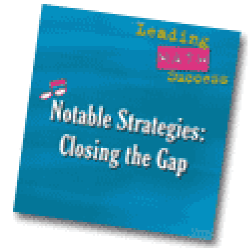
Leading Math Success



Notable Strategies: Closing the Gap

Notable Strategies: Closing the Gap materials are designed to support educational leaders — teachers, principals, professional learning facilitators — in implementing effective mathematics programs in Grade 7, Grade 8, Grade 9 Applied, and combined Grades 7/8. The day-by-day lessons and student sheets align with outlines and lessons in TIPS (Targeted Implementation and Planning Supports: Grade 7, Grade 8, and Grade 9 Applied Mathematics). Strategies for assessing for learning and differentiating instruction are featured.

Grade 7 Year Outline

Term	Cluster of Curriculum Expectation	Reference	Number of Lessons Planned	Lesson Time Available - "Instructional Jazz"	Total Lesson Time
1	Welcome and Problem Solving	TIPS 1 - 4	4		4
	Describing Patterns	TIPS 5 - 9	5	2	7
	Multiples and Factors	TIPS 10 - 12	3	2	5
	Parallelograms and Triangles	TIPS 13 - 16	4	1	5
	3-D Models	TIPS 17 - 22	6	2	8
	Rectangular Prisms	TIPS 23 - 28	6	2	8
	Exponents	TIPS 29 - 31	3	1	4
	Square Roots		1	3	4
	Sub-totals		**32**	**13**	**45**
2	Data Management, Central Tendency		15	1	16
	Solving Equations by Inspection & Systematic Trial		13	3	15
	Adding and Subtracting Fractions	LMS 29 - 38	11	4	16
	Problem Solving with 2-D Shapes and 3-D Figures		13	3	16
	Sub-totals		**52**	**11**	**63**
3	Investigating Trapezoids	LMS 1-5	5	2	7
	Analysis of Transformational Geometry		12	2	14
	Integers	LMS 18 – 29	13	3	16
	Order of Operations		5	2	7
	Sample Space		5	1	6
	Interpreting Graphs and Evaluating Arguments		5	1	6
	Application of Measurement Tools		5	1	6
	Gazebo Investigation – GSP		3	2	5
	Olympic Investigation – Spreadsheets		3	2	5
	Sub-totals		**56**	**16**	**72**
			140 days	**40 days**	**180 days**

Targeted Implementation and Planning Supports (TIPS)
Leading Math Success (LMS)

The number of prepared lessons represents the lessons that could be planned ahead based on the range of student readiness, interests, and learning profiles that can be expected in a class. The extra time available for "instructional jazz" can be taken a few minutes at a time within a pre-planned lesson or taken a whole class at a time, as informed by teachers' observations of student needs. (See software presentation, Differentiated Instruction… and all that jazz.)

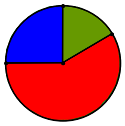
The reference numbers are intended to indicate which lessons are planned to precede and follow each other. Actual day numbers for particular lessons and separations between terms will need to be adjusted by teachers.

Lesson Outline: Days 29 – 38 Grade 7

BIG PICTURE

Students will:
- explore fraction relationships;
- develop an understanding of addition and subtraction of fractions (proper, improper, and mixed);
- develop rules for addition and subtraction of fractions;
- explore the relationship between fractions and decimals;
- solve problems involving fractions and decimals.

Day	Lesson Title	Description	Expectations
29	Fraction Puzzles	• Explore/review fractional parts of geometric shapes. • Order fractions.	7m1, 7m6, 7m26 CGE 3c, 5a, 5e
30	Combining Fractions	• Investigate combinations of fractions using manipulatives.	7m2, 7m12, 7m17, 7m18, 7m26 CGE 3b, 3c, 5a
31	Adding Fractions with Different Denominators	• Add fractions by connecting concrete to symbolic. • Recognize the need for equivalent fractions with common denominators when adding fractions with different denominators.	7m17, 7m18, 7m24 CGE 4b, 5e
32	Fractions Using Relational Rods	• Explore fractions using relational rods.	7m2, 7m17 CGE 3c, 4a
33	Adding and Subtracting Fractions Using Relational Rods	• Practise adding and subtracting fractions using relational rods in a game format.	7m2, 7m17, 7m18, 7m26 CGE 2c, 3b, 3c, 5e
34	Subtracting Fractions Using Equivalent Fractions	• Develop rules for subtracting fractions using equivalent fractions with common denominators. • Practise adding and subtracting fractions.	7m6, 7m18, 7m24, 7m26 CGE 4e, 5g
35	Adding and Subtracting Fractions – Performance Task	• Demonstrate understanding and skills while performing operations with fractions.	7m2, 7m6, 7m7, 7m17, 7m18, 7m24 CGE 2b, 3c
36	Exploring Fractions Further	• Explore repeated addition of fractions and addition and subtraction of mixed numbers. • Develop methods for multiplying a fraction by a whole number.	7m6, 7m7, 7m18, 7m19, 7m24 CGE 3b, 4f, 5a
37	Fractions and Decimals	• Explore the relationships between fractions and decimals.	7m8, 7m9, 7m23, 7m26, 7m27 CGE 2c, 3c
38	Summative Assessment	• Demonstrate understanding of fractions and operations with fractions on an open-ended, problem-solving task.	7m6, 7m7, 7m8, 7m16, 7m18, 7m24, 7m26 CGE 2b, 3c, 4f



Description

- Explore/review fractional parts of geometric shapes.
- Order fractions.

Materials

- pattern blocks
- overhead pattern blocks
- BLM 29.1,29.2, 29.3
- 2 or 3 large Imperial socket wrench sets in cases

Assessment Opportunities

Minds On…

Whole Class → Introducing a Problem

Distribute pattern blocks. Present students with the following area fraction puzzle: With your pattern blocks build two different triangles each with an area that is one-half green and one-half blue.

Have students share their solutions using the overhead pattern blocks. Discuss whether rearranging the blocks makes the solution "different."

TIPS: Section 2 – Fractions, p.3 for teaching fraction concepts.

Virtual pattern blocks are available at http://arcytech.org/java/patterns/patterns_j.shtml

Action!

Pairs → Problem Solving

Explain that all students are to complete questions 1 to 5 on BLM 29.3 then set it aside while they use pattern blocks and BLM 29.1 to solve area fraction puzzles with a partner.

Students must show the graphic solution with each colour labelled with the appropriate fraction of the whole triangle, e.g., colouring on pattern block paper, (BLM 29.2).

While students are working, provide selected pairs with an Imperial set of socket wrenches that are mixed up and ask them to complete number 6 on BLM 29.3.

Ask them to explain why they placed a certain socket between two others.

Learning Skills & Curriculum Expectations/Observation/Anecdotal: Circulate while students are working to assess prior knowledge of fractions. Ask students to explain how they know their solution to the area puzzle is correct.

Briefly review the meaning of *parallelogram* (blue or beige block) and *trapezoid* (red block).

Some methods students may use include physical size of each socket, ordering of the sockets could be accomplished using equivalent fractions, converting to decimals or measuring in millimetres, among other methods.

Consolidate Debrief

Whole Class → Sharing/Discussion

Pairs of students share their solutions to an area puzzle using the overhead pattern blocks and explain how they know their solution satisfies the problem.

Discuss possible answers to question 5 on the student worksheet.

Several different pairs of students share their solutions, even if the solution is merely another arrangement of the same pattern blocks. This allows more students to be recognized and reinforces multiple solutions and explanations.

Discuss the various methods students used to solve the socket set problem.

Concept Practice

Home Activity or Further Classroom Consolidation

Choose one task:

1. Complete the two area fraction puzzles provided and create one area fraction puzzle of your own.
2. Create three area fraction puzzles of your own. Include the solutions, coloured and labelled with their fractional parts on pattern block paper.

Refer students to the virtual pattern block website listed above.

Students can work on pattern block paper, use cardboard cutouts or access the web to complete the assignment

29.1: Pattern Block Area Fraction Puzzles

Name:
Date:

Use pattern blocks to solve each of the area fraction puzzles below. Draw each solution on pattern block paper. Label each colour with its fraction of the whole shape.

1. Build a parallelogram with an area that is one-third green, one-third blue, and one-third red.

2. Build a parallelogram with an area that is one-eighth green, one-half yellow, one-eighth red, and one-quarter blue.

3. Build a trapezoid with an area that is one-tenth green and nine-tenths red.

4. Rebuild each of the puzzles above in a different way.

5. Explain why it is not possible to build a parallelogram with an area that is one-half yellow, one-third green and one-quarter blue.

29.1: Pattern Block Area Fraction Puzzles

Name:
Date:

Use pattern blocks to solve each of the area fraction puzzles below. Draw each solution on pattern block paper. Label each colour with its fraction of the whole shape.

1. Build a parallelogram with an area that is one-third green, one-third blue, and one-third red.

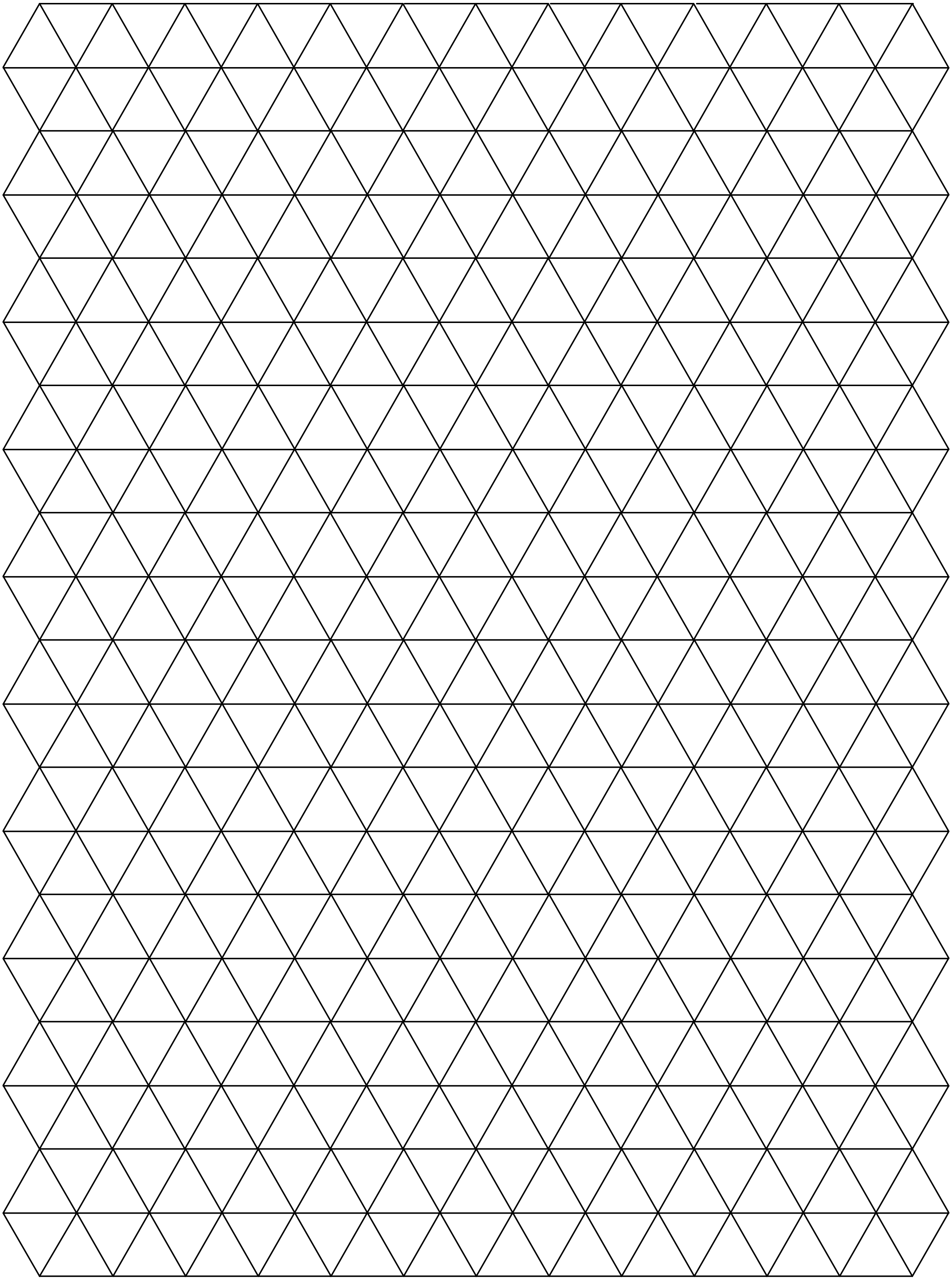
2. Build a parallelogram with an area that is one-eighth green, one-half yellow, one-eighth red, and one-quarter blue.

3. Build a trapezoid with an area that is one-tenth green and nine-tenths red.

4. Rebuild each of the puzzles above in a different way.

5. Explain why it is not possible to build a parallelogram with an area that is one-half yellow, one-third green and one-quarter blue.

29.2: Pattern Block Paper



29.3: Socket to you!

Name:
Date:

1. $\frac{20}{32}$ is an equivalent fraction for $\frac{5}{8}$. Write two more equivalent fractions for $\frac{5}{8}$.

2. Write two equivalent fractions for $\frac{3}{4}$.

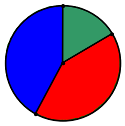
3. Circle which is larger: $\frac{3}{8}$ or $\frac{3}{16}$.

4. Circle which is smaller: $\frac{7}{16}$ or $\frac{9}{16}$.

5. Circle the fraction that fits between $\frac{7}{16}$ and $\frac{9}{16}$.

   $\frac{13}{32}$ $\frac{1}{4}$ $\frac{3}{8}$ $\frac{1}{2}$ $\frac{5}{8}$ $\frac{3}{4}$ $\frac{19}{32}$

6. Often mechanics use socket wrench sets with openings measured in fractions of an inch. These fractions are stamped on the fronts of the sockets.
   Think about how to restock the socket wrench set in the correct order in the case.
   Explain how you decided on the order you chose.



Description

- Investigate combinations of fractions using manipulatives.

Materials

- pattern blocks
- overhead pattern blocks
- BLM 29.2, 30.1

Assessment Opportunities

Minds On…

Whole Class → Introducing Problems

Using pattern blocks, students show that $\frac{1}{6}+\frac{1}{2}=\frac{2}{3}$. Several students share their methods.

Students show that $\frac{1}{3}+\frac{1}{3}=\frac{2}{3}$. Which pattern block did the student choose to represent one whole? Discuss the possibility of using a different pattern block to represent one whole.

One way: Using the hexagon as one whole, the triangle can be one sixth, three triangles (or the trapezoid) can be one half and together they form four sixths (two thirds).

Action!

Pairs → Exploration

Provide students with several questions involving combining fractions that can be modelled with pattern blocks. For example, $\frac{1}{2}+\frac{5}{6}$; $\frac{1}{3}+\frac{1}{6}$; $\frac{1}{3}+\frac{5}{6}+\frac{4}{3}$

(Fractions, both proper and improper, that have denominators of 2, 3, or 6 work well with pattern blocks.)

Students explain each solution, including which pattern block they used to represent the whole.

Initially include activities in which students think about the number of one shape that need to be combined to make a second shape.

Curriculum Expectations/Demonstration/Rubric: Observe students' skills as they solve the problems. Assessment can focus on problem solving, use of appropriate language and clarity of explanation.

Consolidate Debrief

Whole Class → Sharing/Discussion

Students share the strategies they used to combine fractions. Students demonstrate their strategies for the class using overhead pattern blocks.

Discuss the idea of equivalent fractions with common denominators as it relates to the pattern blocks, e.g. using smaller blocks helps to combine fractions with different denominators.

For example, to add $\frac{1}{2}+\frac{5}{6}$, students may choose to use the hexagon as the one whole. They would use the trapezoid to represent $\frac{1}{2}$ and five triangles to represent $\frac{5}{6}$. To combine the fractions, students would need to express the answer in triangles (one whole and two triangles, or one and two sixths, which can be simplified to one and one third using the blue rhombi).

Curriculum Expectations/Observation/Mental Note: Assess students' current level of understanding of fractions, noting particularly their solutions to qestion 4.

The intent of this lesson is to allow students to come to a deeper understanding of equivalent fractions through exploration and discussion. Students should arrive at the standard algorithm for determining a common denominator (multiply numerator and denominator by a constant) through conceptual understanding. Students should use a variety of methods to determine the common denominator.

Concept Practice

Home Activity or Further Classroom Consolidation

Complete the worksheet, Combining Fractions.

For virtual pattern blocks and related activities see:
http://arcytech.org/java/patterns/patterns_j.shtml
http://math.rice.edu/~lanius/Patterns/

30.1: Combining Fractions

Name:
Date:

Use pattern blocks to solve each problem. Record your solutions on the pattern block paper. Include the symbolic fractions as well as the drawings.

1. Show that:

   a) $\frac{1}{6}+\frac{1}{6}=\frac{1}{3}$ b) $\frac{1}{6}+\frac{2}{3}=\frac{5}{6}$ c) $\frac{1}{6}+\frac{1}{3}+\frac{1}{2}=1$

2. Add $\frac{1}{6}$ and $\frac{1}{3}$.

3. Add $\frac{1}{2}+\frac{2}{3}$.

4. Show three different ways of adding three fractions to get two wholes.

5. Show that $\frac{2}{3}+\frac{1}{6}$ is less than 1. How much less than 1 is this sum?

Name:
Date:

Use pattern blocks to solve each problem. Record your solutions on the pattern block paper. Include the symbolic fractions as well as the drawings.

1. Show that:

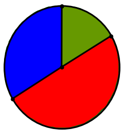
   a) $\frac{1}{6}+\frac{1}{6}=\frac{1}{3}$ b) $\frac{1}{6}+\frac{2}{3}=\frac{5}{6}$ c) $\frac{1}{6}+\frac{1}{3}+\frac{1}{2}=1$

2. Add $\frac{1}{6}$ and $\frac{1}{3}$.

3. Add $\frac{1}{2}+\frac{2}{3}$.

4. Show three different ways of adding three fractions to get two wholes.

5. Show that $\frac{2}{3}+\frac{1}{6}$ is less than 1. How much less than 1 is this sum?



Description

- Add fractions by connecting concrete to symbolic.
- Recognize the need for equivalent fractions with common denominators when adding fractions with different denominators.

Materials

- BLM 31.1a, 31.1b
- pattern blocks

Assessment Opportunities

Minds On…

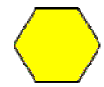
Whole Class → Teacher Directed Instruction

Ask a few students to share their solutions to question 3 from the previous day's Home Activity ($\frac{1}{2} + \frac{2}{3}$) using overhead pattern blocks.

On the board, record the symbolic form of each solution, i.e., the fractions. Discuss how to get the solution without using pattern blocks.

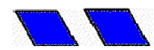
Through questioning, students consider the use of equivalent fractions with a common denominator, in this case, 6. They may determine the common denominator different ways.

BLM 31.1b shows scaffolding.



1 whole

1/2



2/3

Action!

Pairs → Think/Pair/Share

Allow some time for students to think individually about solving each of the questions from the previous day's Home Activity using equivalent fractions with a common denominator. Then, students discuss their strategies for finding equivalent fractions with a common denominator with a partner. Pairs share their strategies with a small group and/or the whole class.

Curriculum Expectations/Observation/Mental Note: Circulate to assess which students understand adding using equivalent fractions with common denominators and which require further support.

Consolidate Debrief

Whole Class → Notemaking

Create a note together that outlines the process for adding fractions using equivalent fractions with a common denominator. Students determine the amount of detail, i.e. number of steps to follow, and develop the steps in the process to the degree that this is practical.

Include the multiples method of finding common denominators as practised in the Home Activity for this lesson.

Differentiated Concept Practice

Home Activity or Further Classroom Consolidation

Complete the worksheet, Adding Fractions with Different Denominators.

Students work independently on differentiated practice, based on the teacher's observations in Action. See worksheets 31.1a, 31.1b.

31.1a: Adding Fractions with Different Denominators

Name:
Date:

1. Use multiples to find three common denominators for the following pair of fractions:

$\frac{1}{2}, \frac{5}{8}$

Multiples of 2:

Multiples of 8:

My three common denominators are _______, _______, and ______

2. Find a common denominator for the following fraction pairs:

a) $\frac{1}{4}, \frac{2}{3}$

Common Denominator: ___________

b) $\frac{3}{5}, \frac{3}{8}$

Common Denominator: ___________

Rewrite each pair as equivalent fractions with a common denominator.

3. Rewrite each of the following expressions using equivalent fractions with a common denominator. Add the fractions.

a) $\frac{1}{3}+\frac{1}{5}$

b) $\frac{5}{6}+\frac{1}{4}$

c) $\frac{3}{5}+\frac{1}{8}$

31.1b: Adding Fractions with Different Denominators

Name:
Date:

1. Use multiples to find two common denominators for the following pair of fractions.

$\frac{1}{2}, \frac{5}{8}$

Multiples of 2: 2, 4, ___, ___, ___, ___, ___, ___, ___, ___, ___

Multiples of 8: 8, 16, ___, ___, ___, ___, ___, ___, ___, ___

My two common denominators are _______ and _______

2. Find a common denominator for the following fraction pairs.

a) $\frac{1}{4}, \frac{2}{3}$

4: 4, ___, ___, ___, ___, ___, ___, ___

3: 3, ___, ___, ___, ___, ___, ___, ___

Common Denominator: _________

b) $\frac{3}{5}, \frac{3}{8}$

5: ___, ___, ___, ___, ___, ___, ___

8: ___, ___, ___, ___, ___, ___, ___

Common Denominator: _________

Rewrite each pair as equivalent fractions with a common denominator.

a) $\frac{1}{4} = \frac{\quad}{\square}$

$\frac{2}{3} = \frac{\quad}{\square}$

Common Denominator

b) $\frac{3}{5} = \frac{\quad}{\square}$

$\frac{3}{8} = \frac{\quad}{\square}$

Common Denominator

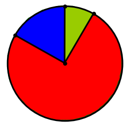
3. Rewrite the following expression using equivalent fractions with a common denominator. Add the fractions.

$\frac{5}{6} + \frac{1}{4}$

6: ___, ___, ___, ___, ___, ___, ___

4: ___, ___, ___, ___, ___, ___, ___

$\frac{5}{6} = \frac{\quad}{\square}$ $\quad$ $\frac{1}{4} = \frac{\quad}{\square}$ $\longrightarrow$ $\frac{\quad}{\square} + \frac{\quad}{\square} = \frac{\quad}{\square}$



Description

- Explore fractions using relational rods.

Materials

- overhead relational rods
- sets of relational rods
- centimetre grid paper
- BLM 32.1,32.2, 32.3

Assessment Opportunities

Minds On…

Whole Class → Introducing the Problem

Distribute BLM 32.2. As pairs of students follow along with their own sets of relational rods, place the blue and black overhead relational rods together to form one whole. Ask students how they would determine the value of a particular coloured rod in relation to this blue-black "whole." Invite a student to demonstrate on the overhead that the brown rod (8 units) is one-half of the blue-black whole (16 units).

Repeat with the dark green rod. Students determine the fractional value of the dark green rod in relation to the blue-black whole. Write this relation as a fraction ($\frac{6}{16}=\frac{3}{8}$). Guide their thinking with questions: What rod(s) may represent one unit for this whole? How many units is the dark green rod? Encourage them to use other rods to determine equivalent fractions in lowest terms.

If students have not worked with relational rods before, some time should be allocated to exploration. They may benefit from some discussion of "unit" in the rods.

If sets of relational rods are not readily available, use BLM 32.1.

Action!

Pairs → Exploration

Students work in pairs to explore the fractional value of each of the relational rods relative to the blue-black whole.

Students organize their work in a table so that it is clear that they have determined the fractional value of all of the coloured rods in relation to the blue-black whole and their relationships to each other (fractions less than one only). (See BLM 32.3)

Curriculum Expectations/Observation/Mental Note: Circulate to observe students' understanding of fractions.

Consolidate Debrief

Whole Class → Sharing/Discussion

Students share how they determined the fractional value for each coloured rod in relation to the blue-black whole and to each other. Several different pairs share their strategies.

Pairs share their methods for organizing the information to show the relationships among the rods.

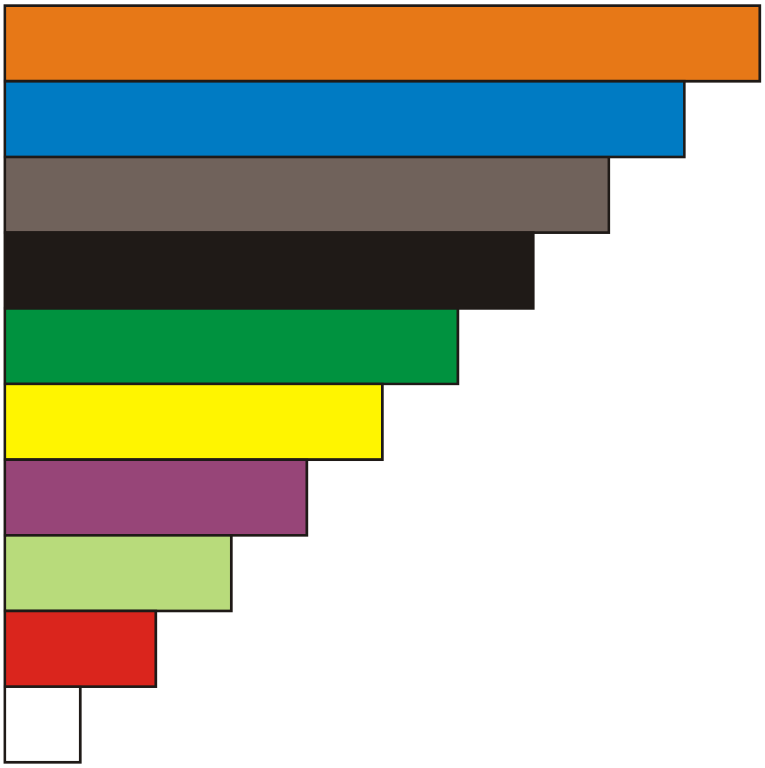
Concept Practice

Home Activity or Further Classroom Consolidation

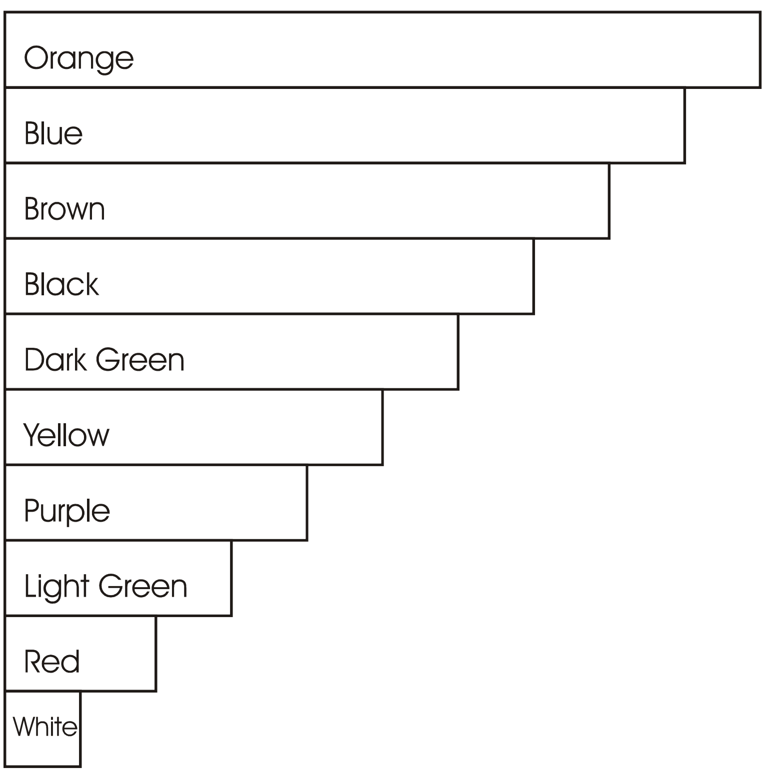
Complete the worksheet in preparation for the next day's activity.

