



# Are You in the

# ZONE

The ability to estimate quantity is an essential skill in today's increasingly technological world if an individual is to feel in control of technology rather than controlled by it. As with other mathematical concepts, the notion of estimating, or knowing whether one's answer is "in the zone," should be introduced at an early age. The skills required by confident estimators need to be included incidentally in a young child's education and then be refined as the child's mathematical thinking matures.

Chester Carlow (1986, p. 94) describes two quite different skills that people must acquire if they are to become confident estimators:

1. *Approximating* skills for judging the reasonableness of computational outcomes involving complex numerical expressions. The solver mediates such judgments by rounding off, compensating, or using other mental operations.
2. *Estimating* skills for making direct, intuitive judgments of quantity in relation to objects in the real world.

In a similar vein, Jennifer Taylor-Cox (2001, p. 209) describes four types of estimation:

1. *True approximations*, used when an estimate is acceptable
2. *Overestimating*, used when rounding up might be beneficial

3. *Underestimating*, used when rounding down is applicable
4. *Range-based estimation*, used to broaden the applicability and understanding of estimation

The latter of each of these descriptions of estimation will be the focus of this article—what Carlow has described as making *intuitive* judgments within a *holistic* framework, and what Taylor-Cox calls *range-based* estimation. We have used this notion of estimating when working with young children in two distinct mathematical environments. The first of these is a Family Math environment, in which parents learn strategies to foster their children's mathematical development. The six-week Family Math programs take place in community settings, such as churches, libraries, and community centers. The second mathematical environment is the classroom setting, in which children are given the opportunity to refine their estimation processes over a longer time frame.

## The Promotion of Estimation in Family Math

For the previous three years, we have been developing range-based estimation as an integral part of parental programming for Family Math. Our goal has been to provide parents and their children with strategies that are useful for estimating quantities in the real world, thereby increasing the participants' comfort level and sophistication with various forms of measurement and number sense. Early in the development of the program, we noticed that every participant—whether adult or child—wanted to offer an exact numerical answer when estimating. Quite often, the youngest children made very wild guesses. As Leutzinger, Rathmell, and Urbatsch explain, a young child perceives little difference between 35 and 1000, thinking of both quantities as large numbers (1986, p. 82).

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# me?

In the Family Math program presented in this article, young children are asked to estimate using a number line that has a range of zero to thirty. For recording their estimates, families are provided with a blank sticky note that corresponds to an amount represented by three to four numbers on the number line. To foster a safe environment in which risk-taking is encouraged, participants do not put their names on their sticky notes. Instead they are invited to anonymously place their estimate, along with those of the other families, on a number line at the front of the room (see **fig. 1**).

Children and adults are discouraged from assigning a definitive numerical response when indicating their estimate. Instead, they are asked to decide on a range, or “zone,” for their responses, and as children’s mathematical skills develop, families explore ways of *refining* the zone. The rationale for this approach is based on the belief that if an exact answer is required then individuals should be doing something other than estimating; for example, using mental mathematics may be more appropriate. The art or process of estimating is often lost when children believe that the person who guesses closest to the exact number is the best estimator. Estimation should be used when finding an exact answer is not important or when calculating one quickly is too difficult.

We use what Chester Carlow (1986) refers to as a “light touch” when introducing estimating. At first, parents and their children are reluctant to take risks, wanting to produce the *correct* answer. Once families have attended a few Family Math sessions, however, they gradually become less threatened or intimidated by not knowing an exact answer, and we have seen an increase in risk-taking, as well as an increase in communication among family members about the strategies they should use to be “in the zone.” Everyone involved needs to appreciate the value of *not*

**Figure 1**

**Children use a sticky note to record their estimation zone on the number line.**



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*knowing*, thereby using these occasions as opportunities for growth rather than anxiety. Family Math leaders attempt to foster such an environment by modeling risk-taking behaviors and conveying their own comfort when not knowing the exact answer. Estimation has helped parents and their children recognize the benefits associated with taking risks in mathematics. Gradually, they begin to enjoy the subject. As one parent remarked, “You don’t have to be good at math to enjoy this program. Math can be so much fun. In our house, math is now more about helping and less about criticism.”

