation, students are **communicating** ideas that make sense to them, thereby demonstrating another mathematical process.

Finally, students are asked to examine the various solutions in order to make comparisons, point out overlapping parts of solutions (perhaps moving solutions to other columns as a result of a specific rationale), discuss preferred approaches, and so on.<sup>4</sup> This represents a form of higher-order **reasoning and proving**—our final mathematical process—which, if practised as a regular part of problem-based teaching/learning, will encourage students to routinely look for multiple-solution strategies en route to a final and justifiable answer. Note carefully, that if all solution strategies are identical and are thus visually displayed in one single, long column, this is usually a clear sign that the problem selected by the instructor is much less “rich” than one that would allow for two or more approaches. (The best problems are collected over time, based on feedback and reflection.) Finally, the verbal expression of solutions is paramount—students need to know that you value their work and ideas, as demonstrated by your willingness to listen.

## Extend the Problem

In this final stage, students can work on a similar problem for their home assignment and/or can even be given the opportunity to create or pose their own related problems, to be shared with their peers in subsequent lessons. Although practising basic skills is still an important part of any mathematics program, home assignments/practice should also feature—on occasion at least—the tackling of similar “rich” problems so that students may further build upon the new knowledge and understanding that they have acquired during the PBL process. Double exposure to a problem (i.e., both inside and outside of school) will help reinforce concepts and will raise an awareness of mathematics in the student's everyday world. Once again, the mathematical process of **connecting** is being reinforced at this stage. Often, problems will require more than one lesson for students to adequately explore the content. In this case, the problem-solving activity can be extended over two or more days, allowing students the time to seek out further information, gather input from other people, and revisit the problem in different ways.

## Closing Thoughts

Bluntly put, problem-based learning can be very uncomfortable at first, both for the teacher and for the students. If the former is used to “delivering” lesson content from well-polished, teacher-centred lesson plans, and prefers a quiet, non-interactive classroom, this approach to learning can be quite unnerving. Further, if students have not had adequate experience in productive, co-operative group-work contexts, a gradual introduction of such tasks will be required, moving from short, specific problems to much more involved tasks and/or problems later on.

Helping students to think outside the rectangular prism (i.e., the *Thinking Kit*) as they solve rich problems and share their results with peers represents a worthwhile, tangible, and reform-oriented goal. By explaining, exploring, expressing, and extending (see *Figure 4*) carefully selected problems, teachers can begin to compile a fantastic collection of resources that will engage students and foster their critical-thinking skills. And it is these very skills, forged through the *hard fun* of Problem-Based Learning experiences, that may turn out to be the most useful and beneficial for our students throughout their future lives.

---

<sup>4</sup> For a description of a similar, yet much more elaborate, version of this type of board-based consolidation activity, the reader may wish to consult the work of Takahashi (Japanese *Bansho*) and Kubota-Zarivnij (Ontario *Bansho*).

EXPLAIN
<input type="checkbox"/> Q <sub>T</sub> : Questions posed by teacher regarding real-life interests and/or prior knowledge
<input type="checkbox"/> P <sub>T</sub> : Teacher presents the problem in both visual and verbal ways
<input type="checkbox"/> P <sub>S</sub> : Problem is rephrased by students in their own words, without reading it
<input type="checkbox"/> Q <sub>S</sub> : Questions from students regarding clarifications or possible misunderstandings
EXPLORE
<input type="checkbox"/> R <sub>F</sub> : Fast rotation to make sure everyone is on-task
<input type="checkbox"/> R <sub>S</sub> : Slow, multiple rotations to assist, assess, and ask good questions
EXPRESS
<input type="checkbox"/> Communicate: Student representatives present their group's solution strategy
<input type="checkbox"/> Compare: Students organize solutions into a visual histogram and compare strategies
EXTEND
<input type="checkbox"/> Continued thinking about the problem that evening, if extended over two or more days
<input type="checkbox"/> Similar practice questions/problems assigned to (or created by) students that evening/week
<i>Thinking Kits for elementary/middle school problem-based mathematics learning might include the following items: linking cubes (small and large sizes), two-sided counters with 5- and 10- frames, counting bugs/objects, pattern blocks, square tiles, fraction pie pieces, number cubes, tape measures, Canadian currency, sorting hoops, algebra tiles, spinners, and calculators (for certain activities).</i>

Figure 4. Checklist of the four E's in the Problem-Based Learning approach

## References

- Burns, M. (2004). 10 big math ideas. *Instructor*, 113(7), 16–19.
- Consortium of Ontario School Boards. (2005). *Targeted implementation and planning supports for revised mathematics*. Toronto: Queen's Printer for Ontario.
- National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.
- Ontario Ministry of Education. (2005a). *The Ontario curriculum, grades 1–8: Mathematics, revised*. Toronto: Queen's Printer for Ontario.
- Ontario Ministry of Education. (2005b). *The Ontario curriculum, grades 9 & 10: Mathematics, revised*. Toronto: Queen's Printer for Ontario.
- Ontario Ministry of Education. (2007). *The Ontario curriculum, grades 11 & 12: Mathematics, revised*. Toronto: Queen's Printer for Ontario.
- Papert, S. (n.d.). Hard fun. Retrieved August 25, 2008, from [www.papert.org/articles/HardFun.html](http://www.papert.org/articles/HardFun.html).
- Small, M. (2008). *Making math meaningful to Canadian students, K–8*. Toronto: Nelson.
- Van de Walle, J., & Folk, S. (2008). *Elementary and middle school mathematics: Teaching developmentally* (2nd Canadian ed.). Toronto: Pearson. ▲