Virtual rods are available at http://www.arcytech.org/java/integers/integers.html . A template is available at http://mason.gmu.edu/~mmankus/Handson/crods.htm

32.1: Template for Relational Rods



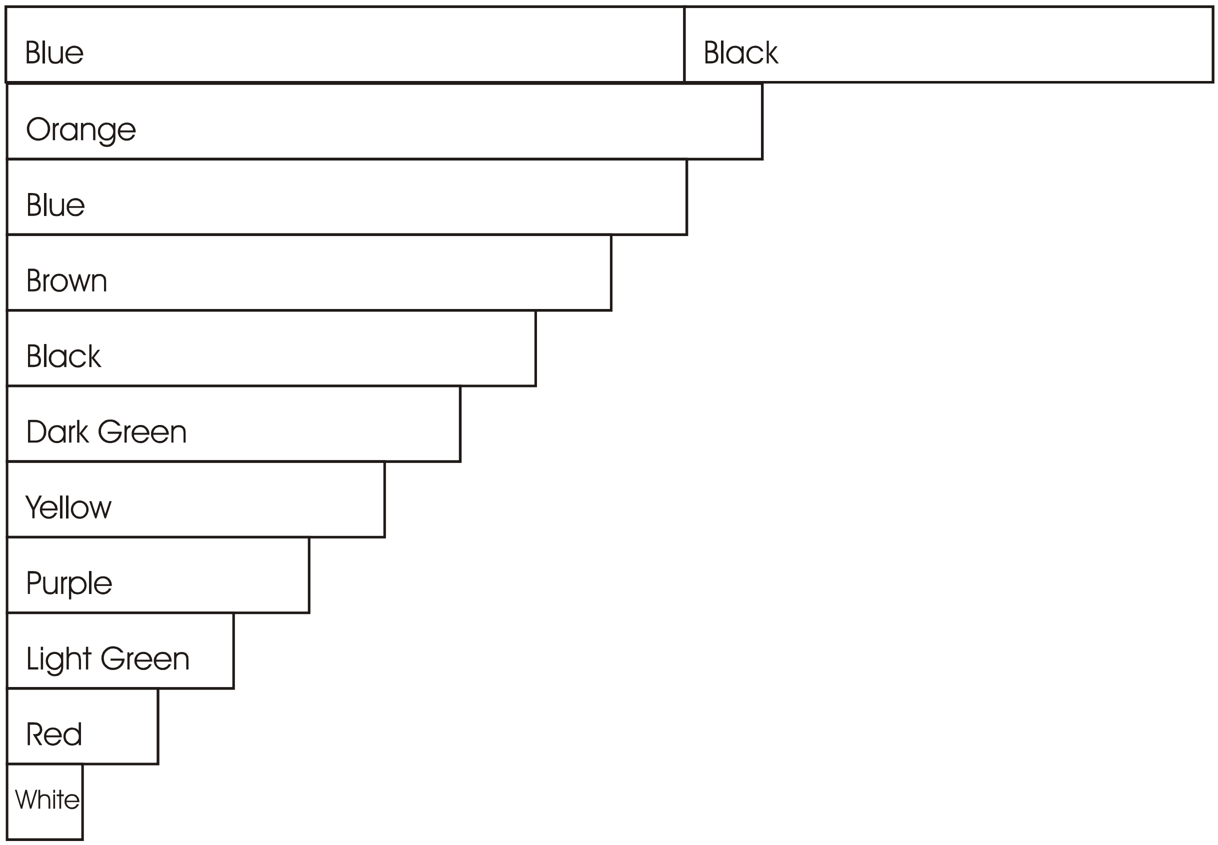
Teachers may want to print the coloured rods on acetate and cut them apart to use on the overhead transparency.



Students can colour the rods as indicated and cut them apart to make their own set of relational rods.

32.2: Relational Rods as a Fraction of One Blue-black Whole

Name:
Date:



Write the value of each coloured rod as a fraction of the blue-black rod. Simplify any fraction that is not in lowest terms.

32.3: Fractions Using Relational Rods

	White	Red	Light Green	Purple	Yellow	Dark Green	Black	Brown	Blue	Orange
White										
Red										
Light Green										
Purple			$\frac{3}{4}$							
Yellow										
Dark Green										
Black										
Brown	$\frac{1}{8}$									
Blue										
Orange								$\frac{4}{5}$		
Blue/ Black	$\frac{1}{16}$									

Description

- Practise adding and subtracting fractions using relational rods in a game format.

Materials

- set of relational rods for each student pair
- BLM 33.1

Assessment Opportunities

Minds On…

Whole Class → Introducing Activity

Students share their fractions from the previous day's work (BLM 32.3). Discuss strategies that different students used for expressing one rod as a fraction of another.

Introduce the activity that students will complete to practise adding and subtracting fractions using relational rods (BLM 33.1).

Action!

Pairs → Activity

Students work with the relational rods to create and complete addition and subtraction problems (BLM 33.1). Students use various strategies to prove that their statement is correct — modelling with the rods, using symbolic manipulation and equivalent fractions, using a calculator. Teachers can modify the winning score, as appropriate (lower to 10, change the scoring methods).

Curriculum Expectations/Demonstration/Rubric: Observe students' skills as they create and solve fraction equations. Assessment can focus on problem-solving skills and use of appropriate mathematical symbols.

Teachers may choose to have all students play one game as a trial or play one game as a whole class before they work in pairs. This would allow for some reinforcement of appropriate language and problem–solving skills.

Consolidate Debrief

Whole Class → Discussion

Each pair of students shares one addition or subtraction expression they created from the activity for the class to solve. Discuss students' strategies for solving, e.g., using rods, mentally, finding equivalent fractions with common denominator.

Application
Reflection
Exploration

Home Activity or Further Classroom Consolidation

Complete one of the following tasks.

1. Create a new game that would require the use of relational rods to add or subtract fractions.
2. Create a new whole based on two or more rods combined (not blue-black). Find the fractional value that each rod is of the whole.
3. Find two equivalent fractions for each of the fractional values found on worksheet 32.2.
4. Complete the following questions about determining equivalent fractions.

Assign these tasks to specific groups of students based on their skill levels.

Select questions involving equivalent fractions from the student textbook

33.1: Fraction Activity with Relational Rods

Name:
Date:

Work with a partner.

Use the handout *32.2: "*Relational Rods as a Fraction of One Blue-black Whole" to help you with the fractional value of each rod.

1. One partner randomly selects 5 rods from the set and lays them out on the table.

2. Individually, create two addition-of-fractions equations and two subtraction-of-fractions equations using the rods. Record your equations using the colours as well as the fractional values in terms of the blue-black rod.

   For example,

dark green	+	purple	=	orange
$\frac{6}{16}$ blue-black	+	$\frac{4}{16}$ blue-black	=	$\frac{10}{16}$ blue-black

   Use equivalent fractions to reduce to:

$\frac{3}{8}$ blue-black	+	$\frac{1}{4}$ blue-black	=	$\frac{5}{8}$ blue-black

orange	+	purple	=	dark green
$\frac{10}{16}$ blue-black	+	$\frac{4}{16}$ blue-black	=	$\frac{6}{16}$ blue-black,

   Use equivalent fractions to reduce to:

$\frac{5}{8}$ blue-black	+	$\frac{1}{4}$ blue-black	=	$\frac{3}{8}$ blue-black

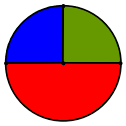
3. Compare your two sets of equations.
   For every correct equation that you have in common, award one point each.
   For every correct equation that your partner doesn't have you receive two points.
   No points are awarded for incorrect equations.

4. Record each person's score for that round.

5. For each round, take turns randomly selecting 5 rods from the set.

6. Play continues until one person reaches 20 points.

Day 34: Subtracting Fractions Using Equivalent Fractions — Grade 7



Description

- Develop rules for subtracting fractions using equivalent fractions with common denominators.
- Practise adding and subtracting fractions.

Materials

- BLM 34.1, 34.2

Assessment Opportunities

Minds On…

Whole Class → Game

Use BLM 34.1 to create an overhead acetate for a game.

BLM 34.2 describes how the game is played.

Play the game with the class.

Consider including visual representations of the fractions on the game board, e.g., coloured rods, pattern blocks.

Action!

Whole Class → Notemaking

Students summarize their understanding of subtracting fractions using equivalent fractions with a common denominator.

Pose questions, create examples related to the questions, work the examples, then add to the note. Highlight different methods that students have developed for determining equivalent fractions and subtracting fractions.

Students determine the amount of detail (number of steps to follow) and develop the steps in the process to the degree that this is practical.

Alternatives to this whole class activity include pairs or small groups creating poster notes.

Consolidate Debrief

Individual → Practice

Students work independently to add and subtract fractions.

Curriculum Expectations/Observation/Checklist: Circulate to observe and record whether students have met the proficiency target in adding and subtracting fractions.

(Developing Proficiency, TIPS: Section 2 – Fractions, p. 8)

Application
Concept Practice
Reflection
Skill Drill

Home Activity or Further Classroom Consolidation

1. Complete the assigned questions.

Select questions requiring an understanding of operations in all four mathematical processes from Developing Mathematical Processes, TIPS: Section 2 – Fractions, p. 5

2. Create a game based on fractions.

Assign for selected students.

34.1: Concentration Game Board (Teacher)

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

34.2: Instructions for a Concentration Game (Teacher)

This overhead game can be used to introduce a topic or to help students consolidate a concept.

For example:

- equivalent fractions
- fractions in simplest form
- converting between fractions and decimals, decimals and percent, or fractions and percent
- converting between mixed numbers and improper fractions

To prepare the game:
Randomly write eight fractions in different boxes on an acetate copy of the game board. In the remaining 8 boxes, write the match to the original eight. Cut out and number 16 paper squares to hide the contents of each box as the game is projected on the overhead screen. Number the blank squares that you use to cover the boxes.

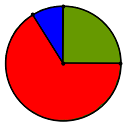
To play the game:
The class forms two teams. A student from the starting team requests that two boxes be uncovered. The student tells if there is a match. If the two items revealed match, the team gets a point. If not, the boxes are covered again and a student from the next team gets a turn. Play continues until all matches have been found.

Alternate playing suggestions:

- If a team makes a match, they get another turn.

- All students must have at least one turn before anyone can take a second turn.

- To prevent students from automatically saying that everything revealed is matching, the team loses a point if a student declares an incorrect match.



Description

- Demonstrate understanding and skills while performing operations with fractions.

Materials

- BLM 35.1 35.2

Assessment Opportunities

Minds On…

Whole Class → Review/Introducing the Performance Task

Discuss and resolve any difficulties students had with the questions from the home activity, Developing Mathematical Processes. (See Day 34.)

Use an overhead acetate to pre-view the performance task and rubric.

Alternative
Set up discussion groups in the four corners of the class. Students go to the corner where the question they are most interested in discussing further is under consideration.

Action!

Individual → Assessment Task

Curriculum Expectations/Performance Task/Rubric: Assess students' demonstration of their learning using a rubric.

Students work independently to complete the Fraction Flag task (BLM 35.1). Students may measure using a ruler or use manipulatives to cover the area. Allow students to use any of the manipulative materials they have been using to add and subtract fractions if they choose. For some students, the flag could be superimposed on grid paper (or grid paper on acetate could be used) to provide an additional option of counting squares to determine area.

Refer to *Think Literacy: Cross-Curricular Approaches – Mathematics*

Consolidate Debrief

Whole Class → Sharing

Students share their strategies in completing the task. Collect students' work.

Home Activity or Further Classroom Consolidation

Exploration
Reflection

Create your own flag using fractional sections.

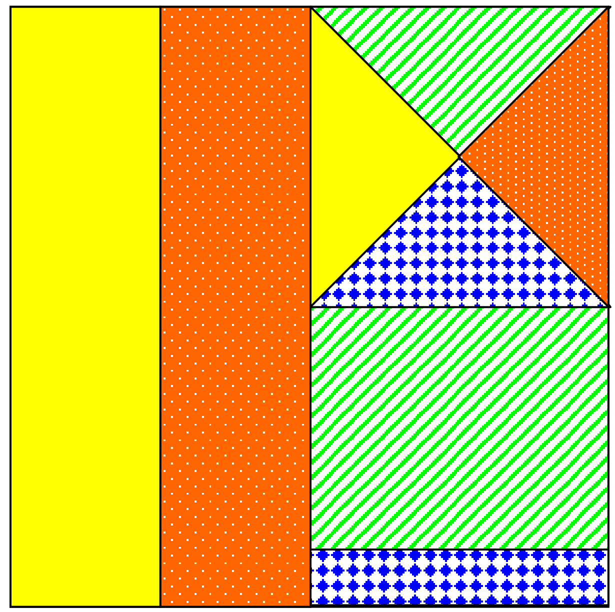
Include solutions.

See BLM 35.2.
Post flags in the classroom.

35.1: Fraction Flag Performance Task

Name:
Date:

The flag to the right was designed with four colours.



1. Determine the fraction of the flag that is:
   a. Orange
   b. Blue
   c. Yellow
   d. Green

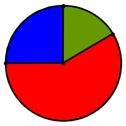
2. What fraction of the flag is not green? Explain your thinking.

3. How much more of the flag is orange than blue? Show all of your work.

35.2: Create Your Own Fraction Flag

______________'s Fraction Flag   Date: ____________________

Note: Your flag must have at least 8 sections and use only straight lines.



Description

- Explore repeated addition of fractions and addition and subtraction of mixed numbers.
- Develop methods for multiplying a fraction by a whole number.

Materials

- BLM 36.1, 36.2

Assessment Opportunities

Minds On…

Whole Class → Introducing the Problems

Brainstorm types of fractions and operations with fractions that have not been addressed in the unit so far (mixed numbers, multiplication and division of fractions, etc.). Accept all responses initially, then ask students to question and support their answers. Students may find it useful to build the fractions referenced with manipulatives as well as represent them symbolically.

Action!

Pairs → Explore

Students work in pairs to develop solutions to the various fraction problems (BLM 36.1).

Learning Skills & Curriculum Expectations/Observation/Rubric: Circulate while students are working to assess problem-solving skills.

Rather than intervening, allow students to grapple with the problems with their partners.

Consolidate Debrief

Whole Class → Sharing

Students share the strategies they used to solve the problems. Encourage each pair of students to provide a complete explanation of how they attempted the solution and how they know their solution is correct.

Record all of the different methods used for students to see and discuss. Is any one method more correct? …easier? …more efficient? Acknowledge all strategies as valid, even if the solution is not correct. Ask probing questions to get the students to explain why their solution is not correct.

Sometimes an incorrect solution can be of more value for students than a correct one as long as the discussion allows them to figure out why a particular strategy did not work.

Application
Concept Practice

Home Activity or Further Classroom Consolidation

Complete worksheet.

See BLM 36.2.

36.1: Challenging Fraction Party Problems

Solve the following problems involving fractions. Show or explain your strategies.

1. A recipe for Pink Party Punch calls for $4\frac{2}{3}$ cups of raspberry juice, $3\frac{1}{4}$ cups of ginger ale, and $2\frac{1}{2}$ cups of raspberry sherbet. How many cups of punch will the recipe make?

2. Sam filled 6 glasses with $\frac{2}{3}$ L of juice in each glass. How much juice did he use?

3. Xia has 16 metres of rope. She cuts off $\frac{1}{6}$ of the rope to use as a skipping rope for a party activity. How long is Xia's skipping rope?

4. Tyson cut some bagels in half and some apples into eighths. At the end of the party, there were 5 pieces of bagel and 11 slices of apple left. How many bagels and how many apples were not eaten?

36.2: Food Fractions

Solve the following problems involving food and fractions. Show and/or explain the strategies you used.

1. Three people shared a mega nutrition bar.
   Which of the following statements are possible?

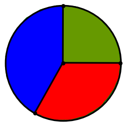
   a. Greg ate $\frac{3}{8}$ of the bar, Gursharan ate $\frac{1}{4}$, and Mo ate $\frac{1}{2}$.

   b. Greg ate $\frac{1}{5}$ of the bar, Gursharan ate $\frac{3}{10}$, and Mo ate $\frac{1}{2}$.

   c. Greg ate $\frac{1}{3}$ of the bar, Gursharan ate $\frac{1}{2}$, and Mo ate $\frac{1}{6}$.

   d. Greg ate $\frac{1}{6}$ of the bar, Gursharan ate $\frac{1}{4}$, and Mo ate $\frac{1}{3}$.

2. Ms. Legume wants to use $\frac{1}{3}$ of her garden for radishes and $\frac{1}{2}$ for beans.
   What fraction of the garden does she have left for each of her carrots and her peas if they both are to get the same amount of space?



Description

- Explore the relationship between fractions and decimals.

Materials

- BLM 37.1

Assessment Opportunities

Minds On…

Whole Class → Review and Introduce New Problem

Students share their strategies for solving the questions from last days' Home Activity (BLM 36.2). Discuss the various strategies and methods students used to arrive at the solutions.

Ask students to think of any two fractions that are "really close." Record a few of their suggestions on the board.

Challenge them to choose one pair of fractions from the board and to find two numbers that are between the two listed. Brainstorm as a whole class what types of numbers students might use, e.g., fractions, decimals, but do not discuss solutions at this point.

Allow students to use a calculator and concrete materials if they wish.

Action!

Pairs → Problem Solving

Students work with partners to find two numbers between one pair of fractions listed on the board.

Students should develop their own strategies and methods rather than guiding them.

Consolidate Debrief

Whole Class → Sharing

Students share their solutions to the problem, as well as their methods for finding the two numbers. Ask students to explain how to use the calculator to convert fractions to decimals. Some discussion around fractions and decimals would be useful, including number systems, common relationships that students are familiar with and applications/appropriateness of each in daily contexts.

Learning Skills & Curriculum Expectations/Observation/Anecdotal: Focus on students' oral communication and use of mathematical language during sharing.

Two methods to change a fraction to a decimal on a calculator are:
- divide the numerator by the denominator.
- enter the fraction using the fraction key ($a\frac{b}{c}$), press ENTER, then press the fraction key again.

Pairs → Practice

Reinforce understanding of the fraction-decimal relationship (BLM 37.1).

Home Activity or Further Classroom Consolidation

Concept Practice
Exploration
Reflection

Create three Determine the Decimal questions. Each one should have either two or three clues and all the clues should be needed to determine the decimal.

Complete questions exploring the relationship of fractions to decimals.

Provide questions selected from the textbook.

37.1: Determine the Decimal

Determine the mystery decimal number from the clues listed.

1. The decimal is… Clue #1: greater than $\frac{1}{8}$

   Clue #2: less than $\frac{1}{5}$

   Clue #3: a multiple of $\frac{1}{20}$

2. The decimal is… Clue #1: between $\frac{2}{5}$ and $\frac{3}{5}$

   Clue #2: greater than $\frac{1}{2}$

   Clue #3: a multiple of 0.11

3. The decimal is… Clue #1: a multiple of $\frac{3}{4}$

   Clue #2: between 2 and 3

4. The decimal is… Clue #1: less than $\frac{7}{8}$

   Clue #2: greater than $\frac{3}{4}$

   Clue #3: a multiple of 0.17

5. The decimal is… Clue #1: greater than $\frac{4}{5}$

   Clue #2: a multiple of 0.22

   Clue #3: less than 1

6. The decimal is… Clue #1: between $\frac{1}{5}$ and $\frac{6}{10}$

   Clue #2: closer to $\frac{1}{4}$ than to one-half

   Clue #3: a multiple of $\frac{1}{10}$

7. The decimal is… Clue #1: multiple of $\frac{1}{2}$

   Clue #2: closer to 6 than to 3.5

   Clue #3: not a whole number

Determine the Decimal

(Answers)

Determine the mystery decimal number from the clues listed.

1. The decimal is… (0.15)
   - Clue #1: greater than $\frac{1}{8}$
   - Clue #2: less than $\frac{1}{5}$
   - Clue #3: a multiple of $\frac{1}{20}$

2. The decimal is… (0.55)
   - Clue #1: between $\frac{2}{5}$ and $\frac{3}{5}$
   - Clue #2: greater than $\frac{1}{2}$
   - Clue #3: a multiple of 0.11

3. The decimal is… (2.25)
   - Clue #1: a multiple of $\frac{3}{4}$
   - Clue #2: between 2 and 3

4. The decimal is… (0.85)
   - Clue #1: less than $\frac{7}{8}$
   - Clue #2: greater than $\frac{3}{4}$
   - Clue #3: a multiple of 0.17

5. The decimal is… (0.88)
   - Clue #1: greater than $\frac{4}{5}$
   - Clue #2: a multiple of 0.22
   - Clue #3: less than 1

6. The decimal is… (0.3)
   - Clue #1: between $\frac{1}{5}$ and $\frac{6}{10}$
   - Clue #2: closer to $\frac{1}{4}$ than to one-half
   - Clue #3: a multiple of $\frac{1}{10}$

7. The decimal is… (5.5)
   - Clue #1: multiple of $\frac{1}{2}$
   - Clue #2: closer to 6 than to 3.5
   - Clue #3: not a whole number

Description

- Demonstrate understanding of fractions and operations with fractions on an open-ended problem-solving task.

Materials

- calculators
- manipulatives

Assessment Opportunities

Minds On…

Whole Class → Review

Students exchange their Determine the Decimal questions with a partner, to solve at the end of the period or as a Home Activity.

Discuss some or all three of the statements in: Is This Always True?, Grade 7, TIPS: Section 2 – Fractions, p. 2.

To show that a statement is not always true, only a single counter example is required.

Action!

Whole Class → Introducing the Task

Tell students they will need to draw on all that they've learned about fractions to solve just one problem. The emphasis should be on explaining their thinking and showing all strategies used.

If students complete this task before time is called, they should work further on Is This Always True? from Minds On.

Before students begin the performance task, provide them with direction for what to do if they complete the task before their peers.

Individual → Performance Task

Students work independently to show three different methods for solving the problem: Extend Your Thinking, Grade 7, TIPS: Section 2 – Fractions, p. 11.

Curriculum Expectations/Performance Task/Rubric: Use a rubric to assess students' understanding of fractions, accuracy in fraction computations, use of mathematical language, clarity of communication, and ability to solve the problem in three ways.

Students can use calculators and any other manipulative materials they have been using during the fractions unit.

Consolidate Debrief

Whole Class → Sharing

Collect students' work.

Discuss the strategies students used to solve the problem.

Students solve their partner's Determine the Decimal question, if time permits.

Concept Practice

Home Activity or Further Classroom Consolidation

Complete your partner's Determine the Decimal questions.

Fractions Unit Diagnostic — Grade 7

Description

Students will:

- activate prior knowledge of fractions;
- demonstrate understanding of equivalent fractions;
- demonstrate understanding of relational size of fractions.

Materials

- BLM D1
- manipulatives
- calculators

Assessment Opportunities

Minds On…

Small Group → Placemat

Working in groups of fours, students complete a placemat by writing in their area what they know about fractions. Students work independently for two minutes, then discuss their notes with their group. The group recorder indicates facts known by all group members in the centre of the placemat.

For more information on Placemat see Teacher TIPS: Placemat (Section 4, TIP 9).

Whole Class → Mind Map

Groups share one fact from their sheet in a mind map with FRACTIONS at the centre. Facilitate this discussion allowing the students to critique and defend the information presented. Ensure that key language is introduced, e.g., numerator, denominator, equivalent fractions, improper fraction, mixed fraction.

Curriculum Expectations/Observation/Mental Note: Determine the level of students' understanding of fractions by listening to facts that were commonly known within each group.

For more information on prior knowledge and curriculum content of Fractions see Continuum and Connections: Fractions (Section 2)

Action!

Individual → Diagnostic

Explain that BLM D.1 is diagnostic and helpful for you to plan their work on fractions. Read through all the questions before students begin.

Students can access manipulatives and calculators, as required. Circulate, clarifying the questions, as needed.

Collect BLMs upon completion.

Curriculum Expectations/Paper-and-Pencil/Holistic: Mark each diagnostic sheet for correctness. Group students according to areas of strengths. (Level 3 or 4 responses on #1, Level 3 or 4 responses on #2, Level 3 or 4 responses on #3, Level 3 or 4 responses on #1 and #2, Level 3 or 4 responses on all three).

Consolidate Debrief

Small Group → Sharing/Discussion

Students rejoin in their small groups and discuss their solutions to the diagnostic questions clearly explaining their thinking.

Whole Group → Debrief

Groups share their solutions questioning each other and critiquing and defending solutions.

This discussion allows students to confirm or refute their understanding of fractions.

Home Activity or Further Classroom Consolidation

Application
Concept Practice
Differentiated
Exploration
Reflection
Skill Drill

BLM D.1

Name:

Complete each question. Show any diagrams, numbers or words used.

1. Write an equivalent fraction for each of the following fractions.

   a) $\frac{5}{10}$

   b) $\frac{3}{7}$

   c) $\frac{1}{4}$

00

2. Write either < (less than), > (greater than), or = (equal to) between the fractions to make a true statement. Indicate your reasoning beside the statement.

   a) $\frac{5}{6}$ $\frac{5}{9}$

   b) $\frac{7}{11}$ $\frac{5}{11}$

   c) $\frac{3}{4}$ $\frac{7}{16}$

   d) $2\frac{1}{2}$ $\frac{5}{2}$

3. Identify whether each statement is always true, sometimes true, or never true?
   Explain your reasoning.

   a) Fractions get bigger when the numerator gets larger.

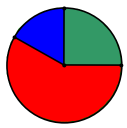
   b) Fractions get smaller when the denominator gets larger.

   c) Fractions stay the same when you make both the numerator and denominator larger.

4. Circle the word that best represents your understanding of fractions:

   Limited Fair Good Excellent

Day 29: Fraction Puzzles – Differentiated Instruction with Centres — Grade 7



Description
- Explore/review fractional parts of geometric shapes.
- Order fractions.

Materials
- pattern blocks
- overhead pattern blocks
- word wall templates
- BLM 29.1-29.5
- 2 or 3 large Imperial socket wrench sets

Assessment Opportunities

Minds On…

Whole Class → Introducing Vocabulary

Students work in pairs to complete one word wall template. (*Think Literacy: Cross-Curricular Approaches–Mathematics*). They use their textbook, dictionary, personal knowledge, and other resources to complete it.