## Making Use of a Referent

Many writers have discussed the importance of making use of referents when estimating. Carlow



## Children and adults are asked to decide on a “zone” for their responses

(1986) uses the term *perceptual anchor*, for example, knowing what 10, 20, or 100 things might look like or that a door handle is approximately one meter from the floor. Siegel, Goldsmith, and Madison (1982) used the term *benchmarks* to describe nonstandard referents, such as “How many glasses of water are in a bucket?” or “It takes about the same time to walk to school as to the park,” that

people can use to assist their estimation skills. *Chunking* uses the notion of a perceptual anchor, referent, or benchmark to visualize how many *chunks* might be in a whole. Reys et al. (1998) refer to this refinement of chunking as *unitizing*. Carter (1986, p. 78) discusses the importance of chunking when decomposing or recomposing a whole into, or from, its various parts. These part-whole schema are crucial components as children develop their ability to decompose and recombine elements

to support their estimation skills and expand their understanding of quantity (Resnick 1983). We have encouraged this use of referents and chunking in the Family Math estimation activities. For example, useful anchors include (a) the width of your little finger for one centimeter, (b) the bottom section of a two-liter milk carton measuring 10 cm by 10 cm by 10 cm for the capacity of one liter, and (c) the mass of one kilogram when a one-liter container is filled with water.

### Estimating in Family Math Settings

This activity is taken from week 5 of the Early Years Esso Family Math program. The theme for this session concentrates on the world outside the child’s home and is titled “Don’t Bug Me!” Between 10 and 15 candy gummy worms are placed in a small plastic container, and families are asked to estimate the quantity by placing a blank sticky note on a number line (0–30) that has been prepared by joining three legal-sized pieces of paper horizontally. The sticky notes cover a range of three or four numbers (see **fig. 1**). Families are encouraged to base their estimates on their previous experiences (estimating a number of teddy bear-shaped snacks, buttons, socks, and rocks from previous weeks), some of which were in a same-sized container, whereas others were in

a larger jar. At this stage of the program, families are usually comfortable taking risks and talking about the strategies they are using. A typical family conversation includes such statements and questions as “I think a gummy worm would be about the length of my finger”; “A curled-up gummy worm is about the same [size] as two teddy snacks”; and “How many gummy worms are around the outside?”

Toward the end of the session, the team leader returns to the estimation activity and asks how families determined where to place their sticky notes. A transparent overlay is placed in the zone where most of the sticky notes are located (see **fig. 2**). The leader takes approximately half of the gummy worms out of the jar (to serve as an anchor) and positions them next to their corresponding numeral on the number line, counting them with the assistance of the children. The children are invited to move the transparent overlay on the basis of the new knowledge they have obtained. They are always asked to explain their reasoning, and the overlay is then shifted to the right or left or is kept in the same position, depending on the opinion of the majority of children. In this way, children begin to learn the process of refining their estimates on the basis of increased knowledge and understanding. The remaining gummy worms are then counted onto the number line to determine whether the group’s estimate—not that of an individual or family—is in the zone. Usually, the refined estimate is within the range of the transparent overlay. We deliberately omit names or any other forms of identification from the sticky notes. The goal is to encourage mathematical thinking and worthwhile discussion, not to determine who has given the correct answer. As the children explain their reasons for the selected strategies, the leaders offer clarification and positive feedback to reinforce useful approaches and encourage future risk taking.

For ten minutes, at the end of each session, the Team Leader talks to the parents about the mathematics their children were exploring, how it matches the mathematics in their child’s school curriculum, and how they can reinforce the activities at home. During this parent talk, the team leader reinforces the idea that the more parents estimate with their children and listen to the strategies their children are using, the better they will understand their children’s thinking and be able to reinforce effective estimation strategies. “The more we do it, the better we become” is a common expression at Family Math sessions.

Each family is given an *At Home* book with activities to complement those done during the Family Math session. During parent-talk time, the children have a snack with one of the other volunteers, who generally reads a mathematics-related story to them.

## Estimating in the Classroom Setting

The second mathematical environment to be discussed is the regular elementary school classroom. In this setting children have increased opportunities for refining their estimation processes over a longer time frame. Students begin estimating in the kindergarten classroom just as they estimate in the Early Years Family Math program. Weekly involvement is encouraged in kindergarten, grade 1, and grade 2 by giving students the opportunity to take home the estimating jar on weekends. Parents of kindergarten children are asked to limit the collections they place in the jar to fewer than thirty similar items, and at grades 1 and 2 the jar contents are not to exceed one hundred items of any one kind. The sticky-note method is used at the beginning of the year to encourage risk-taking and the group development of estimating strategies.