## ▲ MATHEMATICS LITERACY IN THE MEDIA

WAYNE ERDMAN

E-MAIL: [mathman2@sympatco.ca](mailto:mathman2@sympatco.ca)

Wayne is the curriculum leader of Mathematics and Computer Studies at Eastern Commerce C.I., president of the TEAMS chapter of OAME, and senior mathematics author for McGraw-Hill Ryerson. He believes that numeracy and literacy are strongly linked, and that communication should be a focus in the mathematics classroom.

While drinking my morning coffee and reading my local newspaper, it struck me how poorly members of the media—and society in general—use mathematical terminology. I'm not talking about misinterpreting mathematics, or making mathematical errors. All too often, terminology is used to convey a mathematical message, either where none exists, or in a way that is improperly stated. So, I decided to list some of the ways in which mathematics terminology is misused by the media. This is not a rebuttal of the use of math to prove a point, such as the existence of global warming, or whether there is life on other planets; it is more a rant about mathematical illiteracy in the media.

So, here is my list, in descending order, of math terminology that is frequently misused by members of the media:

- **“Giving 110%”** – For example, during the 2008 Summer Olympics, a TV announcer stated that swimmer Michael Phelps “gave 110%” during the 100 m race. Although 110% is achievable (e.g., through a ratio of two quantities), I defy anyone to be able to give any more than 100% of his or her abilities! So “giving 110%” has simply become an expression for doing one's best.
- **“Amount of”** instead of **“number of”** – This results from the failure to differentiate between continuous and discrete variables. How often have we heard the phrase, “The amount of people...”? The term “amount of” should be used for measurements, such as “the amount of water,” while “number of” should be used for counting, such as “the number of cookies.”
- **“Less than”** instead of **“fewer than”** – This may also

# Ontario Mathematics Gazette

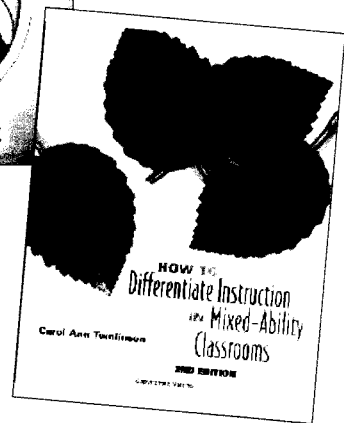
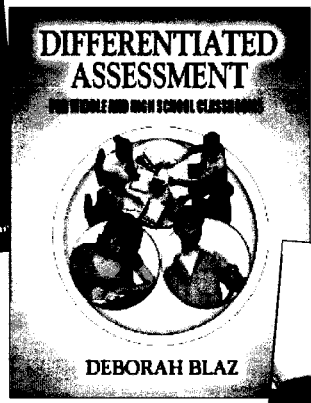
OAME - ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

AOEM - ASSOCIATION ONTARIENNE POUR L'ENSEIGNEMENT DES MATHÉMATIQUES

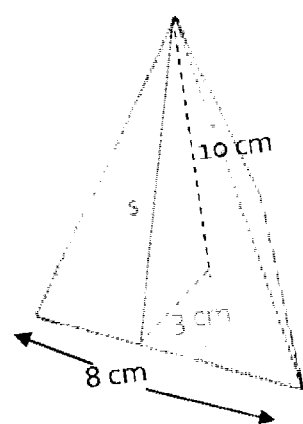
Vol.47 #2  
Dec 2008

## IN THIS ISSUE

- ▲ Editor's Message
- ▲ President's Message
- ▲ Technology Corner - Multiple Versions with Excel
- ▲ What's the Problem? Pretty Pythagorean Problem
- ▲ Data Rocks - New Math Resources from Statistics Canada
- ▲ OAME/NCTM Report
- ▲ OAME Financial Statement 2007/08
- ▲ Book Review: A Three-Book Study of Differentiated Instruction
- ▲ Math: The Cereal Killer
- ▲ Thinking Outside the Rectangular Prism: Fostering Problem-Based Mathematics Learning
- ▲ Mathematics Literacy in the Media
- ▲ Calculus and Vectors: The Connections
- ▲ Directors' Dialogue



◀ See Book Review: A Three-Book Study of Differentiated Instruction



▼ See Thinking Outside the Rectangular Prism: Fostering Problem-Based Mathematics Learning

▲ See Technology Corner - Multiple Versions with Excel