Students share their word and the components of the template. Post all words on the classroom math word wall.

TIPS: Section 2 – Fractions, p. 3 teaching fraction concepts.

Virtual pattern blocks are available at http://arcytech.org/java/patterns/patterns_j.shtml.

Action!

Small Groups → Centres

Group students according to their diagnostic results. Students work at the indicated centre for the action portion of the lesson. (See next page for description of centres.)

While students work at the centres, give selected individuals BLM 29.3 and an Imperial socket wrench set that has been mixed up. Ask them to explain why they placed a certain socket between two others.

Learning Skills & Curriculum Expectations/Observation/Anecdotal: Circulate while students are working to assess prior knowledge of fractions. Ask students to explain how they know their solution to a question is correct.

Briefly review the meaning of *parallelogram* (blue or beige block) and *trapezoid* (red block).

Some methods students may use include: physical size of each socket, ordering of the sockets using equivalent fractions, converting to decimals, measuring in millimetres.

Consolidate Debrief

Whole Class → Sharing/Discussion

Ask students to share what they learned at their centre. Students interested in exploring the mathematics at their centres should be encouraged to do so either as a homework task or as an additional exploration day.

Discuss the various methods discovered by students to solve the socket set problem.

Concept Practice

Home Activity or Further Classroom Consolidation

Write a summary of your current understanding of fractions. It can be a mind map, a written paragraph, diagrams or any other appropriate format.

Refer students to the virtual pattern block website listed above.

Assign related questions from the textbook or BLM 29.9 based on the centre the students worked at.

Differentiated Instruction: Centres-Based Day 29

Following the administration of the diagnostic, students should be grouped according to their readiness to proceed. The centres are somewhat progressive, so students who complete one centre could move to the next. It is not the intention that all students complete all centres.

Group A: students who were unable to answer most or all of the assessment correctly and who demonstrated little fractional understanding in their mind map and dialogue.

Group B: students who were able to answer only question 1 of the assessment correctly and who demonstrated little fractional understanding in their mind map and dialogue.

Group C: students who were able to answer most or all of the assessment correctly and who demonstrated grade-level fractional understanding in their mind map and dialogue.

Group D: students who were able to answer most or all of the assessment correctly and who demonstrated above grade-level fractional understanding in their mind map and dialogue.

Description of Centres

The first centre, **Fraction Puzzles**, provides students with an opportunity to explore equivalent fractions and compare fractional values using the concrete manipulative, fraction circle pieces. Students record their work on BLM 29.4. Questioning students about their reasoning and any conclusions they may be drawing aids in deepening their understanding of fractions.

The second centre, **Virtual Manipulatives: Fraction Circles**, engages students as they explore equivalent fractions and compare fractional values using virtual fraction circles. This centre requires comfort with technology as students turn shapes to fit into a whole. Students are prompted to draw conclusions about the relationships they are exploring. (See BLM 29.5.)

The third centre, **Pattern Block Area Fraction Puzzles**, is the original Day 29 activity. Students who encounter difficulty are provided with hint cards (BLM 29.10). The hint cards should be photocopied on card stock, labelled on the back with the question number and hint number and cut into separate cards. Students are encouraged to access one or more hints as they see appropriate.

The fourth centre, **Explorations**, consists of extension questions for students who finish the third centre quickly. BLM 29.6 and 29.7 could be photocopied on cardstock and laminated, so students can work on the card using non-permanent overhead markers. BLM 29.8 could be separated into single cards and laminated. BLM 29.9 may be considered for a home activity for selected students or included in the centre. Students should have a choice of question that they work on following the completion of the centre.

Group Assignments

Activity	Fraction Puzzles	Virtual Manipulatives: Fraction Circles	Pattern Block Area Fraction Puzzles	Explorations
Materials	• fraction circles • one copy of BLM 29.4 per student	• computer with Internet access • one copy of BLM 29.5 per student	• pattern blocks • one copy of BLM 29.1 and 29.2 per student • one set of Hint Cards (BLM 29.10)	• copies of BLM 29.6, 29.7, 29.8, 29.9 as required (see note above) • graph paper • manipulatives
Group A	✓			
Group B		✓		
Group C			✓	
Group D			✓	✓

29.1: Pattern Block Area Fraction Puzzles

Name:
Date:

Use pattern blocks to solve each of the area fraction puzzles below. Draw each solution on pattern block paper. Label each colour with its fraction of the whole shape.

1. Build a parallelogram with an area that is one-third green, one-third blue, and one-third red.
2. Build a parallelogram with an area that is one-eighth green, one-half yellow, one-eighth red, and one-quarter blue.
3. Build a trapezoid with an area that is one-tenth green and nine-tenths red.
4. Rebuild each of the puzzles above in a different way.
5. Explain why it is not possible to build a parallelogram with an area that is one-half yellow, one-third green and one-quarter blue.

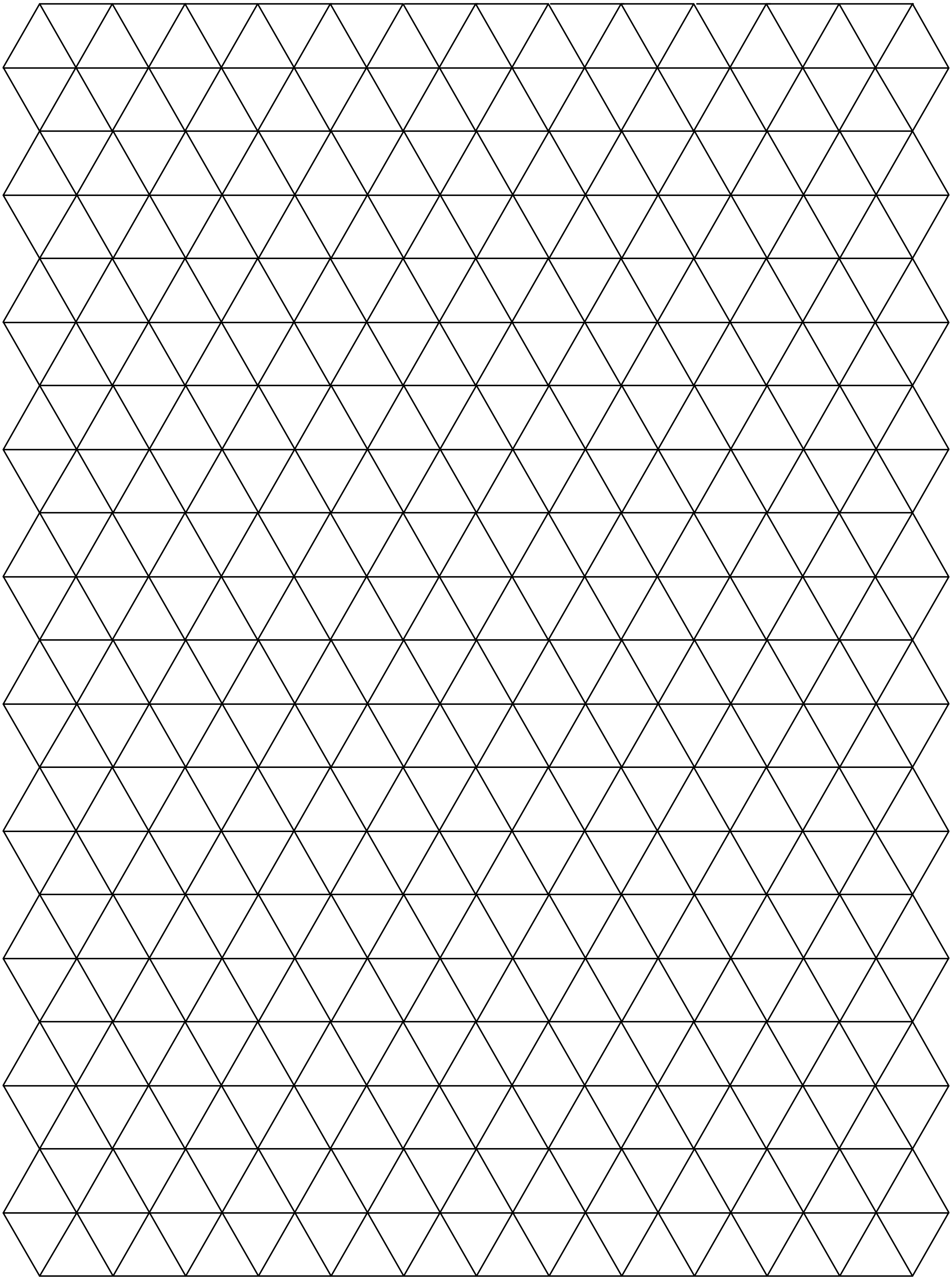
29.1: Pattern Block Area Fraction Puzzles

Name:
Date:

Use pattern blocks to solve each of the area fraction puzzles below. Draw each solution on pattern block paper. Label each colour with its fraction of the whole shape.

1. Build a parallelogram with an area that is one-third green, one-third blue, and one-third red.
2. Build a parallelogram with an area that is one-eighth green, one-half yellow, one-eighth red, and one-quarter blue.
3. Build a trapezoid with an area that is one-tenth green and nine-tenths red.
4. Rebuild each of the puzzles above in a different way.
5. Explain why it is not possible to build a parallelogram with an area that is one-half yellow, one-third green and one-quarter blue.

29.2: Pattern Block Paper



29.3: Socket To You!

Name:
Date:

1. $\frac{20}{32}$ is an equivalent fraction for $\frac{5}{8}$. Write two more equivalent fractions for $\frac{5}{8}$.

2. Write two equivalent fractions for $\frac{3}{4}$.

3. Circle which is larger: $\frac{3}{8}$ or $\frac{3}{16}$.

4. Circle which is smaller: $\frac{7}{16}$ or $\frac{9}{16}$.

5. Circle the fraction that falls between $\frac{7}{16}$ and $\frac{9}{16}$.

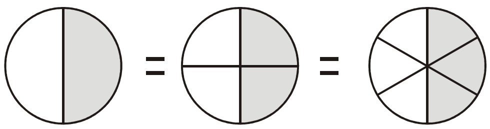
   $\frac{13}{32}, \frac{1}{4}, \frac{3}{8}, \frac{1}{2}, \frac{5}{8}, \frac{3}{4}, \frac{19}{32}$

6. Often mechanics use socket wrench sets with openings measured in fractions of an inch. These fractions are stamped on the fronts of the sockets.
   Think about how to restock the socket wrench set in the correct order in the case. Explain how you decided on the order you chose to the teacher.

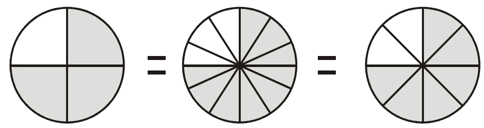
29.4: Fraction Puzzles

1. Review the following fraction sentences that identify equivalent fractions:

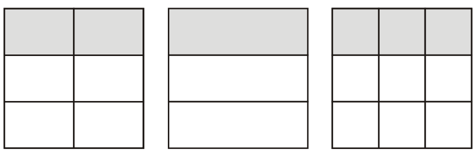
a) $\frac{1}{2} = \frac{2}{4} = \frac{3}{6}$



b) $\frac{3}{4} = \frac{9}{12} = \frac{6}{8}$

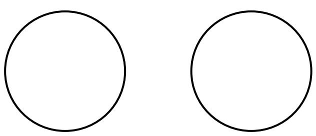


c) $\frac{2}{6} = \frac{1}{3} = \frac{3}{9}$

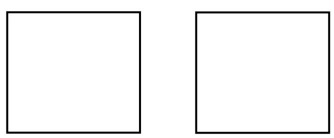


2. Represent the following fractions using FRACTION CIRCLES or FRACTION TILES.

a) $\frac{3}{4} = \frac{6}{8}$



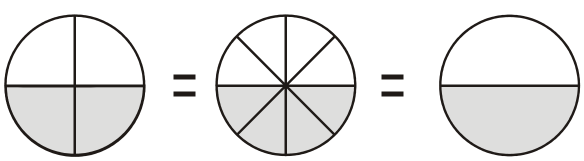
b) $\frac{2}{3} = \frac{6}{9}$



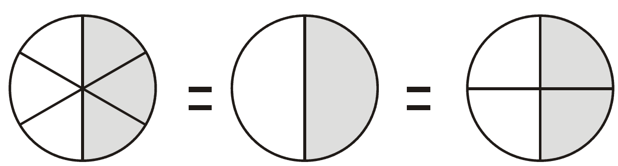
c) $\frac{4}{5} = \frac{8}{10}$

3. Write fraction sentences to represent the following illustrations. Use your fraction circles to recreate the illustration if necessary.

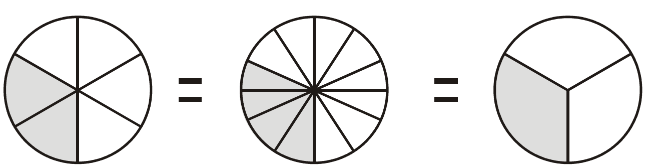
a)



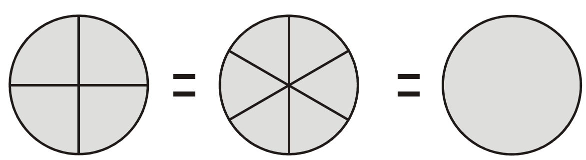
b)



c)



d)



29.4: Fraction Puzzles (continued)

4. State whether the following statements are TRUE or FALSE.
   Use your fraction circles or tiles to recreate the diagrams to compare.

a) $\frac{4}{10} = \frac{2}{5}$

b) $\frac{3}{4} = \frac{6}{10}$

c) $\frac{2}{3} = \frac{2}{5}$

d) $\frac{3}{5} = \frac{2}{5}$

5. Identify the following as GREATER THAN (>) or LESS THAN (<) or EQUAL TO (=).
   Use your fraction circles or tiles to create a diagram, if necessary.

a) $\frac{7}{10} \quad \frac{5}{6}$

b) $\frac{2}{8} \quad \frac{3}{8}$

c) $\frac{2}{5} \quad \frac{2}{4}$

d) $\frac{1}{3} \quad \frac{4}{12}$

6. How many $\frac{1}{10}$ tiles does it take to cover four $\frac{1}{5}$ tiles? Find two ways to determine this.

29.5: Virtual Manipulatives: Fraction Circles

Use the virtual fraction circles at
http://matti.usu.edu/nlvm/nav/frames_asid_274_g_3_t_1.html?open=activities

Investigating Filling a Whole

1. a. Select a coloured piece.
   Colour: ________________
   b. Click on the piece to move it into your work area. Then, click and drag it into the whole shape.
   c. Repeat using the same coloured piece until you have filled the whole shape.
   It takes ________ pieces to fill a whole shape.

   Click on **Show Labels**. Point at each piece.

2. a. Select a different coloured piece.
   Colour: ________________
   b. Repeat the instructions from #1 to fill the whole shape with these pieces.
   It takes ________ pieces to fill a whole shape.

   Click on **Show Labels**. Point at each piece.

   It is _____ of the whole shape.

3. a. Select a different coloured piece.
   Colour: ________________
   b. Repeat the instructions from #1 to fill the whole shape with these pieces.
   It takes ________ pieces to fill a whole shape.

   Click on **Show Labels**. Point at each piece.

   It is _____ of the whole shape.

4. Identify any relationships you notice between the number of pieces required to fill the whole and the denominator.

29.5: Virtual Manipulatives: Fraction Circles (continued)

Investigating Fractional Relationships

5. a. State the fractional value of a yellow piece.

   b. State the fractional value of a black piece.

   c. Determine how many black pieces fill one yellow piece.

6. a. State the fractional value of a purple piece.
   b. Determine what other colour pieces will completely fill one purple piece.

   Example: 3 blues ($\frac{3}{9}$) will completely fill one purple piece ($\frac{1}{3}$)

7. Find other pieces that will completely cover one piece.
   Draw these equivalent fractions and record the fractional values.

8. Find a fraction that is equivalent to $\frac{2}{12}$ (two orange pieces).

   An equivalent fraction is

29.5: Virtual Manipulatives: Fraction Circles continued

Ordering Fractions

9. Use the fraction circle pieces and your knowledge about fractions to determine which fraction is larger, $\frac{1}{6}$ *or* $\frac{1}{9}$. Explain your thinking.

10. Identify the colour and value of the largest fraction piece.

11. Identify the colour and value of the smallest fraction piece.

12. Use the fraction circle pieces and your knowledge about fractions to determine which fraction is larger. Circle the larger fraction.

a. $\frac{1}{5}$ $\frac{1}{3}$

b. $\frac{2}{5}$ $\frac{2}{3}$

c. $\frac{3}{9}$ $\frac{1}{2}$

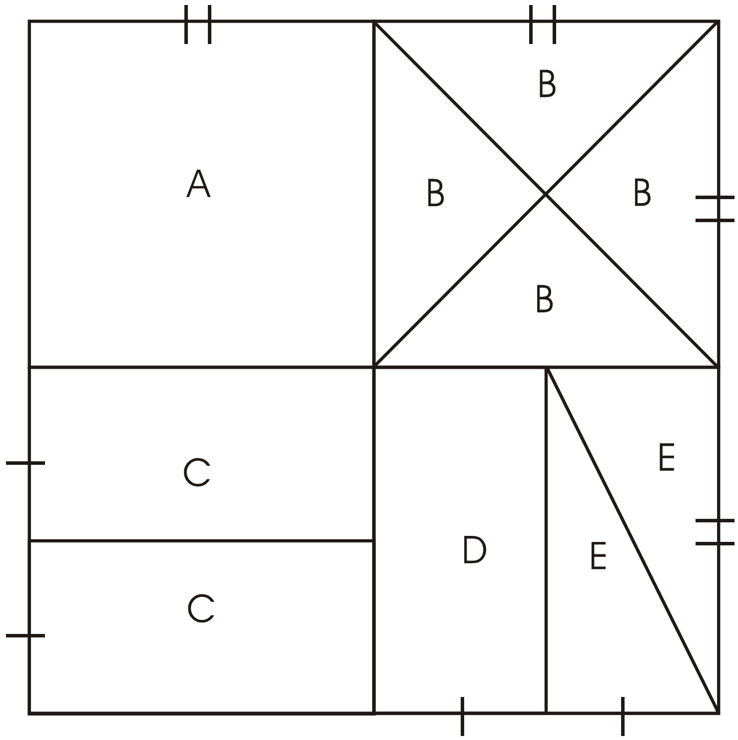
13. Using the information above and your knowledge about fractions, identify ways that you can tell the size of a fraction from the fraction.

14. Put the following fractions in order from the largest to the smallest. Explain your thinking.

$\frac{1}{3}$ $\frac{1}{5}$ $\frac{1}{2}$ $\frac{1}{10}$ $\frac{1}{4}$

29.6: Fractional Names

The large outer square represents 1 whole unit. It is partitioned into pieces. Each piece is identified with a letter.



1. Decide what fraction of the whole square each piece is and write it on the shape.

2. Explain how you know the fractional name for each of the following pieces:

   A

   C

   D

   E

3. What piece or collection of pieces from the square will give you an amount close to:

   a) $\frac{1}{5}$

   b) $\frac{2}{3}$

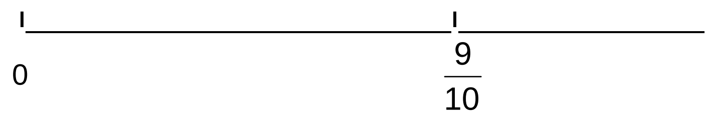
4. Pose your own fraction questions using the above diagram.

29.7: Fractional Sense

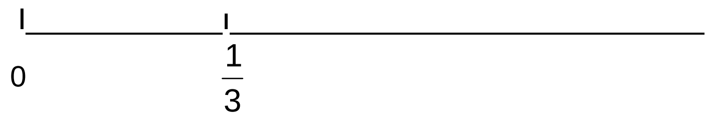
1. Estimate and mark where the numeral 1 would be.

a)

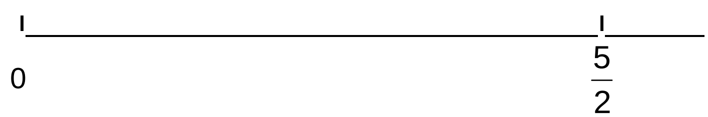
b)



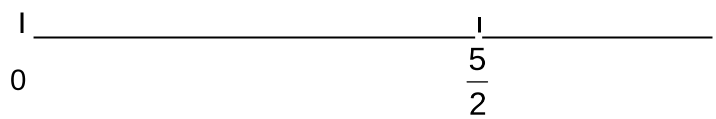
c)



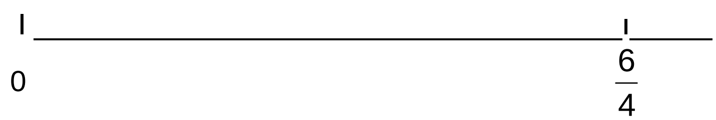
d)



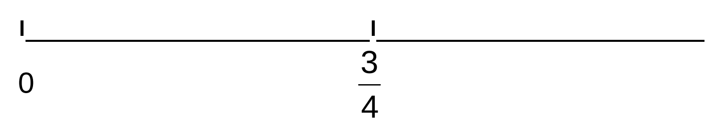
e)



f)



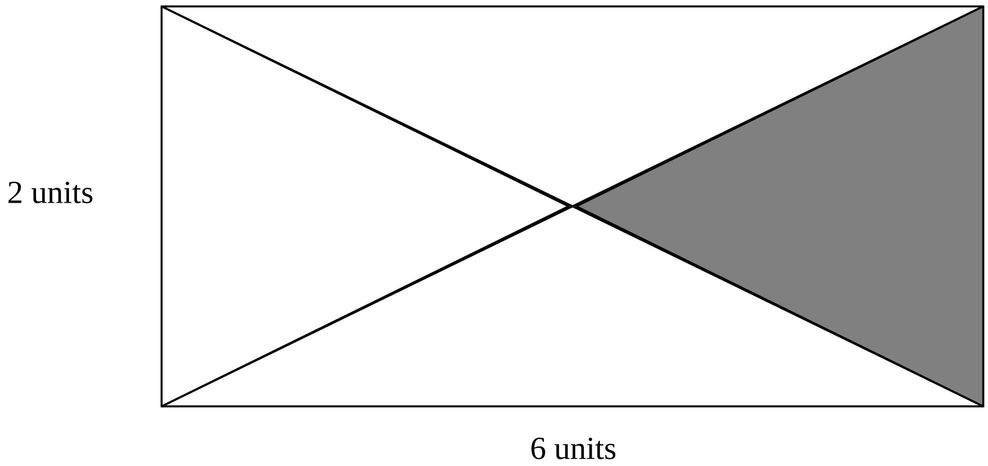
g)



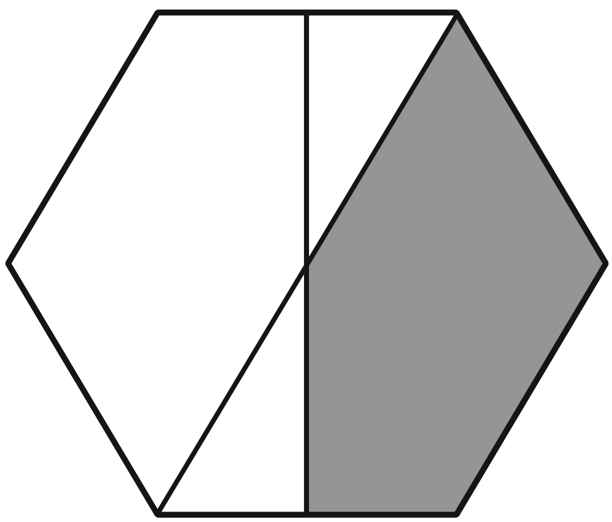
2. Explain how you estimated the position of 1.

29.8: Exploring Some Fractional Relationships

1. Determine what fraction of the area of the rectangle is shaded.

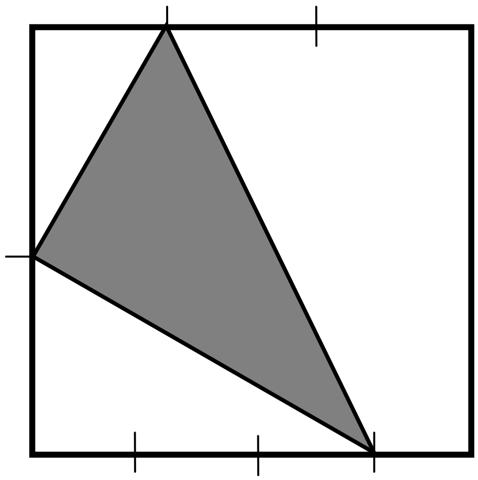


2. Determine what fraction of area of the hexagon is shaded.
   (Leave your answer as the simplest possible fraction.)



29.8: Exploring Some Fractional Relationships (continued)

3. The sides of a square were divided into different numbers of equal sections. As the diagram shows, three divider points are connected. Determine the fraction of the square that is represented by the shaded triangle.



4. Arrange these fractions from smallest to largest.

$$\frac{3}{8} \quad \frac{(3+1)}{(8+1)} \quad \frac{(3+2)}{(8+2)} \quad \frac{(3+12)}{(8+12)} \quad \frac{(3-2)}{(8-2)}$$

5. Suppose *x, y*, and *z* represent whole numbers different from 0. Suppose that $x > y > z$. If possible, tell which fraction is larger. Justify your thinking.

a. $\frac{x}{z}$ or $\frac{y}{z}$

b. $\frac{x}{y}$ or $\frac{y}{y}$

c. $\frac{x}{y}$ or $\frac{x}{z}$

29.9: Working with "Nice" Fractions

Sometimes we round off a fraction to a "nice" fraction, such as $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{2}{3}$, *or* $\frac{3}{4}$, that is more easily understood.

For example:

Chin made 4 out of 15 free throws.

$\frac{4}{15}$ *is close to* $\frac{4}{16}$*, and* $\frac{4}{16}$ *equals* $\frac{1}{4}$*.*

Thus she made about $\frac{1}{4}$ *of her shots.*

Represent each of these with a nice fraction.

1. Matthew took 26 shots and scored 8 baskets.

2. Sandra was at bat 73 times and made 24 base hits.

3. Jason completed 22 out of his 41 passes.

4. Cindy was at bat 73 times and made 15 base hits.

5. Luciano took 17 shots and scored 8 baskets.

6. Daniel was at bat 35 times and made 12 hits.

7. Percival took 35 shots on goal and scored 6 goals.

29.10: Hint Cards

Question 1 **Hint #1** You will need 11 blocks.	**Question 1** **Hint #2** The green piece is 1 unit.
Question 1 **Hint #3** The blue piece is 2 units.	**Question 1** **Hint #4** The red piece is 3 units.
Question 2 **Hint #1** You will need nine blocks.	**Question 2** **Hint #2** Begin by constructing the yellow area.
Question 2 **Hint #3** The green area is equal to the red area.	**Question 3** **Hint #1** You will need twelve red blocks.
Question 3 **Hint #2** You will use more than one green block.	

Lesson Outline: Days 1 – 5 Grade 7

BIG PICTURE

Students will:
- investigate trapezoids with and without dynamic geometry software;
- develop a formula for the area of a trapezoid;
- solve problems involving trapezoids;
- apply measurement tools.