Later in the school year, students in grades 1 and 2 are given a recording sheet that individualizes the use of the zone as well as provides feedback for the teacher. One strategy that has proved particularly effective in helping children refine their zone is a hundred chart (see **fig. 3**). The students cross out those numbers on the hundred chart that they are sure are *fewer than* their estimate. The students are directed to stop when they reach a *maybe* number. Then they cross out the numbers they know for certain are *more than* their estimate, working backward from one hundred until they reach another *maybe* number. The range of numbers remaining on the chart becomes their zone. Children have opportunities to start with a fairly large range that meets their comfort level, then gradually refine that range as they gain confidence through experience. Those who are risk-takers, or those with advanced mathematical maturity, will attempt to narrow the range more quickly than the hesitant or less mathematically mature child. Everyone can work at his or her own comfort level, furnishing the teacher with the diagnostic information needed for planning future activities

**Figure 2**

Parents become aware of the importance of listening to their children's estimation strategies.



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in estimation. Taylor-Cox (2001, p. 214) uses a skillful-visualization technique to help students understand the abstract concept of range. She teaches her students to “sandwich” their estimate, and this visualization tactic has proved extremely meaningful for young students.


Following the establishment of their number zone, students are asked to share their strategies with other class members. Everyone in the class then counts the number of objects being estimated, and records whether the resulting number is in their zone or is lower or higher than their zone. Objects from the estimating jar should be counted in a variety of ways. Using a number line reinforces one-to-one correspondence. Counting out groups of ten (for example, giving a group of ten to each student until only the ones are remaining) not only reinforces place value but also demonstrates what a *chunk* of ten items looks like for future reference. Students are encouraged to reflect on the strategies they used when estimating, record these strategies in the section titled “Reflections,” then consider what strategies they might want to use in the future (see **fig. 4**).

Students are encouraged to discuss their strategies with their peers. They need to understand that a variety of strategies can successfully



**Figure 3**

Children use a hundred chart to find and refine their zone.



**Learning to Estimate**

Are you **“IN THE ZONE”**?

Draw or describe what is in the **Estimating Jar**.

**Instructions:**

- Start at 1. Cross out the numbers on the 100-chart that you think are definitely **NOT** the number of objects in the estimating jar. **STOP** when you reach a “maybe” number.
- Work backward from 100. Cross out the numbers that you think are definitely **NOT** the number of objects in the estimating jar. **STOP** when you reach a “maybe” number.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

The numbers left are **“the zone,”** or the range of reasonable guesses. My “zone” is more than \_\_\_\_\_ but less than \_\_\_\_\_.

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be used for the same estimation activity. For example, one student might consider what a “chunk of ten” looks like, whereas another student might think of how many objects are on each layer in the estimating jar. Sharing ideas encourages the use of diverse strategies on future occasions. Collecting their recording sheets over a period of time in an estimation portfolio helps students reflect on previously used strategies and supplies teachers with diagnostic information for future planning. Estimating should be an ongoing activity, from its introduction in kindergarten to practice with the more sophisticated processes taught in subsequent grades. A well-planned program includes a variety of estimating activities for different levels

of mathematical maturity, using such contexts as time, distance, capacity, and area.

## Some Final Thoughts

Children develop a strong sense of number and measurement when they have real-world referents to visualize mathematical abstractions. Estimation presents a starting point for constructing some of these early reference points. Developing a comfort level with *holistic*, or *range-based*, estimation requires a variety of experiences in many different contexts. The types of estimation activities discussed in this article help students of different ages develop a sense of number, size, and quantity. Discussing with children and parents how they use estimation in their everyday lives (for example, how much they are spending at the grocery store, how far they can drive without stopping for gasoline, how much spaghetti will feed the family) will help them realize that estimation is a skill that people use and depend on without always realizing its importance. This skill of estimating is developed through practice and reflection. The more individuals estimate, and the more they think about how they do it, the better they become at it. To be confident and successful estimators, children need numerous opportunities to practice estimation and to learn from their experiences.

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**Figure 4**

Children reflect on and communicate their estimation strategies.

**The Count**



Count the items in the estimating jar. Record the count below.

**We counted** \_\_\_\_\_ .

(number)                      (items we counted in the jar)

The number counted was---

in the zone.  
 less than the zone.  
 greater than the zone.

**Reflections**

- What strategies helped you to be “in the zone”?
- What strategies could you use next time?

Explain your thinking.

Remember:  
The more you estimate, the better you become at finding “the zone”!

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*Authors’ note: The Esso Family Math Project, based on the work started at Berkeley in the 1980s,*

*has two programs—one for families whose children are between the ages of 3 and 6 years old and one for families whose children are between the ages of 7 and 10. Estimation strategies are refined as families progress through the programs. These programs can be downloaded, free of charge, at the Esso Family Math Web site, [www.edu.uwo.ca/esso/familymath](http://www.edu.uwo.ca/esso/familymath). The project is supported by the Imperial Oil Charitable Foundation and the Maurice Price Foundation. ▲*