Day	Lesson Title	Description	Expectations
1	2-D or 3-D?	• Distinguish between 2-D shapes and 3-D figures. • Reflect on various properties of trapezoids. • Estimate areas of triangles and quadrilaterals. • Explore concepts of perimeters and areas of triangles • Review, consolidate, and calculate perimeters and areas of triangles, rectangles and parallelograms.	7m32, 7m33, 7m35, 7m36, 7m37, 7m47 CGE 3c, 4a, 4f
2	How to Trap a Zoid with *The Geometer's Sketchpad 4* ®	• Explore and/or review the use of *The Geometer's Sketchpad 4* ® for constructing points, segments and shapes. • Practise constructing and measuring trapezoids using *The Geometer's Sketchpad 4* ®.	7m28, 7m40, 7m50, 7m52 CGE 3c, 5a
3	Reducing Taxes	• Understand that a trapezoid can have zero or two (but not one, three or four) right angles. • Develop the formula for the area of a trapezoid containing two right angles.	7m32, 7m33, 7m35, 7m36, 7m37, 7m38, 7m39 CGE 2b, 4e
4	Paying Taxes	• Develop a formula to calculate the area of any trapezoid.	7m32, 7m33, 7m35, 7m37, 7m38, 7m39 CGE 3b, 3c, 5a, 5g
5	Applying Trapezoid Knowledge	• Apply the trapezoid area calculation as a culminating activity.	7m32, 7m33, 7m34, 7m37, 7m39, 7m40 CGE 2b, 3c, 4f

Day 1: 2-D or 3-D? — Grade 7

Description

- Distinguish between 2-D shapes and 3-D figures.
- Reflect on various properties of trapezoids.
- Estimate areas of triangles and quadrilaterals.
- Explore concepts of perimeters and areas of triangles
- Review, consolidate, and calculate perimeters and areas of triangles, rectangles and parallelograms.

Materials

- cm grid paper
- sticky notes
- geometric models
- BLM 1.1, 1.2, 1.3

Assessment Opportunities

Minds On…

Individual → Review
Students complete Part 1 of BLM 1.1. Circulate and assist students having difficulty with the difference between 2-D shapes and 3-D figures.

Whole Group → Discussion
Take up Part 1 of BLM 1.1 with the class. Discuss any issues that students raised.

Whole Group → Activate Prior Knowledge
Students select 1-2 shapes from the list in Question 2 (Part 1, BLM 1.1). On sticky notes, students make a sketch of the shape(s) chosen and write one or two properties of the shape that are not included in its definition. Post the notes on a KWL classroom chart. Read aloud the students' prior knowledge of terms/sketches. Discuss all terms in depth and add to the word wall.

Have models of geometric objects prominently on display.

Select a student to add vocabulary to the word wall.

Action!

Pairs → Investigation
In pairs, students complete Parts 2 and 3 of BLM 1.1.

Whole Group → Discussion
Take up Parts 2 and 3 of BLM 1.1 with the class. Ask: "Is a trapezoid a 2-D shape or a 3-D figure?" Individual students respond including a brief justification. Ask if anyone knows where there is an approximate isosceles trapezoid shape on a computer system. (The "D" connector for the monitor on the back of the CPU.)

Pairs → Investigation
Students read from *Impact Math Measurement* and complete questions 3 to 5 on BLM 1.2 (perimeter and area of 2-D shapes).

Curriculum Expectations/Observation/Rating Scale: Focus on fluent, accurate and effective use of mathematical vocabulary.

The glossary of *The Ontario Curriculum, Mathematics, Grades 1-8* defines trapezoid as a quadrilateral with exactly one pair of opposite sides parallel.

Use the rating scale on BLM 1.3.

A general debriefing of the activity allows the teacher to see where the students are in their understanding of the formulas.

Consolidate Debrief

Whole Class → Discussion
Ask pairs to explain how they calculated the areas of the various shapes. Students could explain how they decomposed larger shapes into simple shapes such as right triangles. Others may explain how a right triangle is half of a rectangle.

Students demonstrate how they applied the area formulas. Encourage all possible answers and ask students whether they think there is more than one method of solving these types of problems.

Concept Practice

Home Activity or Further Classroom Consolidation
Write in your Math journals about the major difference between trapezoids and the group of quadrilaterals that contains squares, rectangles, rhombuses and parallelograms. Illustrate each of the shapes.

1.1: So What is a Trapezoid Anyway?

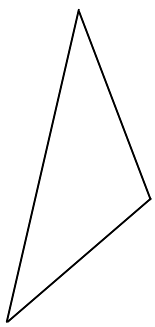
Name:
Date:

Part 1

Think about two-dimensional (2-D) shapes and three-dimensional (3-D) figures. A 2-D shape, such as a triangle, lies on a flat surface while a 3-D figure, such as a rectangular prism, projects above or below the surface.

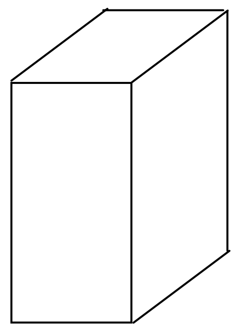
1. Write the following geometric objects in the correct column of the table:

   rhombus, right triangle, cylinder, parallelogram, triangular prism, square, cone, polygon, rectangle, sphere, cone, circle, quadrilateral, pyramid, scalene triangle



Triangle (2-D)

Two-dimensional Shapes	Three-dimensional Figures



Rectangular Prism (3-D)

2. Draw a line from each 2-D shape name to its definition. Some definitions could represent more than one shape so be sure to select the <u>most appropriate</u> definition in each case.

Polygon	A quadrilateral with both pairs of opposite sides parallel.
Triangle	A three-sided polygon.
Quadrilateral	A rectangle with all four sides equal.
Parallelogram	A 2-D closed shape whose sides are straight line segments.
Rectangle	A quadrilateral with all four sides equal.
Rhombus	A four-sided polygon.
Square	A quadrilateral with four right angles and both pairs of opposite sides equal.

1.1: So What is a Trapezoid Anyway? (continued)

Part 2

The definition that most North American mathematicians use for a **trapezoid** is: *a four-sided shape with exactly one pair of opposite sides parallel*. Jake used the Internet to find out about trapezoids. He found several conflicting definitions. They are:

a) a quadrilateral with two sides parallel.
b) a four-sided polygon which has one pair of opposite sides parallel.
c) a four-sided 2-D shape having one pair of opposite sides parallel, the other pair not.
d) a four-sided 2-D closed shape with at least one pair of opposite sides parallel.

Write the letter representing the definition that best answers each question below.

1. Which definition is the same as the one that is used by North Americans? _____
2. Which definition is ambiguous, that is, it could include quadrilaterals with one or both pairs of opposite sides parallel? _____
3. Which definition includes other quadrilaterals such as a parallelogram or a square? _____

Note: In the following questions, the definition of trapezoid will be the North American version given at the beginning of part 2 above.

Part 3

An isosceles trapezoid is one whose non-parallel sides are equal.

1. Could an isosceles trapezoid be a parallelogram? Explain.

2. Can the parallel sides of an isosceles trapezoid be equal? Explain.

3. Can the parallel sides of any trapezoid be equal? Explain.

4. Can a trapezoid ever have

i)	no right angles?	YES	NO
ii)	only one right angle?	YES	NO
iii)	exactly two right angles?	YES	NO
iv)	exactly three right angles?	YES	NO
v)	exactly four right angles?	YES	NO

1.2: Gulliver Dines with the Mathematicians

(*Impact Math – Measurement*)

> "We had two courses of three dishes each. In the first course, there was a shoulder of mutton, cut into an equilateral triangle; a piece of beef into a rhombus and a pudding into a cycloid … The servants cut our bread into cones, cylinders, parallelograms and several other mathematical figures.…Their ideas are perpetually expressed in lines and figures. To praise the beauty of an animal, they describe it in terms of rhombuses, circles, parallelograms, ellipses and other geometric terms."

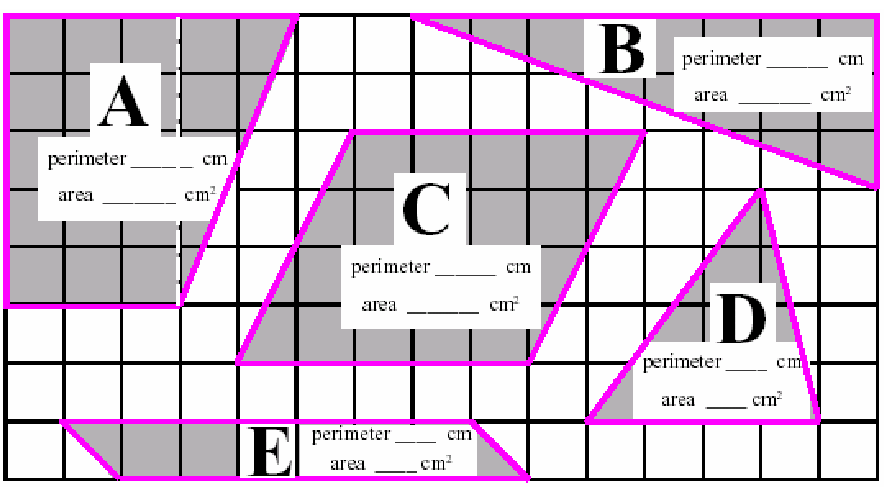
Gulliver's Travels is a popular tale of a traveller named Gulliver who sailed the oceans to strange and distant lands. Most people know of his visit to Lilliput, the land of the little people. Some know of his visit to Brobdingnag, island of the giants. But few have read the chapter about Gulliver's visit to Laputa, the land of the mathematicians. Some small excerpts from that visit are presented here in a slightly modified form, to modernize the old English in which this manuscript was written almost three centuries ago!

Activity 1 – Student Page

1. In his description of the dinner, Gulliver confused some 2-dimensional shapes with 3-dimensional figures. Make a list of the 2-dimensional shapes he named and another list of the 3-dimensional figures. Then rewrite Gulliver's first paragraph using the appropriate terms.

2. Write a sentence and draw a sketch to explain the meaning of each term.
   a) parallelogram
   b) trapezoid
   c) equilateral triangle
   d) rhombus
   e) rectangular prism
   f) triangular prism

3. Name the 2-dimensional shapes drawn on the centimetre grid below. Count squares to estimate the perimeter and area of each. Record your estimates.



1.2: Gulliver Dines with the Mathematicians (continued)

4. Write as many of these area formulas as you know.
   a) The area of a rectangle given its length *l* and width *w*.
   b) The area of a triangle given its height *h* and the length *b* of its base.
   c) The area of a parallelogram given the length *l* of one side and the perpendicular distance *d* from it to the other parallel side.

   Use the formulas you know to check your estimates in Exercise 3.

5. Draw each of these 2-dimensional shapes on centimetre paper.
   a) a rectangle of area 20 cm^2 and perimeter 18 cm.
   b) a parallelogram of area 24 cm^2 and perimeter 22 cm.
   c) a quadrilateral of area 20 cm^2 and perimeter 20 cm.

1.3: Rating Scale – Estimating and Calculating Perimeter and Area

Note: *Impact Math – Measurement* p. 26 contains estimation techniques and values and the calculated values using formulas for perimeter and area of the 2-D shapes in exercise 3, BLM 1.2

Student	Unsatisfactory	Fair	Good	Excellent	Criteria
					Excellent (Enjoys a high degree of success) - able to name all shapes in #3. - estimates of perimeter and area of shapes in #3 are reasonable using a reasonable strategy.
					Good (Is quite competent with minor feedback) - able to name all shapes in #3. - estimates of perimeter and area of most of the shapes in #3 are reasonable using a reasonable strategy. - knows the formulas for the area of two of triangles, parallelograms and rectangles. - calculates the areas accurately. - uses correct measurement units. - able to draw most of the shapes requested in #5.
					Fair (Is not fully competent; requires more feedback) - able to name most of the shapes in #3. - estimates of perimeter and area of some of the shapes in #3 are reasonable. - knows the formulas for the area of one of triangles, parallelograms and rectangles. - calculates most of the areas accurately. - does not use correct measurement units. - able to draw some of the shapes requested in #5.
					Unsatisfactory (Is not achieving; needs extra support) - exhibits little effort, or - unable to name most of the shapes in #3. - cannot estimate perimeter and area of the shapes in #3. - does not know any of the area formulas. - cannot calculate the areas given the formulas. - does not use correct measurement units. - unable to draw any of the shapes requested in #5.

Day 2: How to Trap a Zoid with *The Geometer's Sketchpad 4* ® — Grade 7

Description
- Explore and/or review the use of *The Geometer's Sketchpad 4* ® for constructing points, segments and shapes properly.
- Practise drawing and measuring trapezoids using *The Geometer's Sketchpad 4*®.

Materials
- *GSP 4*®
- BLM 2.1

Assessment Opportunities

Minds On…

Small Group → Brainstorm
Ask: "What are the similarities and differences between using a computer to explore geometry and pencil-and-paper work?" Students work in groups to design a Venn diagram to show relationships.

Whole Class → Sharing
Groups share their brainstorming ideas from the previous activity with the entire class.

Action!

Pairs → Guided Exploration
Conduct a teacher-led walkthrough of various functions of *The Geometer's Sketchpad 4*® (BLM 2.1). Students take turns with one student focusing on the instructions and the other using the program.

Students save their completed trapezoids for use on Day 4.

Learning Skills/Observation/Anecdotal: Observe students' ability to work independently and co-operatively throughout the activity.

Consolidate Debrief

Whole Class → Sharing
Lead a discussion based on the students' experience with *The Geometer's Sketchpad 4*®.

- How did using *The Geometer's Sketchpad 4*® help you develop your understanding of trapezoids and/or computers?
- Did you have any issues or challenges with the program?
- What would you like to learn more about?
- What kinds of applications do you think a program like this could be useful for?
- How did you answer Questions 30 and 31 (BLM 2.1)?
- How could you use *The Geometer's Sketchpad 4* ® to construct a parallelogram that would stay a parallelogram when its points are dragged?

Reflection

Home Activity or Further Classroom Consolidation
- Answer the following questions in your math journals:
- How does this program help me to understand geometry better?
- What would I like to explore further?

  OR
- How could this program be useful to me?

2.1: Introduction to *The Geometer's Sketchpad 4*®
Using Trapezoids

Name:
Date:

Getting Started

1. Launch *The Geometer's Sketchpad 4*®.
2. Click the mouse anywhere to close the introductory window.
3. Maximise both of the geometry windows.
4. Notice the six tools at the left of the working area. The second one down is the **Point Tool**. Click on it, then click in four different places in the work area to make four points.
5. The fourth tool down is the **Segment Tool**. Click on it then connect the four points with segments to form a quadrilateral.
6. The first tool is the **Selection Arrow Tool**. Click on it then drag the points and segments to move them around. Try to make your quadrilateral look like a trapezoid.

Follow the directions below to construct a new trapezoid. Once created, the parallel sides of the trapezoid will remain parallel regardless of how you drag the points or segments.

Constructing a Real Trapezoid

7. Select **New Sketch** from the File menu.
8. Construct two points and the segment between them.
9. Construct a third point not on the segment.
10. Using the **Selection Arrow Tool**, select the segment and the third point by clicking on them. They are highlighted in pink. The original two points should not be selected.
11. From the **Construct** menu, select **Parallel Line**. You now have a line constructed and automatically selected.
12. From the **Construct** menu, select **Point on Parallel Line**. This creates a highlighted point which is forced to always stay on the parallel line.
13. Click the background to deselect the new point.
14. Select only the newly-constructed parallel line and select **Hide Parallel Line** from the **Display** menu.

15 Use the **Selection Arrow Tool** to drag the new point around. Notice that you can't drag it off the hidden line.

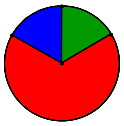
16. Construct three more segments to finish the trapezoid.
17. Use the **Selection Arrow Tool** to drag the vertices (points) and segments of the trapezoid. Note that however you drag each point or segment, the two parallel lines always stay parallel.
18. Drag points and/or segments to make your trapezoid look like:
    a) an isosceles trapezoid   b) a parallelogram
    c) a rectangle   d) a rectangle joined to a right triangle

2.1: Introduction to *The Geometer's Sketchpad 4®*
Using Trapezoids (continued)

Measuring Your Trapezoid

19. Use the trapezoid you created earlier in this investigation.
20. Click the background to de-select everything.
21. Using the **Selection Arrow Tool**, select the four points of your trapezoid in a clockwise or counter-clockwise direction.
22. From the **Construct** menu, select **Quadrilateral Interior**. Notice that the inside of the trapezoid becomes coloured and shaded.
23. From the **Measure** menu, select **Perimeter**. Notice that the perimeter is shown in the upper left corner of the working area.
24. From the **Edit** menu, select **Preferences**. On the **Units** tab, set the **Distance Units** to **cm** and all **Precision** levels to **tenths**. Click **OK**.
25. Drag the points of the trapezoid to adjust its perimeter to:
    a) 25.0 cm.
    b) 40.0 cm.
26. Click the background to de-select everything. Click inside the trapezoid to select it.
27. From the **Measure** menu, select **Area**.
28. Drag the points of the trapezoid to adjust its area to:
    a) 25.0 cm^2.
    b) 40.0 cm^2.
29. Can your create a trapezoid with a:
    a) perimeter of 25.0 cm and an area of 40.0 cm^2?
    b) perimeter of 40.0 cm and an area of 25.0 cm^2?
30. When you drag one of the first three points that you originally created, another point always gets dragged along with it. Explain why this happens.

31. When you drag the fourth point, it moves by itself. Explain why it acts differently than the other points.



Description

- Understand that a trapezoid can have zero or two (but not one, three or four) right angles.
- Develop the formula for the area of a trapezoid containing two right angles.

Materials

- cm grid paper
- scissors
- BLM 3.1, 3.2

Assessment Opportunities

Minds On…

Whole Class → Guided Discussion

Conduct a brief discussion about types of taxes, e.g., property taxes, GST, PST, income tax.

Whole Class → Shared Reading

Consider the “Special Tax” (BLM 3.1) as a class. Prompt students’ thinking by asking probing questions, such as:

- How can you recognise a right angle in a 2-D shape?
- Why did the mathematicians reshape their lots?
- What were the shapes of the lots before and after the tax?
- How many right angles did each lot have?
- Why did the mathematicians want to keep the areas of their lots unchanged?
- Do you think the mathematicians were justified in changing the shapes of their lots? Explain why or why not.

BLM 3.1 and 3.2 are from *Impact Math – Measurement, p. 32, 33*

Action!

Pairs → Problem Solving

In pairs, students complete Questions 1 and 2 in their notebooks. Distribute grid paper and scissors to each pair. Students explain any relationship they found between the length of a lot before the tax and the sum of the lengths of the parallel sides after tax. Prompt them to explain how to use this relationship to calculate the area of a trapezoid, containing two right angles.

Individual → Guided Exploration

Students complete Questions 3 and 4 individually. Students should discover that a line segment drawn through the midpoint of the boundary between A and B divides it into two trapezoids with the same areas as rectangles A and B. Help students who experience difficulty by suggesting that they fold their rectangle in half along a line parallel to its length. The point where the fold intersects the boundary between rectangles A and B is the point through which any line segment joining opposite sides can be drawn to yield the desired result.

Curriculum Expectations/Performance Task/Scoring Guide: Assess students’ demonstration of learning.

For assistance with the implementation of this activity, see p. 34-37 in *Impact Math – Measurement,* containing solutions, ideas, and exemplars.

To find the area of a trapezoid containing two right angles, divide the sum of the lengths of the parallel sides by 2 (finding their average) then multiply by the distance between them.

Consolidate Debrief

Whole Class→ Discussion of Findings

Facilitate student discussions of their findings for Questions 3 and 4, emphasizing that they understand that there are many ways to transform a rectangle into a trapezoid of the same area. Point out that trapezoids can have either zero or two right angles.

Exploration
Reflection

Home Activity or Further Classroom Consolidation

In your math journal explain how to find the area of any right-angled trapezoid. Include an example. Develop a formula for the area of a right-angled trapezoid given the lengths of its parallel sides and the distance between them.

3.1: The Mathematicians Transform Rectangles into Trapezoids

(*Impact Math – Measurement, Activity 2)*

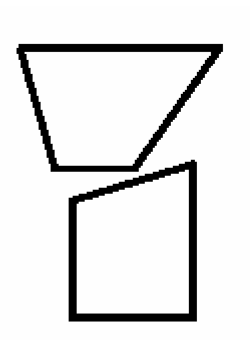
Gulliver observed, with some contempt, that the mathematicians seemed to avoid practical matters. They built their homes without right angles and located their houses on odd-shaped lots. Gulliver was apparently unaware of the reasons why the mathematicians constructed their buildings (and their lots) in unsymmetrical shapes. Legend tells how the king, in his attempt to raise more revenue from his people, levied a special tax on any lot that contained more than two right angles. Two mathematicians, Alpha and Beta, with adjoining rectangular lots, reshaped their lots as shown, to avoid this special tax.

Gulliver proclaimed:

> "These mathematicians are under continual stress, never enjoying a minute's peace of mind, for they are always working on some problem that is of no interest or use to the rest of us. Their houses are very ill built, the walls bevil, without one right angle in any apartment; and this defect ariseth from the contempt they bear for practical geometry. They despise it as vulgar and impure…Although they can use mathematical tools like ruler, compasses, pencil, and paper, they are clumsy and awkward in the common actions and behaviours of life."

By reconstructing their lots as shown on page 32, the mathematicians Alpha and Beta changed each rectangular lot into a *trapezoid*.

1. a) The diagram below shows two trapezoids. Write a sentence to define a trapezoid. Check your definition with a dictionary.
   b) How many right angles has each trapezoid shown here? Do all trapezoids have the same number of right angles? Explain.
   c) Did Alpha and Beta have to pay the special tax on their new lots? Explain.



3.1: The Mathematicians Transform Rectangles into Trapezoids (continued)

2. a) Measure the length and width in millimetres of Alpha's and Beta's lots **before** the special tax was imposed. Record in the table on the left.

Before Tax			
	Length	Width	Area
Alpha			
Beta			

After Tax			
	Sum of the Lengths of the Parallel sides	Distance between the Parallel sides	Area
Alpha			
Beta			

   b) Trace and cut out both lots as they were after the special tax. Place your cut outs on centimetre paper to determine the area of each lot and the lengths of the parallel sides. Record in the table on the right.

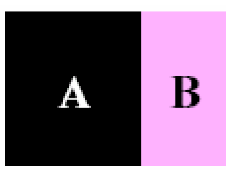
   c) Did Alpha and Beta change the areas of the lots when they reshaped them? Explain.

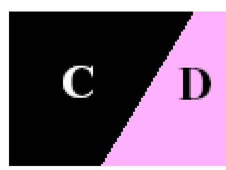
   d) Compare the length of Alpha's rectangular lot to the sum of the lengths of the parallel sides of Alpha's trapezoidal lot. Repeat for Beta's lot. Describe what you discover.

   e) Explain how to calculate the area of a trapezoid containing a right angle, given the lengths of its parallel sides and distance between them.

3. a) Draw two rectangles of length 9 cm and width 6 cm on centimetre paper. Divide one of the rectangles into two rectangles A and B with dimensions 5 cm × 6 cm and 4 cm × 6 cm.

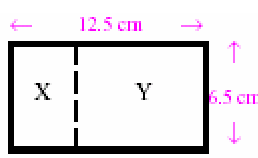
   b) Use what you learned in Exercise 2 to divide the other rectangle into trapezoids C and D so the areas of A and C are the same and the areas of B and D are the same. Explain how you did this. How many ways do you think this can be done?





4. a) Draw a 12.5 cm × 6.5 cm rectangle on a sheet of paper. Divide your rectangle into two other rectangles X and Y and record their areas. Cut out your rectangle and divide it into two trapezoids so that one has the same area as X and the other the same area as Y.

   b) Measure the dimensions of each trapezoid and calculate its area as in 2b. Record the areas of the trapezoids and verify that they are equal to the areas of X and Y.



3.2: Scoring Guide

(*Impact Math – Measurement, p. 35*)

Scoring Guide for Activity 2				
	Level 1	**Level 2**	**Level 3**	**Level 4**
PROBLEM SOLVING				
Selection of an Appropriate Strategy for Constructing a Trapezoid with The Same Area as a Given Rectangle (exercises ❸ & ❹) M 7-6, M 7-7, M 7-8, M 7-9	• unable to describe an appropriate strategy for changing a rectangle into a right trapezoid of the same area.	• describes an appropriate strategy for changing a rectangle into a right trapezoid of the same area. • applies the strategy with minor errors to a given rectangle.	• describes an appropriate strategy for changing a rectangle into a right trapezoid of the same area. • applies the strategy correctly to a given rectangle.	In addition to Level 3: • indicates that there are an infinite number of ways of changing a rectangle into a right trapezoid of the same area.
CONCEPTS				
Understands How To Determine the Area of a Right Trapezoid (exercise ❷) M 7-4, M 7-6, M 7-7, M 7-8, M 7-9.	• The response to Exercise ❷ includes fewer than two of the following elements: • completed tables with measurements correct to within 2 mm and no computational errors. • statement that the areas of the lots did not change. • discovery that the sum of the lengths of the parallel sides after tax is double the length before tax. • appropriate procedure or formula is given for calculating the area of a right trapezoid.	• The response to Exercise ❷ includes two of the following elements: • completed tables with measurements correct to within 2 mm and no computational errors. • statement that the areas of the lots did not change. • discovery that the sum of the lengths of the parallel sides after tax is double the length before tax. • appropriate procedure or formula is given for calculating the area of a right trapezoid.	• The response to Exercise ❷ includes three of the following elements: • completed tables with measurements correct to within 2 mm and no computational errors. • statement that the areas of the lots did not change. • discovery that the sum of the lengths of the parallel sides after tax is double the length before tax. • appropriate procedure or formula is given for calculating the area of a right trapezoid.	• The response to Exercise ❷ includes all of the following elements: • completed tables with measurements correct to within 2 mm and no computational errors. • statement that the areas of the lots did not change. • discovery that the sum of the lengths of the parallel sides after tax is double the length before tax. • appropriate procedure or formula is given for calculating the area of a right trapezoid.

Description

- Develop a formula to calculate the area of any trapezoid.

Materials

- BLM 4.1, 4.2.1, 4.2.2

Assessment Opportunities

Minds On…

Whole Class → Shared Reading

Students follow along as the teacher reads the story and poem on BLM 4.1. Discuss the possible solutions to Question 1, using the following questions:

- Mathematically, what is the meaning for “mean?”
- Why does the tax appraiser use the “mean parallel side?”
- For which other figures is area calculated using base and height?

BLM 4.1 and 4.2.2 are from *Impact Math – Measurement*

Review measures of central tendency: mean, median, mode; especially “mean.”

Action!

Option 1:

Pairs → Guided Exploration

Students use their trapezoid file for *The Geometer's Sketchpad 4®* from Day 2. Guide students, in pairs, as they work through BLM 4.2.1.

Option 2:

Pairs → Exploration

Pairs work through Questions 2, 3 and 4.

Curriculum Expectations/Question and Answer/Rating Scale: Students will hand in completed answers to BLM 4.2.2 at the end of the period.

Curriculum Expectations/Question and Answer/Rating Scale: Assess students' understanding using their completed work.

Option 1 is based in the computer lab using BLM 4.2.1, while Option 2 can take place in the classroom using BLM 4.2.2.

Reference *GSP 4®* files.

Consolidate Debrief

Whole Class → Reflection

Facilitate discussion as students reflect on the day's activities. Students share formulas. Stress similarities and develop a common formula. Reach a consensus that the formula for the area of a trapezoid could be: the average (mean) of the lengths of the two parallel sides times the distance between them.

Note: Formulas may look different but actually have the same value. Understanding of number sense, algebra, order of operations, can be used to confirm equivalence of formulas that appear to be different.

Reflection

Home Activity or Further Classroom Consolidation

Explain to someone one or two strategies for remembering the formula for the area of a trapezoid. Record any questions or discussion items raised during your conversation.

4.1: The King Moves from Angles to Area

(*Impact Math – Measurement, Activity 3*)

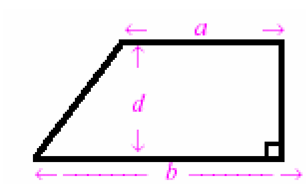
… the king levied a special tax on lots with more than two right angles. In response, the mathematicians reshaped their rectangular lots into trapezoids of the same area. In this way they preserved the size of each lot and escaped the new tax. The king was not amused, and sent his tax appraiser to announce new tax measures.

The king is quite clearly annoyed,
For the taxes you tried to avoid.
By changing your lots
From rectangular plots,
To cleverly-shaped trapezoids.

So the king ordered me to advise
That he will tax each lot by its size;
For he doesn't care
Trapezoid or square,
The area only applies.

Your tax appraiser's no fool,
He calculates fast without tools.
Mean parallel side
Times measurement wide
Is his trapezoid area rule.

1. a) How did the king revise the special tax provision so that taxes would not depend on the shape of the lot?
   b) What does the tax appraiser mean by "mean parallel side?" by "measurement wide?"
   c) Describe in your own words how the tax appraiser calculates the area of a trapezoid.
   d) Write as a formula the tax appraiser's rule for calculating the area of a trapezoid that has parallel sides of length *a* and *b* if the distance between these sides is *d* units. Do you think this formula works for a trapezoid that has no right angles? Give a reason for your answer.



4.2.1: Developing a Formula for the Area of Trapezoids Using *The Geometer's Sketchpad 4*®

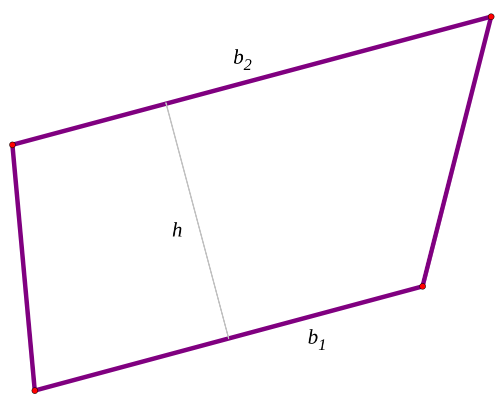
Name:
Date:

What Do Two Trapezoids make?

1. Launch *The Geometer's Sketchpad 4*®.
2. Open the file containing the trapezoid you created in Day 2 of this unit.
3. Select any side of the trapezoid. From the **Display** menu, choose **Color** and pick a colour for that side. De-select the side. Colour each of the other three sides of the trapezoid differently.
4. Select one of the non-parallel sides of the trapezoid. From the **Construct** menu, choose **Midpoint**.
5. With this midpoint selected, choose **Mark Center** from the **Transform** menu (or simply double-click on the midpoint).
6. Use **Select All** from the **Edit** menu. Choose **Rotate** from the **Transform** menu. The angle to rotate the trapezoid is 180º.
7. You have now constructed an exact, congruent copy of the trapezoid. By matching colours, notice to which position each of the original segments was rotated.
8. What type is the resulting shape?
   Test your answer by dragging various points and noting if the type of shape remains the same or changes to a different type.
9. Select all of the vertices (corner points) of the original trapezoid. From the **Construct** menu, choose **Quadrilateral Interior**. Use the **Measure** menu to find its area.
10. Repeat step 9 to find the area of the entire figure.
11. What is the relationship between these two areas? Why does this make sense?

12. Label the two parallel sides b_1 and b_2. Write a formula for the area of the whole shape, in terms of h, b_1 and b_2, where h is the distance between the two parallel sides.

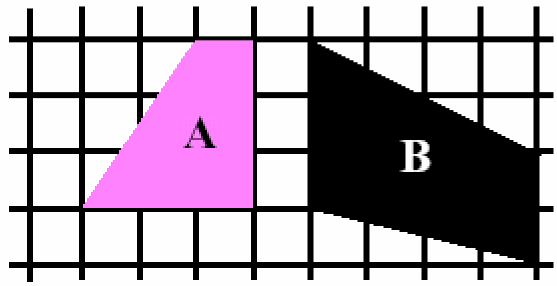
13. Using information from 11 and 12 above, write a formula for the area of the original trapezoid, in terms of h, b_1 and b_2.



4.2.2: The King Moves from Angles to Area (continued)

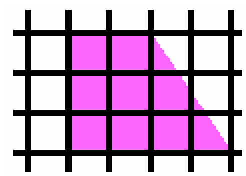
(*Impact Math – Measurement, Activity 3*)

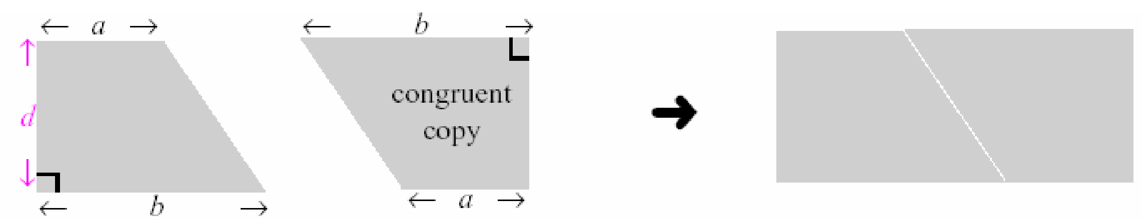
2. a) Is the tax appraiser's rule for calculating the area of a trapezoid the same as the formula you discovered in Activity 2? Explain your answer.
   b) Use the tax appraiser's rule to calculate the areas of the trapezoids drawn on this centimetre grid.



3. a) Draw a line segment to divide trapezoid A in Exercise 2 into a right triangle and a rectangle. Calculate the areas of the rectangle and triangle to find the area of trapezoid A. Compare with your answer in 2b.
   b) Divide trapezoid B in Exercise 2 into two triangles. Then use the formula for the area of a triangle to calculate the area of trapezoid B. Compare with your answer in Exercise 2b.

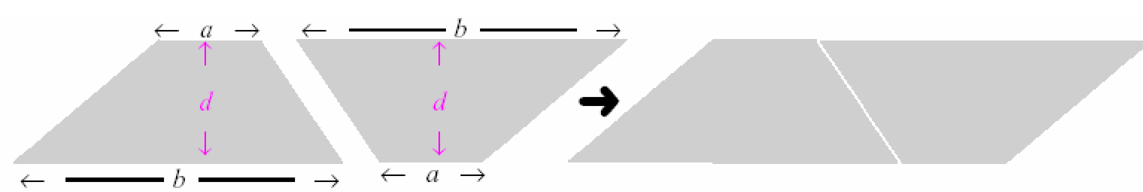
4. a) Draw a trapezoid like the one on the right on centimetre paper and count squares to determine its area. Draw another trapezoid congruent to it. Cut out both trapezoids and fit them together to form a rectangle.
   b) Record the area of the rectangle and the area of each trapezoid in 4a.
   c) A congruent copy of the trapezoid below is made and they are fitted together to form a rectangle as shown.





Write an expression for the area of the rectangle and for the area of each trapezoid in terms of *a*, *b*, and *d*.

   d) A congruent copy of the trapezoid below is made and they are fitted together to form a parallelogram as shown.



Challenge:

Write an expression for the area of the parallelogram and for the area of the trapezoid in terms of *a*, *b*, and *d*. Show your work.

4.2.2: The King Moves from Angles to Area (continued)

(*Impact Math – Measurement*)

ACTIVITY 4 – STUDENT PAGE

❶ a) Complete the tables on the other page, recording each centimetre as a Laputian unit.

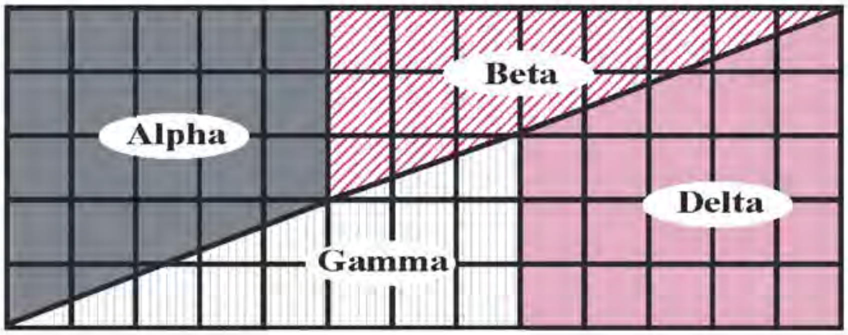
b) What do you notice about the areas of triangles Beta and Gamma?

c) Are Beta and Gamma congruent triangles? Explain why or why not.

d) Are trapezoids Alpha and Delta congruent? Explain why or why not.

e) Add the areas in your table to find the total area of all four lots.

After the tax appraiser computed the areas of the four lots, the mathematicians rearranged their lots on the building plans as shown below.



❷ a) What is the total area of this rectangle?

b) Compare your answers in ❶ e and ❷ a and explain why the tax appraiser became confused.

❸ a) Using centimetre paper, cut out lots Alpha, Beta, Gamma, and Delta with the dimensions given in your tables. Show they have a total area of 64 cm² by arranging them in an 8 cm × 8 cm square.

b) Arrange these lots in a rectangle of length 13 cm and width 5 cm. What is the total area of the lots? Where did the extra unit of area come from?

REPORT

Write a letter on a scroll to the king indicating whether the total area taxed should be 64 square units or 65 square units. Give reasons to support your argument.

Indicate what percent of the total tax should be assigned to each of the four lots.

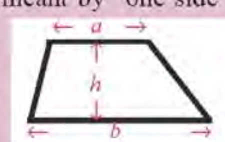
Research:
How is land taxed in your municipality?
By area? By frontage?
By market value?

CHALLENGE

TAX APPRAISER'S LIMERICK

The trapezoid rule 'tis true,
Applies to other shapes too.
Triangle's trapezoid
With one side that's void,
And parallelogram follows the rule.

The tax appraiser's limerick suggests that the formula for the area of a trapezoid applies to triangles and parallelograms. Explain what is meant by "one side that's void." Show how the formula for the area of a trapezoid becomes a) $b \times h/2$ as side length a gets close to 0.
b) $b \times h$ when $a = b$.



Day 5: Applying Trapezoid Knowledge — Grade 7

Description
Apply the trapezoid area calculation as a culminating activity.

Materials
- BLM 5.1, 5.2, 5.3

Assessment Opportunities

Minds On…

Whole Class → Sharing
Student volunteers share their journal entries from the previous day. Students help answer some of the questions posed during the conversation.

Whole Class → Review
Briefly review concepts discussed on the top half of the first page.

BLM 5.1 is from *Impact Math – Measurement*

Action!

Individual → Performance Task
Students complete Questions 1, 2 and 3, including the report, on BLM 5.1. Students complete BLM 5.2. Circulate to ensure students stay on task, and to clarify task requirements.

Curriculum Expectations/Performance Task/Rubric: Evaluate students' problem solving, communication, and ability to make connections (BLM 5.3).

BLM 5.2 is from TIPS - Developing Mathematical Processes – Grade 7

Some time during Action!, you may wish to incorporate brain gym activities such as stretching and blood-flow techniques.

Consolidate Debrief

Whole Class → Sharing
- What did students find difficult?
- What was straightforward?
- How can students improve upon what they did today?

Reflection
Exploration

Home Activity or Further Classroom Consolidation
Record places in your home environment where trapezoids occur. Answer the following questions in your math journal:
- Why are trapezoids common?
- Where do you find trapezoids in your home?

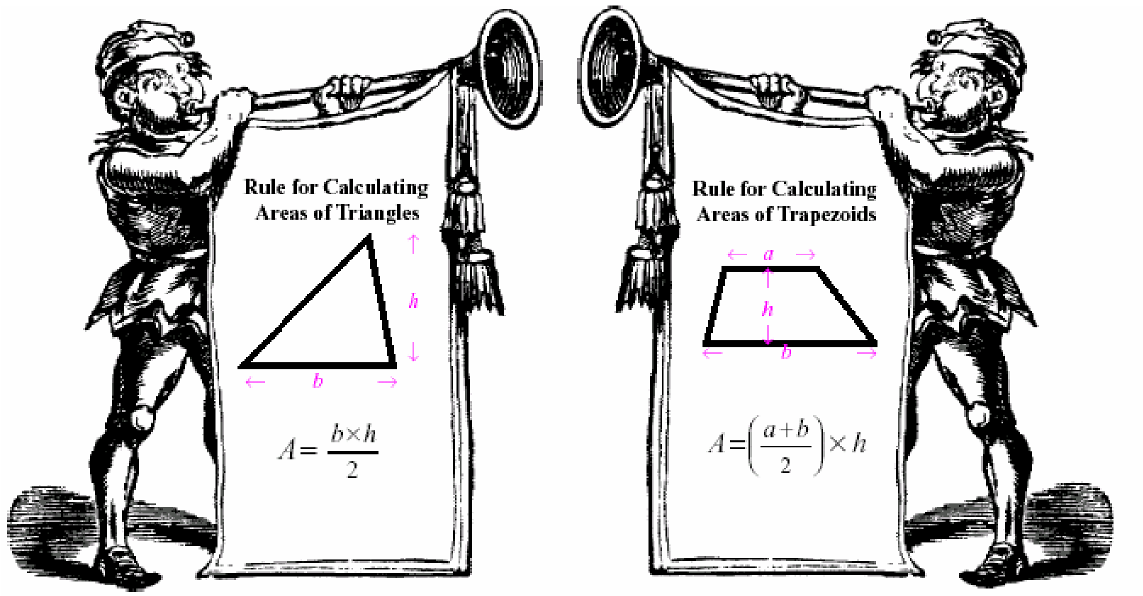
Example of trapezoids: tiles near the edge of angled walls or the area between the rungs of a kitchen chair or other furniture with splayed legs.

Having students identify trapezoids in tiling patterns can serve as an effective segue to the following section.

5.1: Is It Mathematics or Magic?

(*Impact Math – Measurement, Activity 4*)

We learned in Activity 3 that the tax appraiser in Laputa was very good at calculating areas. He was particularly proud of his rules for calculating the areas of triangles and trapezoids.

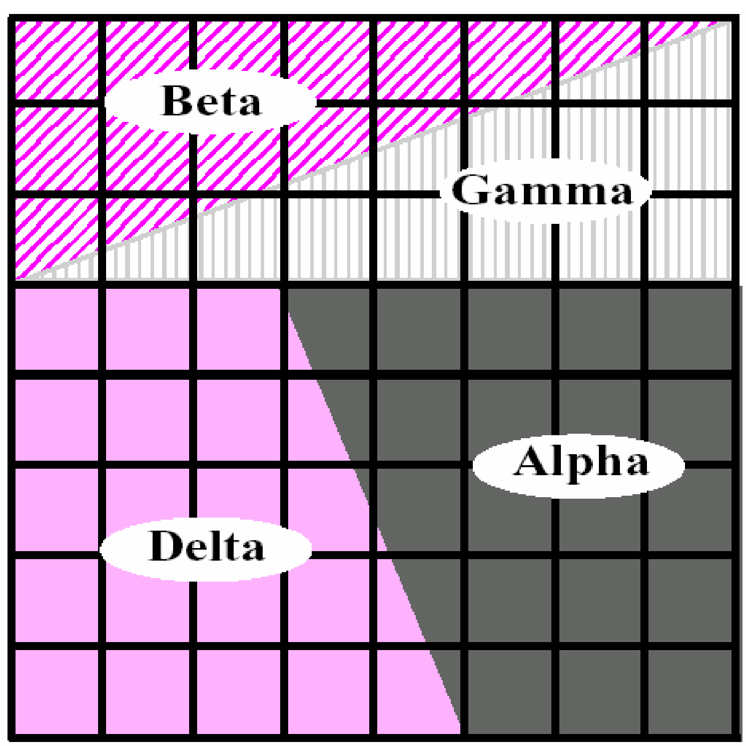


Knowing the tax appraiser's eagerness to apply these rules, the mathematicians Alpha, Beta, Gamma, and Delta constructed their lots as shown here. Each centimetre on the grid stands for a Laputian distance unit.

The tax appraiser recorded the dimensions of each lot in tables like these.

Triangles			
	Base	Height	Area
Beta			
Gamma			

Trapezoids			
	Lengths of Parallel Sides	Distance between Parallel Sides	Area in Square Units
Alpha			
Delta			



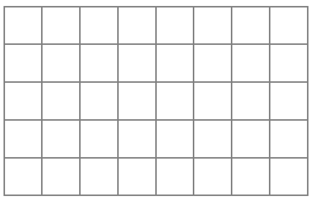
5.2: Application of Trapezoid Area and Perimeter Grade 7

Name:
Date:

1. Reasoning and Proving

Alpha was planning to fence in his pet monkey's play area. He has 16 m of fencing and the area of his trapezoidal garden is 12 m^2.

Draw the shape of the trapezoidal monkey play area. Include all necessary dimensions.



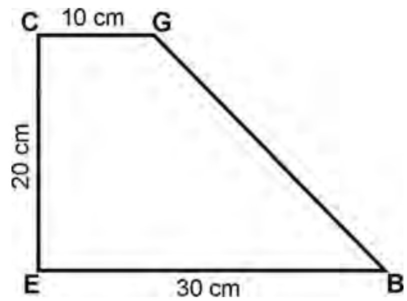
Scale: one grid unit = 1 m

Hint: $A = \dfrac{(a+b)h}{2}$

2. Reasoning and Proving

In order to please the king, Beta baked a cake for him. The king would like to share the cake equally with the queen.

Show where he should make the cut(s). *Explain* your answer.



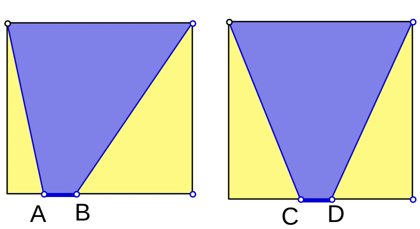
3. Communicating

Gamma has been hired to make ceramic floor tiles for the queen's palace.

Note: The square tiles that are shown are the same size.

AB and CD have the same length.

Explain how Gamma could use the formula for the area of a trapezoid to convince the queen that the inside dark areas are the same size.



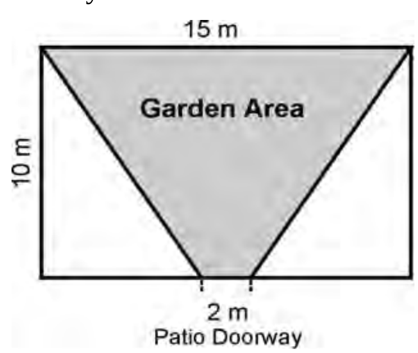
Hint: $A = \dfrac{(a+b)h}{2}$

4. Making Connections

Delta's backyard is rectangular. Its dimensions are 15.0 m by 10.0 m.

Delta's family is making a garden from the patio doors to the corners at the back of the yard. The patio doors are 2.0 m wide. Determine the area of the garden.

Show your work.



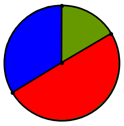
5.3: Performance Task – Evaluation Rubric **Grade 7**

Criteria	Level 1	Level 2	Level 3	Level 4
Problem Solving	- shows limited awareness that there is a paradox in 5.1 - limited awareness that at least one of the lots in 5.1 has increased in size	- is unaware that at least one of the lots in 5.1 has increased in area but otherwise applies problem-solving strategies appropriately - applies some area concepts appropriately	- is aware that at least one of the lots in 5.1 has increased in area - identifies 65 square units as the area on which the total tax of the rearranged lots should be calculated	- is aware that at least one of the lots in 5.1 has increased in area - is able to calculate the correct proportions, 30.77%, 19.23%, 19.23%, and 30.77% of the 65 square units on which tax is to be paid
Making Connections	- shows limited connection of concepts learned to everyday situations	- shows some connection of concepts learned to everyday situations	- connects concepts learned to everyday situations	- demonstrates solid connections to everyday situations
Reasoning and Proving	- explanations of answers are limited - uses limited proper terminology when explaining answers	- demonstrates some competency in explaining answers - uses some proper terminology when explaining answers	- demonstrates competency in explaining answers - uses proper terminology when explaining answers	- demonstrates extensive competency in explaining answers - uses very clear and highly accurate terminology when explaining answers

Lesson Outline: Days 18 – 29 **Grade 7**

BIG PICTURE			
Students will: • develop an understanding of integers (representation, ordering, addition and subtraction); • develop strategies to add integers with and without the use of manipulatives; • develop strategies to subtract integers with the use of manipulatives • recognize the use of integers in everyday life.			
Day	**Lesson Title**	**Description**	**Expectations**
18	What Are Integers?	• Investigate the concept of integers. • Investigate where integers appear in our daily lives.	7m1, 7m12, 7m24, 7m25 CGE 4e, 5e
19	The Zero Principle	• Show that (-5) and (+5) are opposites, and balance each other out when added. • Investigate representing integers with integer tiles. • Recognise that "zero" may be represented as an equal number of positive and negative tiles (e.g., one positive and one negative). • Represent any integer in multiple ways.	7m1, 7m12, 7m21, 7m24 CGE 2a, 4a
20	All Integers Come to Order	• Use proper integer notation (positive/negative, brackets). • Order integers on an integer line.	7m1, 7m7, 7m10, 7m25 CGE 2c, 5a, 5e, 5d
21	Add Some Colour	• Explore the concept of integer addition using integer tiles. • Apply the zero principle to integer addition. • Use proper integer notation (positive/negative, brackets).	7m1, 7m7, 7m10, 7m21, 7m22, 7m23, 7m25 CGE 5a, 4f
22	What's Right About Adding and What's Left to Count	• Consolidate and practise integer addition with integer tiles. • Investigate integer addition using number lines. • Compare the two methods for addition of integers.	7m1, 7m7, 7m10, 7m21, 7m22, 7m24, 7m25 CGE 3c, 5e
23	Adding On!	• Demonstrate integer addition using integer tiles, number lines and symbols. • Investigate mean, median and mode using an integer context.	7m1, 7m7, 7m8, 7m10, 7m16, 7m21, 7m22, 7m81, 7m81, 7m101 CGE 2c, 5a, 5e, 5g
24	Carousel Performance Activity	• Demonstrate individual understanding of the concepts presented so far in this unit.	7m1, 7m7, 7m8, 7m10, 7m16, 7m21, 7m22 CGE 3c, 4e

Day	Lesson Title	Description	Expectations
25	What's the Difference?	• Investigate how subtraction is related to addition. • Use integer tiles to model integer subtraction.	7m1, 7m7, 7m10, 7m21, 7m22, 7m23 CGE 4b, 5e
26	Which Way to Integerland?	• Investigate subtraction of integers using number lines. • Compare subtraction on a number line with subtraction using integer tiles.	7m1, 7m7, 7m10, 7m21, 7m22, 7m24 CGE 4e, 5a
27	I win!	• Practise integer addition and subtraction using a game.	7m1, 7m7, 7m8, 7m10, 7m16, 7m21, 7m22 CGE 3c, 4a
28	Summative Assessment Task – Part 1	• Complete exercises encompassing the knowledge and skills developed in this unit.	7m1, 7m7, 7m8, 7m10, 7m16, 7m21, 7m22 CGE 2b, 3c, 4e, 4f
29	Summative Assessment Task – Part 2	• Complete exercises encompassing the knowledge and skills developed in this unit.	7m1, 7m7, 7m8, 7m10, 7m16, 7m21, 7m22, CGE 2b, 3c, 4e, 4f



Description
- Investigate the concept of integers.
- Investigate where integers appear in our daily lives.

Materials
- overhead projector
- BLM 18.1, 18.2

Assessment Opportunities

Minds On…

Small Groups → Brainstorm
Students suggest where numbers below zero are used, e.g., temperature, golf, above/below sea level, football, stock market, banking, hockey statistics, etc.

Present students with an integer, either positive or negative, and discuss possible meanings of that number. For example, what does it mean if a person's bank account is -\$50.00? If a person were at -5 m in the ocean, what would this mean? Could she breathe? If it were +30ºC, would a jacket be needed?

Placemat (TIP 9) or "Pass the Paper" are effective strategies.

Action!

Whole Class → Investigation
On the overhead projector, display the temperature chart for Canadian cities (BLM 18.1). Ask questions such as: What information does the chart show? Which city had the highest temperature? Which city had the lowest? Which city had the greatest temperature difference? How do you know? Which city had the smallest temperature difference? Explain. Which city had a change in temperature of 50 degrees? How did you figure that out?

Pairs → Activity
Clarify what the +/- ratings on the hockey roster mean. The ratings reflect the season up to the last game played. The + means that a player has been on the ice when more goals were scored for/by the team than against the team. A - means that a player has been on the ice when more goals were scored against the team than for/by the team.

The students are given the team roster with only the +/- rating from one NHL club. Using the data, each pair is required to order the players according to their +/- score (on a number line).

Curriculum Expectations/Observation/Mental Note: Observe students as they complete the activity, watching for understanding of positive and negative integers.

In a game of team A vs. B with the score at 2:1 for team A, an A player who was on the ice for both goals would have +2. But if he were also on the ice when team B scored, his rating would drop to +1 (+2 + (–1)).

Consolidate Debrief

Whole Class → Discussion
Facilitate discussion of the hockey roster activity. Students justify their arguments with guidance from the teacher. Pose the following questions: Your team is trailing by one goal and you have pulled your goaltender. Which six players would you put on the ice? Justify your decision using the +/- rating (and the number line you created). How did the number line help? What does a +/- of zero mean?

Individual → Response Journal
Provide sentence stems that allow students to summarize today's activities, e.g., I learned…, I discovered… Encourage a definition for integers.

Application
Exploration

Home Activity or Further Classroom Consolidation
1. Explore these topics at the school or community library or at a website:
   Find the elevations of the following locations:
   a) Your city or town b) Mount Everest
   c) Grand Canyon d) Amsterdam
   Order them and determine the differences in heights.

   Visit the Environment Canada website (www.ec.gc.ca) to find the highest and lowest temperatures recorded in Canada. Record the date and the community in which these events occurred. Find the highest and lowest temperature recorded for your community. How do they compare?

OR

2. Use the data on worksheet 18.1, to locate the cities and attempt to explain the various results in relation to the geographic location.

Refer to information gathered on BLM 1.3.

Cross-curricular opportunity TIP 8.

18.1: Temperature Extremes in Canadian Cities

April	Extreme high	Extreme low	Change
Calgary	29	-30	
Edmonton	31	-28	
Halifax	25	-13	
Iqualuit	7	-34	
Montreal	31	-15	
Ottawa	31	-17	
Regina	33	-6	
St. John’s	29	-15	
Toronto	31	-17	
Vancouver	25	-3	
Whitehorse	21	-29	
Winnipeg	34	-26	
Yellowknife	20	-41	

18.2: Hockey Statistics (from ESPN)

Name:
Date:

Toronto Player	+ / -
Aki Berg, D	-1
Alexander Mogilny, RW	9
Alexei Ponikarovsky, LW	14
Brian Leetch, D	11
Bryan Marchment	4
Bryan McCabe, D	22
Calle Johansson, D	5
Chad Kilger, LW	2
Darcy Tucker, RW	4
Drake Berehowsky	5
Gary Roberts, LW	9
Joe Nieuwendyk, C	7
Karel Pilar, D	2
Ken Klee, D	-1
Mats Sundin, C	11
Matthew Stajan, C	7
Mikael Renberg, RW	-1
Nathan Perrott, RW	-1
Nik Antropov, C	7
Owen Nolan, RW	4
Ric Jackman, D	-11
Robert Reichel, C	2
Ron Francis, C	3
Tie Domi, RW	-2
Tom Fitzgerald, RW	-2
Tomas Kaberle, D	16
Wade Belak, RW	0

Ottawa Player	+ / -
Antoine Vermette, LW	5
Anton Volchenkov, D	1
Brian Pothier, D	6
Bryan Smolinski, C	22
Chris Neil, RW	13
Chris Phillips, D	15
Curtis Leschyshyn, D	13
Daniel Alfredsson, RW	12
Greg de Vries, D	0
Jason Spezza, C	22
Josh Langfeld, RW	6
Karel Rachunek, D	17
Marian Hossa, RW	4
Martin Havlat, RW	12
Mike Fisher, C	-3
Peter Bondra, RW	1
Peter Schaefer, LW	22
Petr Schastlivy, LW	-1
Radek Bonk, C	2
Shane Hnidy, D	2
Shaun Van Allen, C	6
Todd Simpson, D	-1
Todd White, C	12
Vaclav Varada, LW	2
Wade Redden, D	21
Zdeno Chara, D	33

Montreal Player	+ / -
Alexei Kovalev, RW	-4
Andreas Dackell, RW	8
Andrei Markov, D	-2
Benoit Gratton, C	0
Chad Kilger, LW	2
Craig Rivet, D	-1
Darren Langdon, LW	-2
Donald Audette, RW	-4
Francis Bouillon, D	1
Jan Bulis, C	-8
Jason Ward, RW	3
Jim Dowd, C	6
Joe Juneau, C	-4
Marcel Hossa, LW	-3
Michael Ryder, RW	10
Mike Komisarek, D	4
Mike Ribeiro, LW	15
Niklas Sundstrom, RW	3
Patrice Brisebois, D	17
Pierre Dagenais, LW	15
Richard Zednik, RW	5
Ron Hainsey, D	3
Saku Koivu, C	-5
Sheldon Souray, D	4
Stephane Quintal, D	10
Steve Begin, C	6
Yanic Perreault, C -10	

Description

- Show that (-5) and (+5) are opposites, and balance each other out when added.
- Investigate representing integers with integer tiles.
- Recognise that "zero" may be represented as an equal number of positive and negative tiles (such as, one positive and one negative).
- Represent any integer in multiple ways.

Materials

- overhead projector
- integer tiles: double-sided, two-colour (red +, blue –) disks.

Assessment Opportunities

Minds On…

Whole Class → Guided Exploration

Show students a positive and a negative integer tile. State that they represent +1 (red) and –1 (blue).

Show 3 red tiles and ask what integer is represented. (+3) Show 4 blue tiles and ask what integer is represented. (-4) Continue until students are clear about the concepts of size and sign.

Distribute integer tiles sets to students. Pose the following questions.

1) How could we represent zero (0)?
   Students should offer multiple models using the tiles (+1 and –1, +2 and –2, etc). Recall the hockey roster activity and the meaning of "zero."
2) How would you represent three (+3)?
   Guide students toward not only 3 positive tiles, but also a combination of positives and negatives, e.g., 5 red and 2 blue. Connect solutions to the "zero principle" when it applies.

Students consider the questions posed by the teacher and share answers.

Transparent overhead integer tiles are effective for modelling.

Action!

Pairs → Investigation

Using integer tiles, students model the integers –2 and +1, three different ways each. They record their models.

Learning Skills/Observation/Mental Note: Observe students' co-operation with others as they work through this activity.

Whole Class → Discussion

Ask: How many different ways were there of representing these integers? Guide students toward the understanding that there is an infinite number of ways. Lead students to accept "equal numbers of red and blue tiles" as a model of zero.

Pairs → Activity

Students select three numbers and create three different ways to represent each number. They share their solutions and rationale with their partner. Partners check for correctness.

Consolidate Debrief

Whole Class → Discussion

Partners share one number and representation with the rest of the class. Students who had the same number with different representation can add theirs to the list.

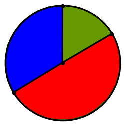
Curriculum Expectations/Observation/Mental Note: Note students' understanding of different models of zero.

Application
Exploration
Reflection

Home Activity or Further Classroom Consolidation

Brainstorm everyday instances of "opposites" which produce a result of "0." For example, in banking a balance of "0" means that deposits and withdrawals are equal.

For each scenario, identify the meaning of positive, negative and zero.



Description
- Use proper integer notation (positive/negative, brackets).
- Order integers on an integer line.

Materials
- Integer cards
- BLM 20.1
- Thermometer on overhead

Assessment Opportunities

Minds On…

Small Group → Thinking Activity
Present groups with the following two questions:
Which is greater: -5 or 2? Justify your response with an everyday example.
When can a two-digit integer be greater that a three-digit integer?

Pairs → Think/Pair/Share
Students take two minutes to think about and to record independently, the process that they will use to find the greater of two integers and share strategies with a partner. They share the strategies with the whole class.

Students should have considered all possibilities, i.e., - -, + -, + +

Action!

Whole Class → Kinaesthetic Activity
Each student is given a different integer card. Students come to the front of the room and order themselves according to the integer on their card. Use a thermometer placed horizontally, with the negative numbers to the left to demonstrate the concept visually. Pose: Which integer is closest to zero? Which side of the zero has the negative integers? Which side has the positive integers?

Students stand, one pair at a time, and hold up the integer cards. Students determine which is greater or smaller and the number of spaces from one integer to the other.

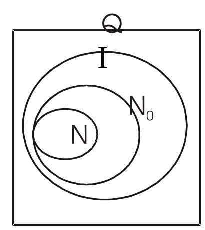
One student stands and shows their number. Ask: Who has the integer 3 less? The student who has that card stands up and the first student sits down. Ask: Who has the integer that is 2 more? And so on.

Illustrate how brackets are sometimes used around integers to keep them separate from the mathematical operations of addition and subtraction. Include a discussion of the inclusionary nature of number sets. (Rational numbers (fractions, decimals) include integers which in turn include whole numbers which in turn include natural numbers). Draw a large Venn diagram, assign different types of numbers to different students. Students show and explain where on the Venn diagram their number belongs.

[Prepare integer cards with enough consecutive integers for entire class, e.g., -15 to +15]

Some students may wish to use calculators throughout this unit. Show them how to use the +/– or the (–) key. Point out that the calculator does not explain how the answer was determined. They should use the calculator as a checking device.

Q represents Quotients, or rational numbers.



Consolidate Debrief

Whole Class → Discussion
Review the concept that integers can represent a comparison between a number and a standard or baseline. Discuss the use of integers to represent above and below, left and right, and less and more.

Discuss the appearance of numbers along a number line. For example, two- and three-digit numbers appear on both the left and right hand sides of the number line but the value of the left-hand side number is very small. Values increase going to the right but decrease going to the left.

Individual → Journal
Students complete a journal entry of their understanding of integers and the number line to this point. They include their explanation of how a set of random integers could be placed in order according to value. Suggest that they use a real context in their explanation.

Curriculum Expectations/Self-Assessment/Checklist: Students self-assess using BLM 20.1.

Exploration
Reflection

Home Activity or Further Classroom Consolidation
Order the Canadian Cities by high temperatures and low temperatures, using worksheet 18.1.

20.1: Integer Check

Name:
Date:

1. Circle the correct response to each of the following questions.

   a) Integers, including zero, can be represented in many ways.

   Always Sometimes Never

   b) Which of the following situations could involve integers?

   banking temperature elevation sports ALL of these

   c) Each integer has an opposite, and the two numbers add to zero.

   Always Sometimes Never

   d) Integers include the following number sets.

   Whole Numbers Natural Numbers Fractions

   e) Three-digit integers are bigger than two digit integers.

   Always Sometimes Never

2. Explain the meaning of “zero” in ***one*** of the following situations.

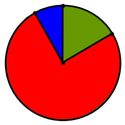
   a) +/- in a hockey roster
   b) golf scores
   c) elevation
   d) a bank account

3. a) Place the following set of integers in order from smallest to largest.

   2, –5, 20, 0, –125, 120, –120, 3, –1, –2, 5

   ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___

   b) How many of the above integers are smaller than –4?



Description

- Explore the concept of integer addition using integer tiles.
- Apply the zero principle to integer addition.
- Use proper integer notation (positive/negative, brackets)

Materials

- overhead projector
- overhead integer tiles
- sets of integer tiles or two coloured counters or two colours of any counting material

Assessment Opportunities

Minds On…

Whole Class → Discussion

Recall from Day 18 that banks use positive numbers to signify amounts deposited into an account, and negative numbers to signify amounts withdrawn. Lead a discussion based on the following scenarios: My bank account has $30.00 in it. If I deposit $5.00 today, how much will be in the bank? If I then withdraw $10.00 tomorrow, how much will be in the bank account then?

Remind students of the hockey rating discussions from Day 18. A player who, at the start of one game had a +1 rating, was on the ice when his team scored three goals. What was his rating after the game? (+4) In the next game, the same player with a rating of +4 had one goal scored against his line. What was his rating then? (+3).

Model these discussions using the overhead integer tiles.

Any two colours can be used for the integer tiles, but once decided, stay with those same colours.

Action!

Pairs → Guided Exploration

In pairs, students use integer tiles to represent the following integer addition questions: (+2) + (+6); (+1) + (+4); (–1) + (–3); (–5) + (–2), etc. Discuss similarities and differences between adding two positive integers and adding two negative integers. Is their sum ever zero?

Curriculum Expectations/Observation/Checkbric: Observe students for competency and confidence level with integers.

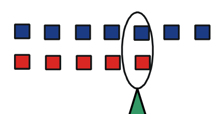
Display questions and model on the overhead for clarity.

Pairs → Investigation

Continue the activity adding integers of different signs.

Remind students that two counters, one of each type, are opposites and balance each other making zero. For example: If there are 7 blues (–) and 5 reds (+) then there are two blues with 5 zero pairs or –2. Teacher (or students) demonstrates further addition questions on the overhead, using two coloured tiles.

Students work in pairs to represent the following integer addition questions: (–2) + (+6); (–1) + (+4); (+1) + (–3); (+5) + (–2), etc.



Line up a + and – tile to make zero before taking the pair away together from the demonstration.

Pairs (A/B) → Exploration

Introduce the number pattern 5, 2, -1, -4, … Model the progression with tiles and note the results, i.e., continually adding three blue tiles. One partner (A) models a new pattern with tiles while the other partner (B) writes the pattern symbolically with integers. Some students demonstrate their work using the concrete materials on the overhead projector.

Learning Skills/Observation/Mental Note: Observe students' co-operation with others.

Encourage variations in patterns such as -5, -4, -2, 1, (add 1, then add 2, then add 3, etc.) or 2, 1, 3, 0, 4, -1, … (add -1, then add 2, then add -3, etc.)

Consolidate Debrief

Whole Class → Discussion

Discuss other models of representing integer addition. Could a hockey player's +/- rating be displayed by a line graph? Could banking deposits and withdrawals be represented by a bar graph? Link the zero principle into each graph.

Differentiated
Exploration
Application
Connections

Home Activity or Further Classroom Consolidation

1. Make up number patterns that can be verified using tiles.

OR

2. Construct graphs to display examples discussed to date, including the number patterns.

Description
- Consolidate and practise integer addition with integer tiles.
- Investigate integer addition using number lines.
- Compare the two methods for addition of integers

Materials
- overhead projector
- BLM 22.1
- sets of integer tiles

Assessment Opportunities

Minds On…

Whole Class → Problem Solving

Pose the problem: If a spider climbs 3 metres up a water spout during the day then slides back down 2 metres every night, how many days does it take to reach the top of a 10 metre spout?

Discuss multiple ways to model and solve this problem. Using the integer addition sentence (+10) + (+20), prompt students to ask a question related to everyday life whose answer could be determined by this addition sentence, e.g., if the spider climbed 10 metres up the water spout today, and 20 m tomorrow, how high will the spider be?

Answer:
It takes eight days for the spider to reach the top.

Diagram, number line, integer tiles, integer addition and graphs are useful.

Pairs → Discussion → Connecting

Write five such symbolic representations of addition sentences on the board. In pairs, students write corresponding questions.

Action!

Whole Class → Visual Activity → Discussion

Have nine volunteers line up at the front, evenly spaced, facing the class to form a human number line. The 5th (middle) person represents "0" (students should display numbers corresponding to their position).

Ask one more student to come up. This student stands in front of the "0" person and then walks three places in the positive direction to stand in front of the person at "+3."

Ask: What integer can represent the move so far? (+3) Record this on the board. This student walks one more place in the positive direction. Ask: What integer can represent this second move? (+1) Record this beside the previous answer. Demonstrate that the "trip" so far can be represented by the addition sentence (+3) + (+1), whose answer can be determined by looking at the volunteer's current location (+4).

Use a similar procedure for demonstrating addition of two negative integers, then a positive and a negative integer.

Encourage the use of a number line to show integer addition: always start at 0, use red arrows pointing to the right for positive integers, and blue arrows pointing to the left for negative integers.

Add further visual cues, such as the +3 person and the +1 person to hold their hands up
OR
model the trip with a visual drawing on the board.

Addition on the number line: start at 0, show first arrow, second arrow begins where first one ends, resulting destination is the sum.

See *Elementary and Middle School Mathematics: Teaching Developmentally* by John A. Van de Walle p. 425. for more information on the coloured arrow techniques.

Individual → Problem Solving

Students complete BLM 22.1, representing the addition questions with blue and/or red arrows, and determining answers.

Curriculum Expectations/Exhibition/Rating Scale: Collect diagnostic information about the students' level of comfort and competency with integer addition.

Encourage the use of the word *sum* as the result of addition.

Consolidate Debrief

Small Groups → Discussion

Students compare each of their answers against those of other group members and share their "rule" for addition. Discuss as a class.

Concept Practice
Reflection
Problem Solving

Home Activity or Further Classroom Consolidation

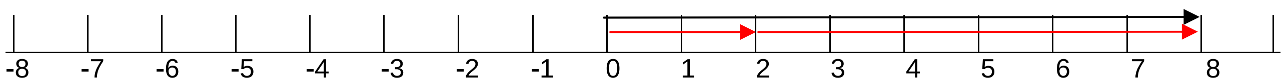
Explain to a parent or friend the similarities and contrasts between using number lines vs. integer tiles to perform integer addition. Record thoughts in your math journal, along with your personal preference.

22.1: Integer Addition Using Number Lines

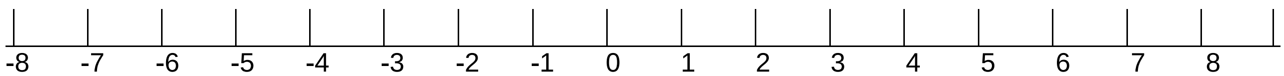
Name:
Date:

Show each number in the addition with a coloured arrow on the corresponding number line. Draw a third arrow to indicate the sum. The first one is done for you.

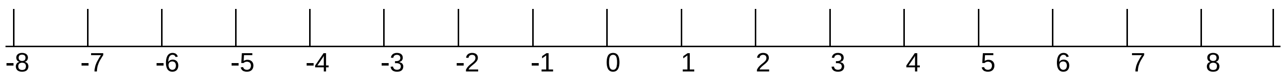
1. (+2) + (+6) = 8



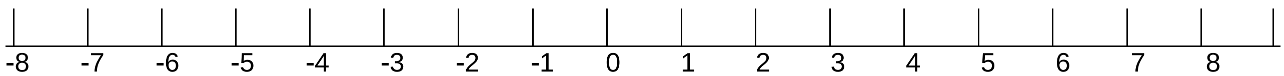
2. (–3) + (–4) =



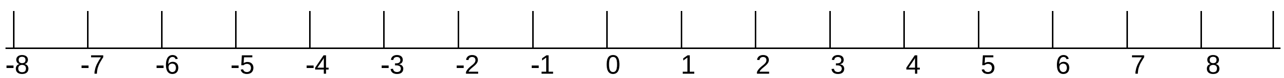
3. +5 + (–6) = (**Note:** Brackets around an integer are needed only after an operation symbol.)



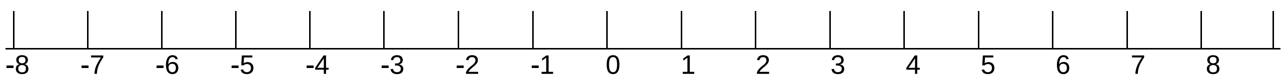
4. –7 + (+2) =



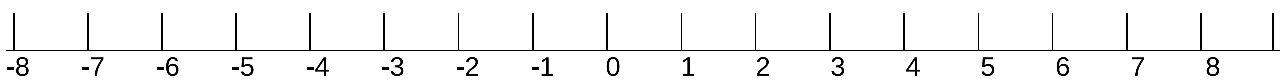
5. +2 + (–2) =



6. –5 + (+4) + (–3) =



7. +1 + (–2) + (+3) + (–4) + (5) =



Make up your own “RULE” for adding integers.

Description

- Demonstrate integer addition using integer tiles, number lines and symbols.
- Investigate mean, median and mode using an integer context.

Materials

- overhead projector
- BLM 23.1, 23.2
- integer tiles

Assessment Opportunities

Minds On…

Whole Class → Instructions

Discuss terminology and general ideas of the game of golf, e.g., par, birdie, bogey. Ask students what is meant by the numbers on the "leader board" at major golf tournaments such as –5, –3, even, +2.

Review the definitions and differences between mean, median and mode.

Mean: arithmetic average;

Median: middle item in an ordered list;

Mode: the most frequently occurring item.

Reference the classroom Word Wall for mean, median, mode.

Action!

Pairs → Investigation

Students work with their partners to complete the "golf" activity. (BLM 23.1) Circulate to clarify and observe progress.

Student volunteers periodically fill in entries on an overhead version of the handout for students to check their progress.

Curriculum Expectations/Observation/Checkbric: Observe students for competency with integers and the calculation of "averages."

Consolidate Debrief

Small Groups →Discussion →Presentations

In groups of three or four each, students share their responses and formalize the justification. Groups present their rationale to the remainder of the class.

Curriculum Expectations/Quiz/Analytical Marking: Assess addition of integers with or without manipulatives.

Note which students are still using manipulatives.

Home Activity or Further Classroom Consolidation

Application
Concept Practice
Reflection

Create a simple game that uses integers. It could include spinners, sets of cards, markers or number cubes.

23.1: Fore!

Name:
Date:

Two students in your class are comparing their golf scores over and under par. Each claims to be the better golfer. Your task is to use your knowledge of integers and statistics to help make the decision of “**Who is the better golfer?”**

Note: All **scores** are relative to PAR on each hole. Therefore, +1 is one over par (bogey) and –1 is one under par (birdie). Gord’s score on the 6th hole was the result of hitting his ball into the marsh.
Comparison to Par **card**:

Hole	1	2	3	4	5	6	7	8	9	PAR (+/-)
Lisa	0	+1	+2	+3	–1	+2	+3	0	–1	
Gord	–1	+1	0	+1	–1	+8	+2	–1	–2	

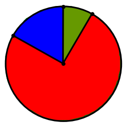
1. Who won the round? (low score wins)
2. Who won the greatest number of holes?
3. Determine the ***mean*** score for each golfer. Lisa: _____ Gord: _____
4. Who had the better mean score?
5. Determine the ***mode*** score for each golfer. Lisa: _____ Gord: _____
6. Who had the better mode score:
7. Determine the ***median*** for each player. Lisa: _____ Gord: _____
8. Who had the better median score?
9. Using the information gathered, decide who you believe is the better player.
   Justify your choice to the players.

23.2: Integer Quiz

Name:
Date:

Use integer tiles, a number line or your rule to complete the following.

1. (+4) + (+2) =
2. (–3) + (–5) =
3. +3 + (–7) =
4. –5 + 0 =
5. –1 + (+6) =
6. +8 + (–8)=
7. –5 + (–2) + (+8) =
8. 4 + (–1) +(–2) + (+3) =
9. The temperature one morning was –6° C. It rose 13 degrees by the end of the day. What was the high temperature?
10. On the first four holes of a golf game, Jamie scored +2, –1, –2 and –1. What was his score at that point in the game?



Description

• Demonstrate individual understanding of the concepts presented so far in this unit.

Materials

• Integer tiles
• BLM 24.1, 24.2, 24.3

Assessment Opportunities

Minds On…

Whole Class → Instructions

Divide the class into groups and explain the carousel activity.

Decide whether groups will be heterogeneous or homogeneous by demonstrated achievement in this unit. If the groups are homogeneous, the size of the groups could vary.

Action!

Small Groups →Carousel Activity

Groups rotate through all 5 stations completing the required tasks and the self assessment.

Stations (See BLM 24.3, Stations 1-5):

1. Integer Tile Addition – five questions for students to model integer addition with tiles.
2. Number Line Integer Addition – five questions for students to model integer addition on a number line.
3. Application Problems –solve four integer problems in context.
4. Make Up a Problem – diagram or graph showing positive and negative values that could represent an everyday situation.
5. Symbolic Integer Addition – five questions for students to demonstrate their understanding of integer addition symbolically

Place answers to one station at the next station so that groups can check their work as they move from station to station.

Curriculum Expectations/Self-Assessment/Check-line: Students self-assess their understanding of integers using BLM 24.1

Curriculum Expectations/Observation/Checkbric: Observe students for competency with integer models and applications.

Select one centre and create a checklist of proficiency skills that can be observed.

Consolidate Debrief

Individual → Reflection→Journal

Students complete their self-assessment reflections. Encourage each student to complete statements similar to the three "stems" provided:

• What am I good at?
• What needs to be improved?
• My plan is to…

Reflection

Home Activity or Further Classroom Consolidation

Students exchange the integer games they created with a classmate. Try the game and write comments suggesting what is good about the game and how it might be improved.

See Day 23.

24.1: Self Assessment: Check Line

Name:
Date:

When you have finished the activity at each station, place an "X" on the appropriate line to indicate your assessment of your proficiency at the given task.

Station 1: Integer Tile Addition

I need more practice | Given time, I can do this | I'm an expert

Station 2: Number Line Integer Addition

I need more practice | Given time, I can do this | I'm an expert

Station 3: Application Problems

I need more practice | Given time, I can do this | I'm an expert

Station 4: Make Up a Problem

I need more practice | Given time, I can do this | I'm an expert

Station 5: Symbolic Integer Addition

I need more practice | Given time, I can do this | I'm an expert

Overall Reflection:

I understand…

I need help with…

My plan is…

24.2: Carousel Activity

Name:
Date:

You may not be starting at Station 1. Therefore, be sure you are addressing the appropriate part of this activity sheet at each station.

Station 1: Integer Tile Addition

Use the integer tiles to determine the sums. Record your answers below.

a) answer: ______

b) answer: ______

c) answer: ______

d) answer: ______

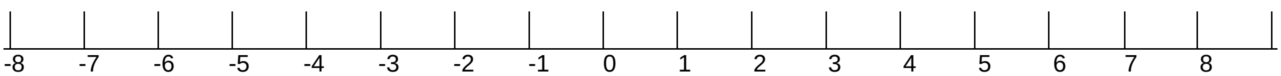
e) answer: ______

Complete the self assessment for Station 1 on handout 24.1. Wait for your teacher's signal before moving to Station 2.

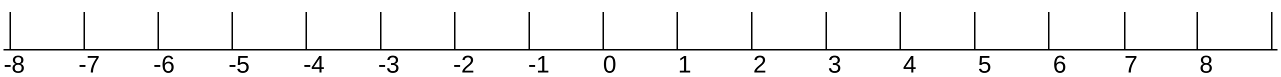
Station 2: Number Line Integer Addition

Use the number lines to determine the sums at this station. Record your answers.

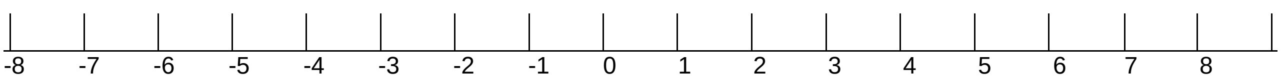
a) answer: ______



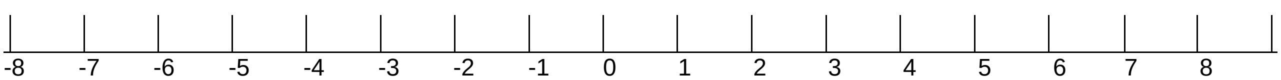
b) answer: ______



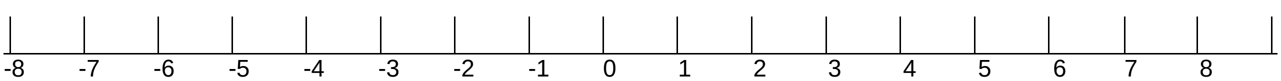
c) answer: ______



d) answer: ______



e) answer: ______

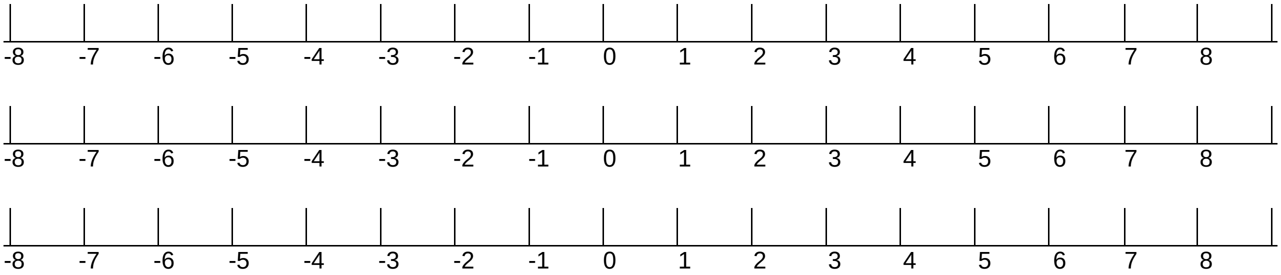


Complete the self assessment for Station 2 on handout 24.1. Wait for your teacher's signal before moving to Station 3.

24.2: Carousel Activity (continued)

Station 3: Application Problems

Use the integer tiles or the number lines below to help solve the application problems. Record your answers.



a) R.U. Ready's score: ______ I.M. Set's score: ______ Wynn Lots's score: ______

The winner is ____________.

b i) The next four integers are: _____, _____, _____, _____

ii) The next four integers are: _____, _____, _____, _____

c i) The temperature in four hours: ______

ii) The temperature four hours ago: ______

Complete the self assessment for Station 3 on handout 24.1. Wait for your teacher's signal before moving to Station 4.

Station 4: Make Up a Problem

Describe below an everyday situation that could be modelled by each graph at this station.

a)

b)

Complete the self assessment for Station 4 on handout 24.1. Wait for your teacher's signal before moving to Station 5.

24.2: Carousel Activity (continued)

Station 5: Symbolic Integer Addition

Use the rule you created for adding integers to determine the sums. Record your answers.

a) answer: ______

b) answer: ______

c) answer: ______

d) answer: ______

e) answer: ______

Complete the self assessment for Station 5 on handout 24.1. Wait for your teacher's signal before moving to Station 1.

24.3: Station 1 … Integer Tile Addition

Do not write on this card.

Use the integer tiles to determine the sums.

a) (–5) + (+3)

b) (–5) + (–3)

c) +2 + (–6)

d) –4 + (–1) + (+6)

e) +1 + (+3) + (–4)

Check your answers at the next station.

24.3: Station 2 … Number Line Integer Addition

Do not write on this card.

Use the number lines for Station 2 on your copy of handout 24.2 to determine the sums.

a) $(2) + (-6)$

b) $(-2) + (-6)$

c) $+4 + (-7)$

d) $+2 + (-1) + (+5)$

e) $-3 + (+8) + (-5)$

Check your answers at the next station.

24.3: Station 3 … Application Problems

Do not write on this card.

Use the integer tiles or the number lines for Station 3 on your copy of handout 24.2 to help solve the application problems.

a) Which person won the four round golf tournament? (lowest score wins)

	Round 1	Round 2	Round 3	Round 4	Final
R.U.Ready	**-2**	**3**	**-1**	**-2**	[redacted]
I.M.Set	**3**	**-2**	**-2**	**-1**	[redacted]
Wynn Lots	**1**	**0**	**-2**	**-2**	[redacted]

b) What are the next four numbers in the following numerical patterns:
   i) 4, 1, –2, –5
   ii) –10, –5, –1, 2

c) The temperature is dropping at a rate of 2C° per hour. If the present temperature is -8°C, determine:
   i) the temperature in 4 hours.
   ii) the temperature 4 hours ago.

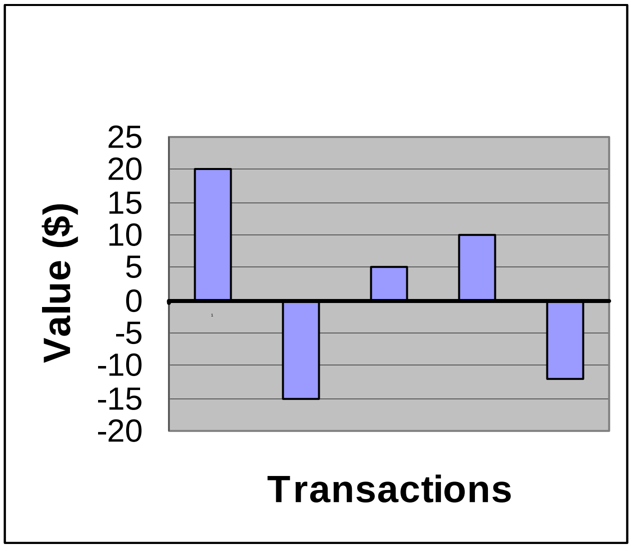
Check your answers at the next station.

24.3: Station 4 … Make Up a Question

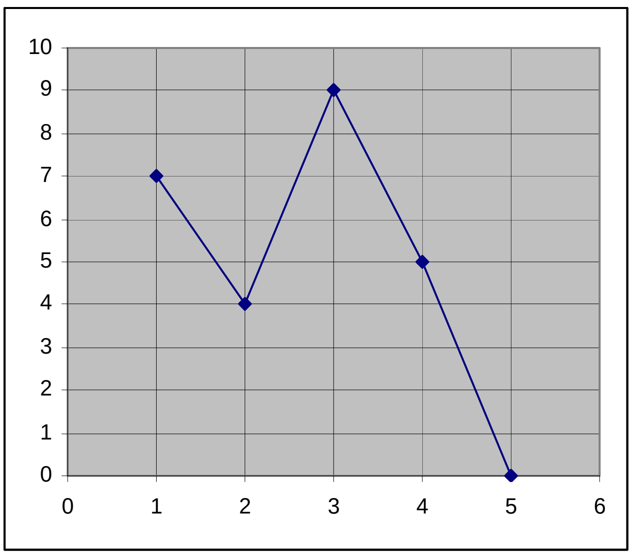
Do not write on this card.

Describe, in the space provided for Station 4 on your copy of handout 24.2, an everyday situation that could be modelled by each graph.

a)



b)



Check your answers at the next station.

24.3: Station 5 … Symbolic Integer Addition

Do not write on this card.

Use the rule you created for adding integers to find the sums.

a) (–8) + (–6)

b) (+5) + (–7)

c) –3 + (+9)

d) +5 + (–2) + (–3)

e) –6 + (+8) + (–4)

Check your answers at the next station.

24.3a: Station 1 … Answers

a) –2

b) –8

c) –4

d) +1

e) 0

24.3b: Station 2 … Answers

a) –4

b) –8

c) –3

d) +6

e) 0

24.3c: Station 3 … Answers

a) –2, –2, –3, Wynn Lots

b) i) –8, –11, –14, –17

ii) 4, 5, 5, 4

c) i) –16ºC

ii) 0ºC

24.3d: Station 5 … Answers

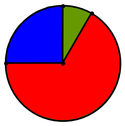
a) –14

b) –2

c) +6

d) 0

e) –2



Description

- Investigate how subtraction is related to addition.
- Use integer tiles to model integer subtraction.

Materials

- overhead projector
- overhead integer tiles
- sets of integer tiles
- BLM 25.1, 25.2
- number cubes

Assessment Opportunities

Minds On…

Whole Class → Discussion

Use real examples to introduce integer subtraction, e.g., banking – withdrawals; football – yard losses, temperature – dropping.

Guide students to understand that subtraction will find the second part if the whole and a first part are known. Use difference to describe the outcome.

Some students may recognize that they have already completed similar questions using addition, e.g., 6 – 4 can be 6 + (–4).

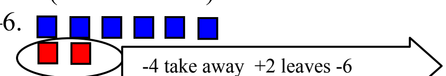
Encourage the use of the word difference for the result of subtraction.

Action!

Whole Class → Demonstration

Model several integer subtraction questions with the overhead integer tiles in the following two steps:

1) +6 – (+4) or –6 – (–2); questions where tiles for the first part of the whole are available to be removed.
2) –4 – (+2) or +5 – (–3); where tiles for the first part of the whole are not immediately available to be removed and must be introduced. Introduce the necessary tiles using the zero principle. For –4 – (+2), the tiles for +2 are not available. So introduce (+2 and –2 = 0) then the +2 can be removed from -4 leaving –6.



Pairs → Game

Students practise subtraction using integer tiles during a game (BLM 25.1). Using two regular number cubes, one blue (-) and one red (+), students take turns rolling and recording the integers on a chart. With the help of integer tiles to model each result, students calculate the difference between the integers and enter this in the difference column of their chart. The student who has the greatest difference (total of the difference column) after a set amount of time is the winner.

Learning Skills/Observation/Mental Note: Observe students' co-operation with others as they work through this activity.

Number cubes can be differentiated as + or – by size or markings if differently coloured ones are not available.

The activity can be differentiated by providing different pairs with various types of number cubes (four-sided, eight-sided, sixteen-sided).

Some students may have discovered that instead of subtracting they could be adding opposites.

Consolidate Debrief

Whole Class → Discussion

Ask: When you were using the number cubes, which two numbers gave you the greatest amount of difference? How did you know? Which two numbers gave you the least amount of difference? Explain. Could you roll two numbers that would give you a difference of "0"? Explain.

Pairs → Think/Pair/Share

Each student records a rule or procedure for subtraction and shares the rule with a partner. As a class, discuss the essential ideas that should be included. Students apply their rules as they complete the questions on BLM 25.2.

Concept Practice
Reflection

Home Activity or Further Classroom Consolidation

Consider the questions:

+3 – (+5)	and	+3 + (–5);
–1 – (+2)	and	–1 + (–2);
+4 – (–3)	and	+4 + (+3)

Draw conclusions regarding the operations and numbers involved and the result. Revise the rule or procedure you developed with your partners to include this outcome, if it doesn't already. Create another three sets of questions (with answers) that follow this same pattern.

Students should be able to model both questions using tiles.

25.1: Rolling for Cubes

Name:

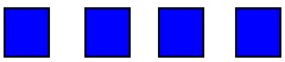
Game Rules

- Shake the two number cubes, and, without looking, release the cubes one at a time.
- Record the value of the first cube in the 1st column and the value of the second cube in the 2nd column. Remember that blue represents negative and red represents positive.
- With the help of your integer tiles, determine the difference between the first integer and the second. The first example has been done for you.

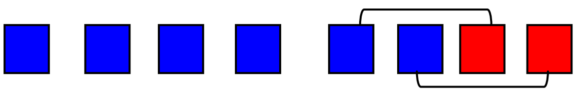
Example

If the 1st cube is blue and the 2nd cube is red, then - 4 goes in the 1st column and 2 or +2 goes in the 2nd column.

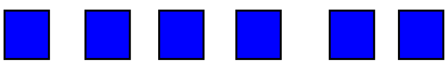
Place four blue integer tiles on your desk to represent –4.



To calculate the difference, you will need to remove two red tiles (representing positive 2) but there are none. So add two zeros (two blue tiles and two red tiles).



Now you can remove the two red tiles (representing positive 2).



What remains represents the difference (–6). Write this number in the third column.

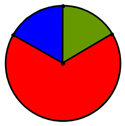
1st Number cube	2nd Number cube	Difference
-4	**+2**	**-4 – (+2) = -6**
Total of differences		

25.2: Integer Subtraction Using Integer Tiles

Name:
Date:

Model the following integer addition and subtraction questions using your integer tiles. Draw a pictorial representation of the operation and write the answer in each case.

1. (+5) – (+3) =
2. (–6) – (–2) =
3. 4 – 1 =
4. 4 + (–1) =
5. –7 – (–3) =
6. –7 + (+3) =
7. 3 + (+2) =
8. 3 – (–2) =
9. –5 – (–4) =
10. –2 – (+4) =
11. –1 – (–3) =
12. 4 – (+5) =



Description
- Investigate subtraction of integers using number lines.
- Compare subtraction on a number line with subtraction using integer tiles.

Materials
- overhead projector
- BLM 26.1
- two-colour counters

Minds On…

Whole Class → Discussion
Use students' experience with the previous day's home activity and parts of BLM 25.1 to establish that, using number tiles, subtracting an integer is the same as adding the opposite integer. Provide students with feedback as they model examples of subtracting an integer with integer tiles. As you circulate, ask what corresponding addition question would give the same result.

Pose the question, "I wonder if subtracting an integer is the same as adding the opposite integer if we use a number line model for integers instead of integer tiles?"

Review representation of positive and negative integers on a number line, blue arrows (negatives) facing left and red arrows (positives) facing right. The first integer starts at zero. Each added arrow begins at the point end of the previous arrow.

Action!

Whole Class → Discovery
Guide a discussion about modelling subtraction on a number line; positive integers are as red arrows facing right and negatives as blue arrows facing left. (It is only the subtraction concept that is being discussed, not the representation of an integer.)

Help students come to the conclusion that if addition is 'moving forward from the end of the first arrow along the second arrow,' then subtraction must be 'moving backwards from the end of the first arrow along the second arrow.'

Continue with questions and discussion until students demonstrate an understanding of integer subtraction on the number line.

Pairs → Modelling on a Number Line
Pair students to work on BLM 26.1 so that at least one student in each pair is sure how to use the number line model.

Students work in pairs to complete BLM 26.1.

Curriculum Expectations/Observation/Checkbric: Observe students as they complete the activity, watching their competency and confidence level with integers.

Consolidate Debrief

Whole Class → Consolidation
Facilitate a discussion that leads students to realize that the number line models confirm the concept illustrated with integer tiles; 'to subtract an integer, you can add the opposite integer.' Ask: "Which type of question do you find easier to do – an addition question or a subtraction question?" Orally, students practise converting a set of subtraction questions to addition questions.

Home Activity or Further Classroom Consolidation
1. Develop a class note to convince an absent classmate that, 'You get the same result when you subtract an integer as when you add the opposite integer.' Include specific examples and integer tile and number line models.
2. Illustrate question and answers to the following questions using integer tile or number lines
   1. $(-5)+(+2)$
   2. $(-3)+(-1)$
   3. $(-6)-(-4)$
   4. $(-6)-(+4)$

Reflection

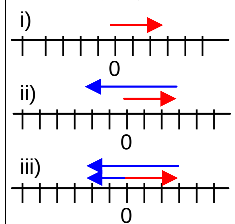
Assessment Opportunities

Students who are still unsure should work on subtraction examples that do not require use of the zero principle.

E.g., $-5-(-3)$

Addition model

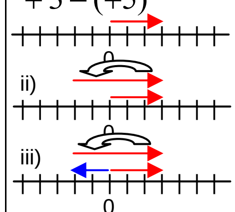
$+3+(-5)$



The result is a –2 arrow from 0 to the end of the 2nd arrow.

Subtraction model

$+3-(+5)$



See Van de Walle p. 425 for coloured arrow techniques.

Encourage students to convert subtraction questions to addition questions.

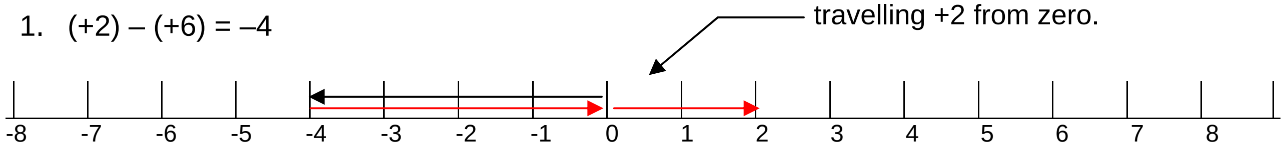
26.1: Integer Subtraction Using Number Lines

Name:
Date:

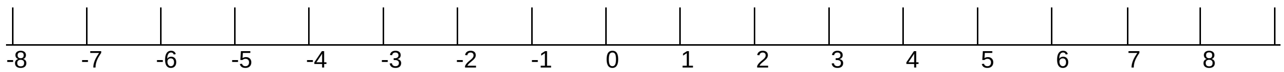
Show each number in the subtraction with a coloured arrow on the corresponding number line. Draw a third arrow to indicate the difference. The first one is done for you.

Travel backwards on the +6 goes after travelling +2 from zero.

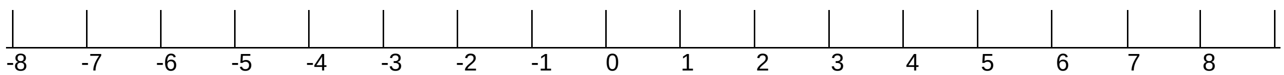
1. (+2) – (+6) = –4



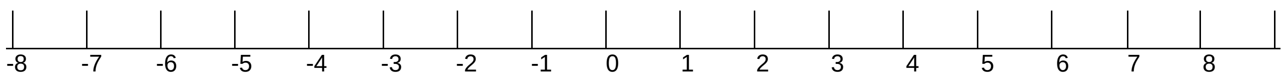
2. (–3) – (–4) =



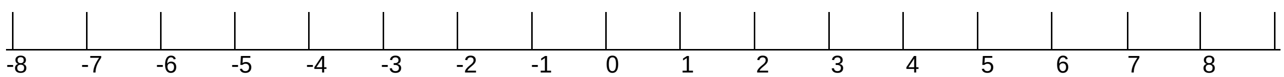
3. +5 – (–2) =



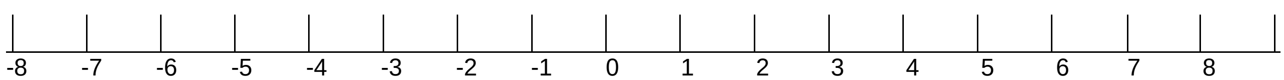
4. –4 – (+2) =



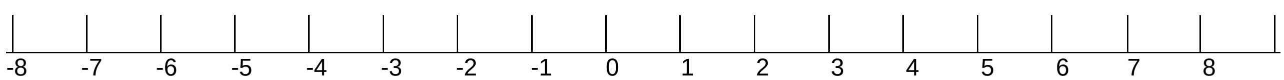
5. +2 – (–2) =



6. –5 – (+3) – (–4) =



7. +1 + (–2) – (+3) – (–4) + (5) =



Make up your own “RULE” for subtracting integers.

Description

- Practise integer addition and subtraction using a game.

Materials

- overhead projector
- integer tiles
- number lines
- markers
- BLM 27.1, 27.2

Assessment Opportunities

Minds On…

Whole Class → Review

Complete two or three practice questions to review addition and subtraction models. Use one or two operations per question. Students should have integer tiles and number lines available to them.

Students fill in their game cards (BLM 27.1) using integers ranging from –20 to +20. No number should be repeated. Students may use one or two cards.

Action!

Individual → Practice

Use BLM 27.2 for questions that could be given for students to mark answers on their card. Using the overhead, display each question allowing students time to calculate the answer in a manner of their choosing. Track each question given before proceeding to the next question.

Learning Skills/Observation/Mental Note: Observe students' perseverance as they work.

Use the random number generator on a scientific calculator to generate the numbers. Skip any repeats. [21 × RND + 1: ignore any decimal part]

To create additional interest, award small prizes during the game.

Consolidate Debrief

Individual → Journal

Ask students to reflect on their understanding of integer subtraction and addition including strengths and concerns. Solutions, using the students' preferred model, should be included. This activity may extend into a home activity or further classroom consolidation. Help students identify the components of an effective record of how to add and subtract integers.

Concept Practice
Reflection
Skill Drill

Home Activity or Further Classroom Consolidation

In your journal, write some specific questions and answers for addition and subtraction of integers. Illustrate each one, either with tiles or a number line.

27.1: Integer Game

Name:
Date:

Rough Work:

27.2: Integer Game Questions

–20: +10 + (–30)

–19: –14 – (+5)

–18: –25 + (+7)

–17: –30 – (–13)

–16: +8 – (+24)

–15: +6 + (–21)

–14: –5 – (+9)

–13: –20 – (–7)

–12: –14 + (+2)

–11: +5 – (+16)

–10: –6 + (–4)

–9: +4 + (–10) + (–3)

–8: +7 – (+3) + (–12)

–7: –3 – (+8) – (–4)

–6: –2 + (–9) + (+5)

–5: +5 + (+5) – (+15)

–4: –9 – (–8) + (–3)

–3: +6 – (+10) + (+1)

–2: –8 – (–1) – (–5)

–1: –10 + (+4) + (+5)

0: +4 – (–7) – (+11)

1: –8 – (+6) + (+15)

2: –7 + (+3) + (+6)

3: –4 – (–6) – (–1)

4: +12 – (+3) + (–5)

5: +4 – (–10) – (+9)

6: –2 – (+8) – (–16)

7: +6 + (+6) – (+5)

8: +1 – (+10) – (–17)

9: –15 + (+20) + (+4)

10: –12 – (–22)

11: –14 + (+25)

12: +6 – (–6)

13: +18 – (+5)

14: +21 + (–7)

15: –5 + (+20)

16: –8 – (–24)

17: +9 + (+8)

18: +20 – (+2)

19: +19 – 0

20: –10 + (+30)

Description

- Complete exercises encompassing the knowledge acquired in this unit.

Materials

- recipe cards
- integer tiles
- number lines
- BLM 28.1, 28.2, 28.3, 28.4

Assessment Opportunities

Minds On…

Whole Class → Data Collection

Mark the height of the average 12-year old (151 cm) on the board. Student volunteers (8-10) each compare their height to the mark on the board. Each student must represent the difference between his/her height and the marked height, using an integer (negative if shorter, positive if taller). Each student records the difference on a recipe card without disclosing their integer.

These same students line up according to height (shorter students on the left from the perspective of those seated). The students reveal their height variations from the average height resulting in a sorted list of integers. Ask: Is it possible for there to be missing integers between some of the students? How would you determine your height's difference from the average using tiles, number lines, or symbols?

Have several measuring tapes taped to the wall. The volunteers should have a partner help measure their height.

Collect the height variation cards for further use in the assessment activities.

Action!

Individual → Assessment

1) Communication (BLM 28.1)
2) Knowing Facts and Procedures (BLM 28.2)
3) Making Connections (BLM 28.3)
4) Reasoning and Proving (BLM 28.4) Write the data from the cards collected in Minds On the overhead or board.

Curriculum Expectations/Written Work: Use the four mathematical processes of Continuum and Connections – Mathematical Processes (Section 2, TIPS), to place a different focus on each of the activities as indicated.

When returning graded work to students, consider photocopying samples of level 3 and 4 responses with student names removed. Select and discuss pertinent examples.

Consolidate Debrief

Small Group→Analysis

From your observation of the activities, choose one question from BLM 28.2 and BLM 28.3 which proved to be more challenging. Assign one of the chosen questions to each group for a place mat activity. The group members complete a solution in their given space then the group writes its "model solution" in the centre. Post the place mats as exemplars.

See Place mat, TIPS, Section — TIP 10.

Based on the results of the activities, it may be necessary to provide additional practice and feedback for some students.

Reflection

Home Activity or Further Classroom Consolidation

In your journal comment on the truth of the following statement.
"Every integer has an opposite integer."

28.1: Summative Task Part 1 … Communication

Name:
Date:

Write a letter aimed at an alien from Mars, named Red, showing him how to add two integers. Assume the alien knows what integers are and can add whole numbers. Include at least one of: pictures of integer tiles, number lines or rules you have developed.

Dear Red;

28.2: Summative Task Part 2 … Knowing Facts and Procedures

Name:
Date:

1 Use integer tiles OR a number line to illustrate each integer sentence. State your answer in the space provided.

(+5) + (–2) = _____	–3 – (–5) = _____

2. Use positive and negative integers to create a true mathematical statement. Use integer tiles, if you wish.

_____ + _____ = +3	_____ – _____ = –2

3. *Circle* the correct response to each part below then justify your answer. Use drawings, examples or words to explain.

a) "A negative number plus a positive number equals a positive number."
This statement is:

Always true   Sometimes true   Never true

b) "A negative number subtract a positive number equals a positive number."
This statement is:

Always true   Sometimes true   Never true

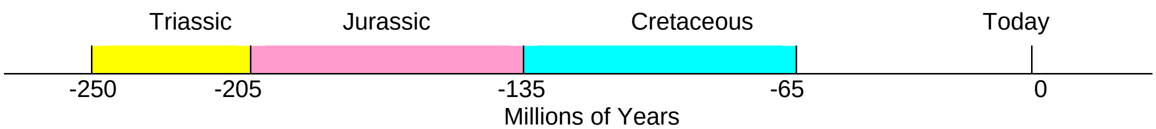
28.3: Summative Task Part 3 … Making Connections

Name:
Date:

1. Model the following problem using the number line below. Use arrows to help answer the question.
"The temperature at 10:00 a.m. was 20ºC. It rose 5Cº by noon then dropped 8Cº by 11:00 p.m. and dropped a further 2Cº in the next hour. What was the temperature at midnight?"

2. When did these dinosaurs live?

Mesozoic Era



a) The **Tyrannosaurus Rex** lived 75 million years ago.

It lived in the ____________________ period.

b) The **Euskelosaurus** lived 90 millions years before the end of the Jurassic period.

It lived in the ____________________ period.

c) The **Brachiosaurus** lived in the Jurassic period, near its end.

It lived about _________________million years ago.

d) The **Triassic period** was ___________________ million years long.

28.4: Summative Task Part 4 … Reasoning and Proving

Name:
Date:

Carefully copy the data (the height data collected earlier today) into the box. Double check that you have not made a copy error. These integers represent the differences of several volunteer students' heights from the average height of 12-year-old students, 151 cm.

1. a) Calculate the mean of the numbers in the box.

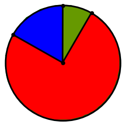
   b) Add your answer in part (a) above to 151. This represents the average (mean) height of the volunteers.

   c) The average height of the volunteers might not be the same as the average height of 12-year old students. Give reasons why this might be so.

2. Calculate the median of the numbers in the box.

3. Calculate the mode of the numbers in the box.

4. Which of the three "averages" would be affected the most if a very tall volunteer's difference in height data were to be added to the box?



Description

- Complete exercises encompassing the knowledge acquired in this unit.

Materials

- integer tiles
- number lines

Assessment Opportunities

Minds On…

Whole Class → Discussion

Students place their Home Activity from Day 28 on their desks. Lead a discussion regarding the statement: Every integer has an opposite integer. Pose questions such as: What is the opposite of –5; 25; 0; the statement **She wants a new car**; the statement: **I don't like chocolate;** the statement**: I don't unwelcome your concern**. Try to establish that the first statement is like a positive integer, the second is like a negative integer and the third is like the opposite of a negative integer, i.e., a positive integer.

Zero is the only integer that does not have an opposite.

Action!

Individual → Assessment Activities

These activities are taken from Continuum and Connections – Integers, TIPS, Section 2

1) Developing Mathematical Processes – Grade 7 (p. 5)
2) Developing Proficiency – Grade 7 (pp. 8 and 9)
3) Extend Your Thinking – Grade 7 (p. 12)
4) Is This Always True? – Grade 7 (p. 24)

Curriculum Expectations/Written Work: Use the four mathematical processes of Continuum and Connections – Mathematical Processes (Section 2), as a focus for the assessment activities.

Copies should be made for each student.

Consolidate Debrief

Whole Class → Discussion

Discuss the answers to part 4 in Action! Point out that to show a statement to be false, as is the second one, only a single counter example is necessary.

Reflection

Home Activity or Further Classroom Consolidation

Answer the questions about order of operations.

To activate prior knowledge and as diagnostic for order of operations provide questions from the textbook.

Description

Students will:

- activate prior knowledge of integers;
- assess students' knowledge of integers – including appropriate contexts, understanding of zero and meaning of positive and negative numbers.

Materials

- BLM D.1
- signs with locations

Assessment Opportunities

Minds On…

Whole Class → Mind Map

Activate prior knowledge by writing the word "integers" on the board and asking students if they know what an integer is. Clarify the definition.

Students generate a mind map with items that they understand about integers. These could include contexts in which integers are used, meaning of + and –, and different ways to represent integers (using pictures, words, symbols).

See Continuums and Connections: Integers for more details about expected prior knowledge.

Action!

Individual → Paper-and-Pencil

Read the information provided on BLM D.1 and clarify any questions. Individually students identify an appropriate location for the first scenario.

Whole Class → Four Corners

Explain that each corner of the room and the centre of the room has been assigned one of the location names from the BLM D.1. On your signal, students move to the spot that represents the location that they selected for the first scenario. On your recording sheet, note the students whose selection is inappropriate. Select some students to justify their decision. Students listen to the reasoning to assist them with their next choice. Identify the intended location so that students will not select it for future scenarios.

Teachers may select students who appear to have followed other students as well as students who made correct and incorrect decisions.

Repeat the individual and whole class activity for the remaining three scenarios.

Curriculum Expectations/Activity/Checklist: Record each student's prior knowledge to inform next steps.

Consolidate Debrief

Whole Class → Discussion

Students share any further thoughts they may have about the activity. Note use of correct mathematical terminology as well as different rationale for the same locations.

Explain that this diagnostic will be helpful for you to plan their work on integers.

Home Activity or Further Classroom Consolidation

Application
Concept Practice
Differentiated
Exploration
Reflection
Skill Drill

BLM D.1

	Fall/Winter Temperature (°C)		Spring/Summer Temperature (°C)		Elevation (m)	Average Hours of Sunshine per month		Average Precipitation per month (mm)	
	High	Low	High	Low		F/W	S/S	F/W	S/S
1 Cranbrook	12	-12	26	0	939	108	263	27	37
2 Fort Chipewyan	5	-29	22	-5	232	102	286	22	41
3 Gillam	3	-30	21	-10	145	105	202	35	57
4 Landsdowne House	6	-27	22	-7	256	99	184	26	73
5 Stephenville	10	-10	20	-1	8	66	174	112	99

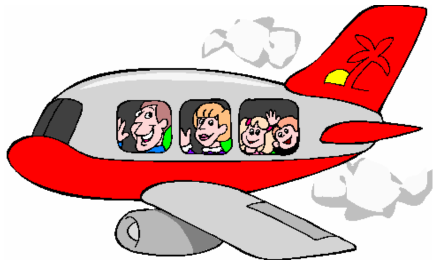
Refer to the table and select a location most appropriate for each trip outlined below. You don't want to visit the same place twice. You may choose to record some of your thinking.

TRIP #1

Your family would like to travel to a location where they will be able to enjoy the outdoors as they walk along the beach at the ocean and cycle on bike trials. A location which has moderate summer temperatures, with little fluctuation between highs and lows, would be most appropriate.

TRIP #2

Mountain biking will be the key activity for your summer vacation. Warm summer temperatures with lots of sunshine and little rain will make your trip more enjoyable.



TRIP #3

You are a winter enthusiast – enjoying cold temperatures in which you can ski, snowshoe, and snowmobile at a relatively low elevation. Some snowfall throughout the winter and lots of sunshine is what you are looking for as you select a place to spend a few months.

TRIP #4

You are not sure what time of year you are taking this trip so you must select a location that would be good year round. You would like to be able to enjoy some outdoor activities and sunshine but will be happy to spend a few days indoors at museums, theatres, and shopping malls.

Grade 8 Year Outline

Term	Cluster of Curriculum Expectations	Reference	Number of Lessons Prepared	Lesson Time Available "Instructional Jazz"	Total Lesson Time
1	Building Social Skills while Solving Problems	TIPS 1 – 6	6	2	8
	Powers and Square Roots	TIPS 7 – 10	4	2	6
	Pythagorean Relationship	TIPS 11 – 16	6	2	8
	Experimental vs. Theoretical Probability	TIPS 17 – 23	7	2	9
	Integers		13	5	18
	Solving Equations		6	2	8
	Patterning		5	2	7
	Writing *n*th terms		4	2	6
	Sub-totals		**51**	**19**	**70**
2	Angle Properties, Order of Operations		13	2	15
	Circles	LMS 16 – 22	7	3	10
	Fractions, Unit Rate	LMS 26 – 37	12	4	16
	Complex Probability, Central Tendency, Census vs. Sample		10	2	12
	As needed			2	
	Sub-totals		**42**	**13**	**55**
3	Order of Operations with Fractions and Exponents		7	3	10
	Review and Extend Solving Equations in Contexts, Inequalities		5	2	7
	Unit Rate and Percents		10	2	12
	Triangular Prisms, Valuing Measurement, and Connect Pythagorean Theorem to 3-D Figures,	LMS 23 – 33	11	2	13
	Comparative Bar Graphs, Bar Graphs vs. Histograms		5	2	7
	Multi-darts Investigation		4	2	6
	Sub-totals		**42**	**13**	**55**
			135 days	**45 days**	**180 days**

Targeted Implementation and Planning Supports (TIPS)
Leading Math Success (LMS)

The number of prepared lessons represents the lessons that could be planned ahead based on the range of student readiness, interests, and learning profiles that can be expected in a class. The extra time available for "instructional jazz" can be taken a few minutes at a time within a pre-planned lesson or taken a whole class at a time, as informed by teachers' observations of student needs.

The reference numbers are intended to indicate which lessons are planned to precede and follow each other. Actual day numbers for particular lessons and separations between terms will need to be adjusted by teachers.

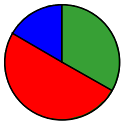
Lesson Outline: Days 16 – 22 Grade 8

BIG PICTURE

Students will:

- draw circles and measure radii, diameters, and circumferences using concrete materials;
- investigate the relationship between the diameter and circumference of a circle to discover the constant ratio π;
- develop and use formulas for circumference and area of circles;
- solve problems relating to the radius, diameter, circumference and area of a circle;
- understand and apply the order of operations and use of π key on calculators.

Day	Lesson Title	Description	Expectations
16	Talking about Circles	• Use circle vocabulary. • Measure parts and features of a circle. • Investigate the relationship between the circumference and the diameter of a circle, i.e., $C \div d = \pi$.	8m44, 8m45, 8m49 CGE 2b
17	Mysterious Circles	• Develop and apply formulas for the circumference of a circle.	8m35, 8m37, 8m39, 8m44, 8m45, 8m46, 8m48, 8m49 CGE4b
18	Circulating Problems	• Develop and apply the formula for the area of a circle. • Use inquiry and communication skills.	8m36, 8m39, 8m44, 8m46, 8m42, 8m31, 8m32 CGE3c, 4f
19	Parts and Wholes	• Apply formulas for circumference and area of circles in problem solving situations.	8m4, 8m6, 8m8, 8m16, 8m31, 8m35, 8m37, 8m39, 8m42, 8m47, 8m48 CGE3c
20	Getting Your Piece of the Pizza!	• Apply formulas for circumference and area of circles in problem solving situations.	8m6, 8m9, 8m14, 8m31, 8m32, 8m33, 8m35, 8m37, 8m39, 8m40, 8m49 CGE2d
21	Composition with Circles	• Apply formulas for circumference and area of circles in problem solving situations involving composite shapes.	8m4, 8m6, 8m8, 8m30, 8m31, 8m37, 8m39, 8m42, 8m44, 8m47 CGE2b
22	Race Track	• Performance task.	CGE4f



Description

- Use circle vocabulary.
- Measure parts and features of a circle.
- Investigate the relationship between the circumference and the diameter of a circle, i.e., $C \div d = \pi$.

Materials

- cylindrical objects
- chart paper
- placemats
- flexible measuring
- tape, string, rulers
- calipers
- BLM 16.1, 16.2
- highlighters

Minds On ...

Small Groups → Placemat

Students form small groups around a placemat with the central question, What are some of the different parts and features of a circle? After some individual response time, students write a collective response in the centre of the placemat. Facilitate a whole class discussion, advising students that after the discussion there will be a paper/pencil assessment.

Students complete a diagnostic (BLM 16.1). Write a question on the board for students who finish the assessment quickly: *How is a circle the same as or different from a 200-sided regular polygon?*

Curriculum Expectations/Quiz/Marking Key: Assess prior knowledge of circle vocabulary.

Whole Class → Discussion

Inform students that they will be doing a measurement investigation involving circles. Discuss different ways that parts of a circle can be measured, e.g., using a ruler, using *The Geometer's Sketchpad 4®*, using a string/ruler combination. Discuss how to accurately measure a diameter.

Show the class a pair of calipers. Students think about how they might be used to measure diameters. Follow up the discussion with a demonstration of how to use calipers to measure the diameter of an object.

Action!

Pairs → Investigation

Number a collection of objects with circular faces and display them in one location in the classroom. Each pair selects one item.

Explain the instructions on BLM 16.2. Students exchange the item after measuring it. Students take measurements for at least 6 items before starting the calculations. This investigation provides 'yes' and 'no' examples of the constant measurement relationship between circumference and diameter ('yes' – division; 'no' – addition, subtraction, multiplication).

Consolidate Debrief

Whole Class → Summarizing

Compare the data collected. Be prepared to discuss outliers (data that doesn't fit due to incorrect measurements or calculations). Discuss responses to questions 6 and 7. Guide students to the discovery that there is a relationship between the circumference and diameter.

Concept Practice
Differentiated
Exploration

Home Activity or Further Classroom Consolidation

Find other circular objects at home in something that has a very large diameter, e.g., vehicle tire, culvert, tree stump, etc. Confirm that the rule works for these objects.

Assessment Opportunities

When returning diagnostic, highlight where different answers are possible for the same diagram, e.g., major arc and minor arc.

Objects with circles: container lids, cans, CDs, coins, circles on gym floor, paper cups, clock faces, etc.

Ways to measure circumference: Flexible tape measure, string/ruler, rolling edge along a ruler

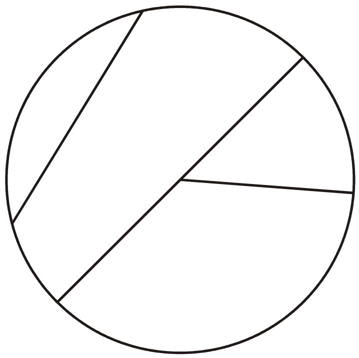
GSP16.1 Circles.gsp can be used to demonstrate parts of a circle and construction and measurement of features of a circle.

Stress careful measurement. Students should record their observations to the nearest tenth of a cm. Student values for $C \div d$ may vary from 2.8 to 3.5.

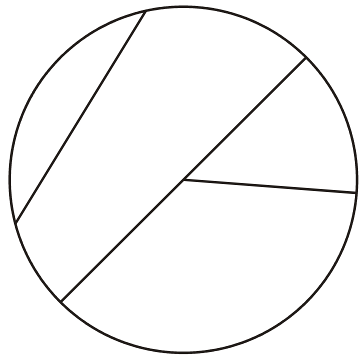
16.1: Looking at Circles

Highlight the indicated feature on each diagram.

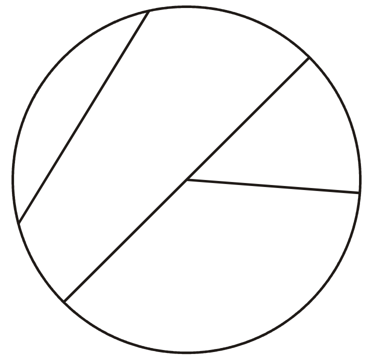
centre



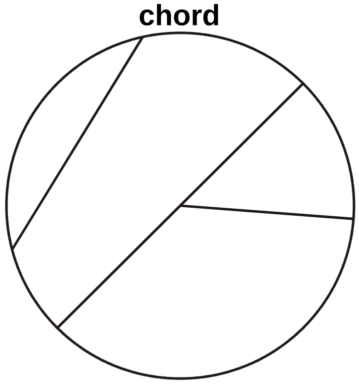
radius



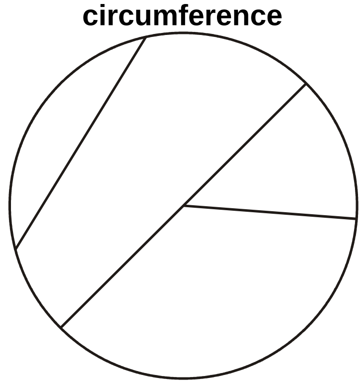
diameter



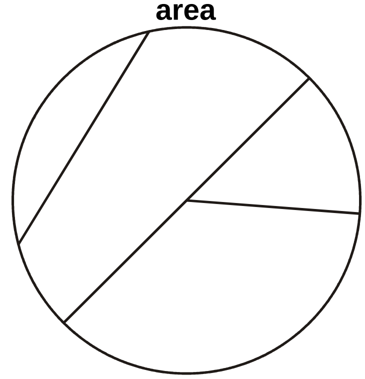
chord



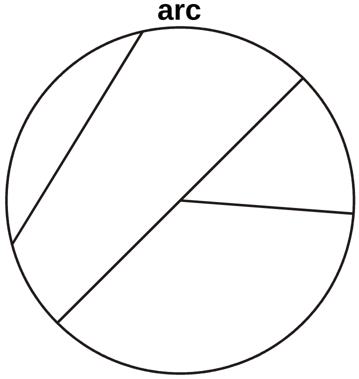
circumference



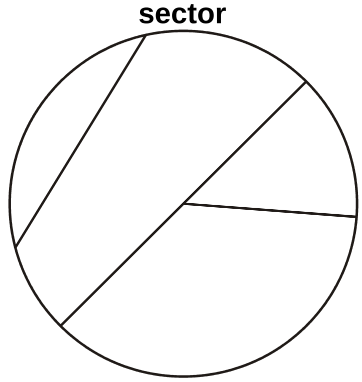
area



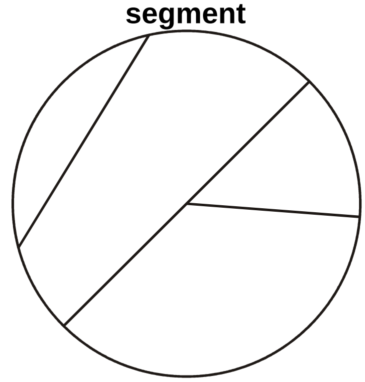
arc



sector



segment



16.2: Exploring Relationships Within the Circle

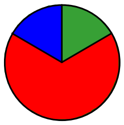
1. Select an object with a circular face.
2. Measure the circumference (C) to the nearest 0.1 cm. Record the measurement.
3. Measure the diameter (d) to the nearest 0.1 cm. Record the measurement.
4. Repeat the first three steps with different objects.
5. Do the calculations indicated in the chart. Use a calculator and round answers to the nearest 0.1 cm.

Object	**C**	**d**	**Calculations**			
			$C - d$	$C + d$	$C \div d$	$C \times d$

6. Examine the four "calculations" columns. What pattern(s), if any, do you see?

7. If possible, state a mathematical rule for any pattern you observed.

Day 17: Mysterious Circles — Grade 8



Description

- Develop and apply formulas for the circumference of a circle.

Materials

- computer
- calculators
- BLM 17.1, 17.2
- GSP 17.1

Assessment Opportunities

Minds On ...

Whole Class → Discussion

Facilitate a discussion about the observations from the previous day's investigation. Guide students to the hypothesis that the ratio of the circumference to the diameter in any circle is a constant. Describe the next activity as a computer exploration that will simulate what students did in the previous day's exploration.

Action!

Pairs → Exploration

Students use the instructions on BLM 17.1 to construct a dynamic model (or alternatively use the pre-made sketch, GSP 17.1 CircumDiam.gsp), to explore the ratio. Students collect data that confirms or denies the hypothesis.

Whole Class → Discussion

Discuss the dynamic exploration and whether it confirmed the hypothesis that the ratio of the circumference to the diameter in any circle is a constant (*yes*). Name the constant as π (pi).

Compare the two different exploration methods. The dynamic exploration results in a greater degree of accuracy. Demonstrate how greater accuracy can be shown on *The Geometer's Sketchpad 4* ® activity by changing the measurement properties.

Facilitate a discussion about how to apply this knowledge about the π ratio. Ask how the information can be used to determine the circumference if the diameter or radius is known and vice versa. Do the first question on BLM 17.2 together. Ensure that students know how to use the π key on a calculator.

Students work in pairs to complete BLM 17.2

Curriculum Expectations/Observation/Mental Note: Circulate to assess for understanding how to use the circumference formula.

Consolidate Debrief

Whole Class → Discussion

Update the word wall with the value of π (pi) and the formula for finding circumference (given π and d) and for finding the diameter (given C and π). Discuss how the formulas would change if the radius was used. Students explain how they could construct a circle if the circumference is given.

Exploration

Home Activity or Further Classroom Consolidation

Start your π project "A Taste of Pi." Your project can take the form of a report, poster, letter to the editor, or introductory letter. Include some interesting π facts and history. List the first twenty digits of π. Include an explanation of why π is called an irrational number. You can find information about π on the web. Two websites you can visit are:

http://www.ualr.edu/~lasmoller/pi.html

http://www.geom.uiuc.edu/~huberty/math5337/groupe/welcome.html

Your project will be assessed for accuracy and effective communication.

3.14 is an approximation for π. Students should also explore how the calculations change when the π button is used on the calculator. Be consistent and use the π button for all calculator calculations.

Describe the home activity.

The TIPS Grade 7 "Gazebo" investigation includes dynamic sketches that can be used to show another method for discovering the value of π.

Did you know? …Einstein's birthday is on π Day – Mar 14. Ask students why March 14 is called π day and why celebrations occur at 1:57 p.m.

17.1: Investigating the Relationship between C and d Using *The Geometer's Sketchpad 4*®

1. Construct a circle.
   Draw a line segment from the centre of the circle to the outside edge and label it "*r*" to represent the radius.
   Construct a line perpendicular to "*r*" through the centre of the circle.
   Construct points where the perpendicular line and circumference meet.
   Hide the perpendicular line.
   Draw a line segment between the two points.
   Label the new line segment "*d*" for diameter.
   Label the circumference "C."

2. Measure the length of the radius (*r*) and diameter (*d*).
   Measure the circumference (C).
   Calculate: $C + d$, $C - d$, $C \times d$, and $C \div d$.
   Highlight the four calculated values and create a table.
   Change the size of the circle by dragging the size control point.
   Double click on the table to enter another value.
   Repeat this step 5 times then look for a pattern in one of the columns.

17.1: Investigating the Relationship between C and d Using *The Geometer's Sketchpad 4*®

1. Construct a circle.
   Draw a line segment from the centre of the circle to the outside edge and label it "*r*" to represent the radius.
   Construct a line perpendicular to "*r*" through the centre of the circle.
   Construct points where the perpendicular line and circumference meet.
   Hide the perpendicular line.
   Draw a line segment between the two points.
   Label the new line segment "*d*" for diameter.
   Label the circumference "C."

2. Measure the length of the radius (*r*) and diameter (*d*).
   Measure the circumference (C).
   Calculate: $C + d$, $C - d$, $C \times d$, and $C \div d$.
   Highlight the four calculated values and create a table.
   Change the size of the circle by dragging the size control point.
   Double click on the table to enter another value.
   Repeat this step 5 times then look for a pattern in one of the columns.

17.2: Circus of Circles

Calculate the missing measures. Use a calculator. Show your work.

a) hoop

d = 75 cm
r =
C =

b) surface of drum

d = 40 cm
r =
C =

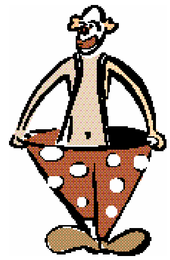
c) surface of circular barrel

C = 2.5 m
r =
d =

d) base of tent

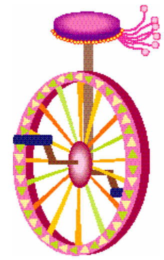
C = 40 m
d =
r =

e) hoop waist



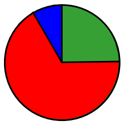
C = 210 cm
d =
r =

f) unicycle wheel



r = 30 cm
d =
C =

The unicycle wheel in part (f) made 1000 complete revolutions as it traveled down a road. What distance did the rider travel?



Description
- Develop and apply the formula for the area of a circle.
- Use inquiry and communication skills.

Materials
- scissors, tape/glue
- grid paper of various sizes
- BLM 18.1, 18.2, 18.3

Assessment Opportunities

Minds On ...

Small Groups → Brainstorm → Investigation

Pose the following problem:

A dog is on a chain that is attached to a stake in the ground. The area that the dog can access is badly damaged and the grass needs to be replaced with sod. How can you estimate the amount of sod needed?

Guide students to conclude that they need to find the area of a circle.
Model this problem so students can see that the dog has access to a circular area.

Assign half of the small groups BLM 18.1 and the rest BLM 18.2. Groups follow the instructions for their problem. The purpose of these two activities is to establish a need for a formula for the area of a circle.

The radius of the signal is the maximum distance the signal will reach. A circle with this radius will show the maximum area the signal will cover.

Action!

Small Groups → Exploration

The first three activities on BLM 18.3 involve the exploration of the area of a circle. The activities vary in difficulty. Each group works on one of the activities. Each group summarizes their activity.

Curriculum Expectations/Observation/Mental Note: Circulate to assess communication and inquiry skills.

Consolidate Debrief

Whole Class → Discussion

Discuss the mathematical reasoning students used in each activity. Discuss the pros and cons of each estimation method for determining the area of a circle. Guide students to the conclusion that the formula for the area of a circle is $A = \pi r^2$. Add the formula to the word wall.

Model how to use the formula by determining the area of the circle found on BLM 18.1 and/or BLM 18.2.

Exploration

Home Activity or Further Classroom Consolidation

Complete Activity 4 on the worksheet.

Explain the instructions for Activity 4.

18.1: Can You Hear the Radio?

The radio station located at the satellite icon on the map broadcasts to most of Central and Southern Ontario. Construct a circle surrounding the station to represent its broadcast area. Assume the signal comes from the tip of the antenna.

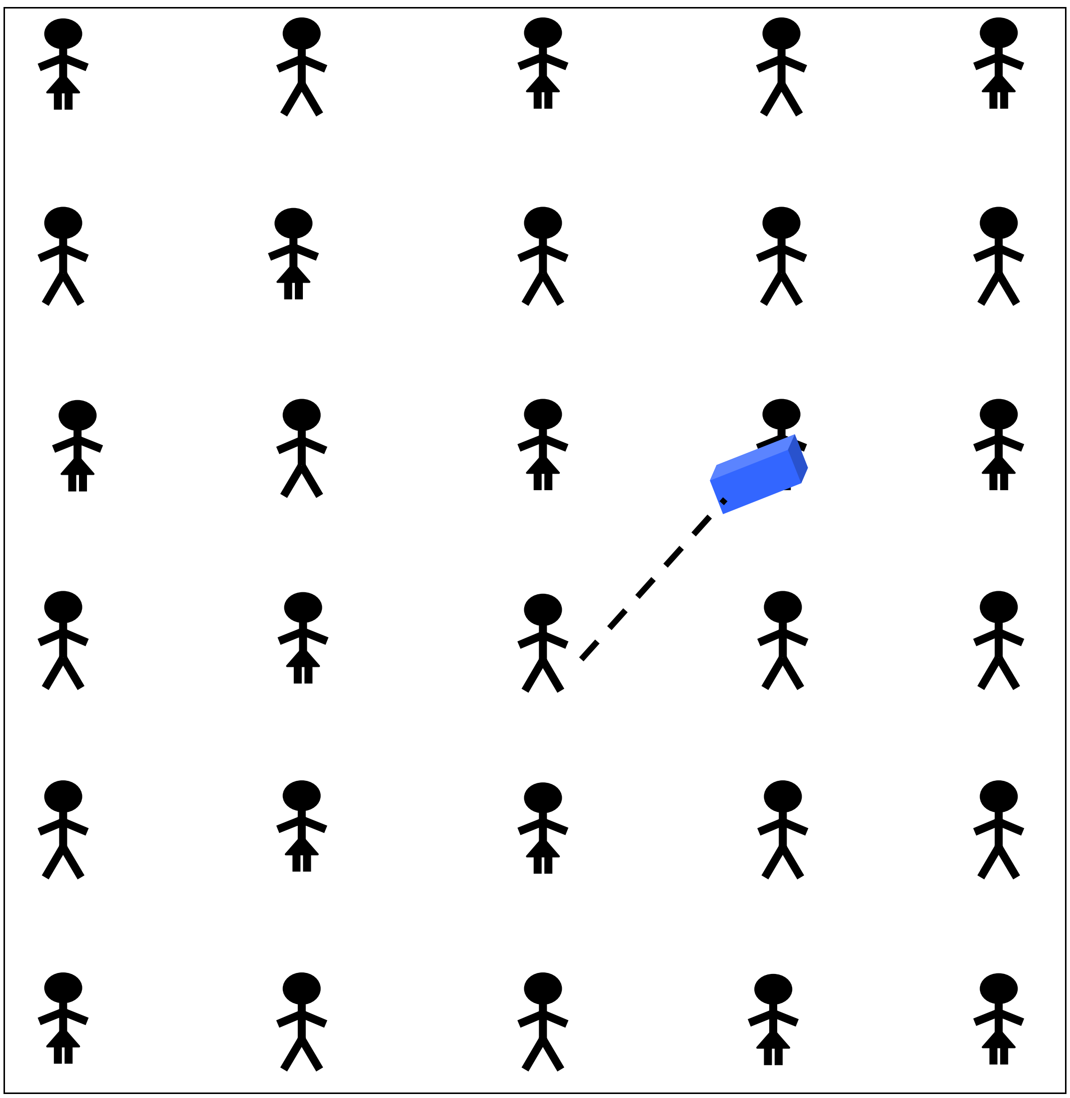
Determine where you would place a radio station in Northern Ontario so it covers the largest possible area in Northern Ontario. The broadcast signal of this new station will have the same strength as the southern station.

18.2: Who Gets Wet?

It is a hot day in June and to cool off, the class decides to use its outside activity break to play a sponge toss. The students line up in rows as shown with 'fair ground' anywhere within the rectangular outline shown. Students stretch on the spot. The person near the middle of the field gets to throw out the first toss. He can throw it far enough to reach the person standing one row ahead of him, one place to the right. Construct a circle to show all possible 'fair' landing locations for the sponge.

The person standing in the upper right corner of the diagram can throw the sponge the same distance. Construct a model of the possible 'fair' landing area for this student's sponge.

What is the total possible 'fair' landing area for the sponge from tosses of the two sponges? How many students might get wet from these two sponges?

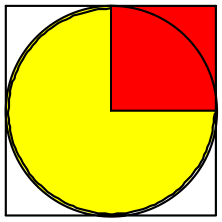


18.3: Estimating the Area of a Circle

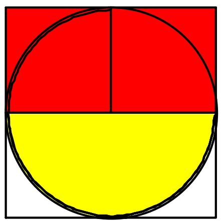
Activity 1: Circles Inscribed in Squares

<u>Materials:</u>
Circle inscribed in a square, scissors, tape or glue, highlighter

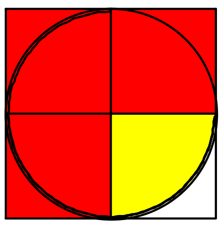
<u>Instructions:</u>



Shade the square as illustrated in the diagram.
What is the length of one side of the square?
What is the area of the square?
Use words to compare the area of the square to the area of one-quarter of the circle, i.e., greater than, less than, equal to.



Shade a congruent square adjacent to the first square as illustrated in the diagram. Use words to compare the area of the two squares to the area of one-half of the circle.



Shade a congruent square adjacent to the second square as illustrated. Use words to compare the area of the three squares to the area of three-quarters of the circle.

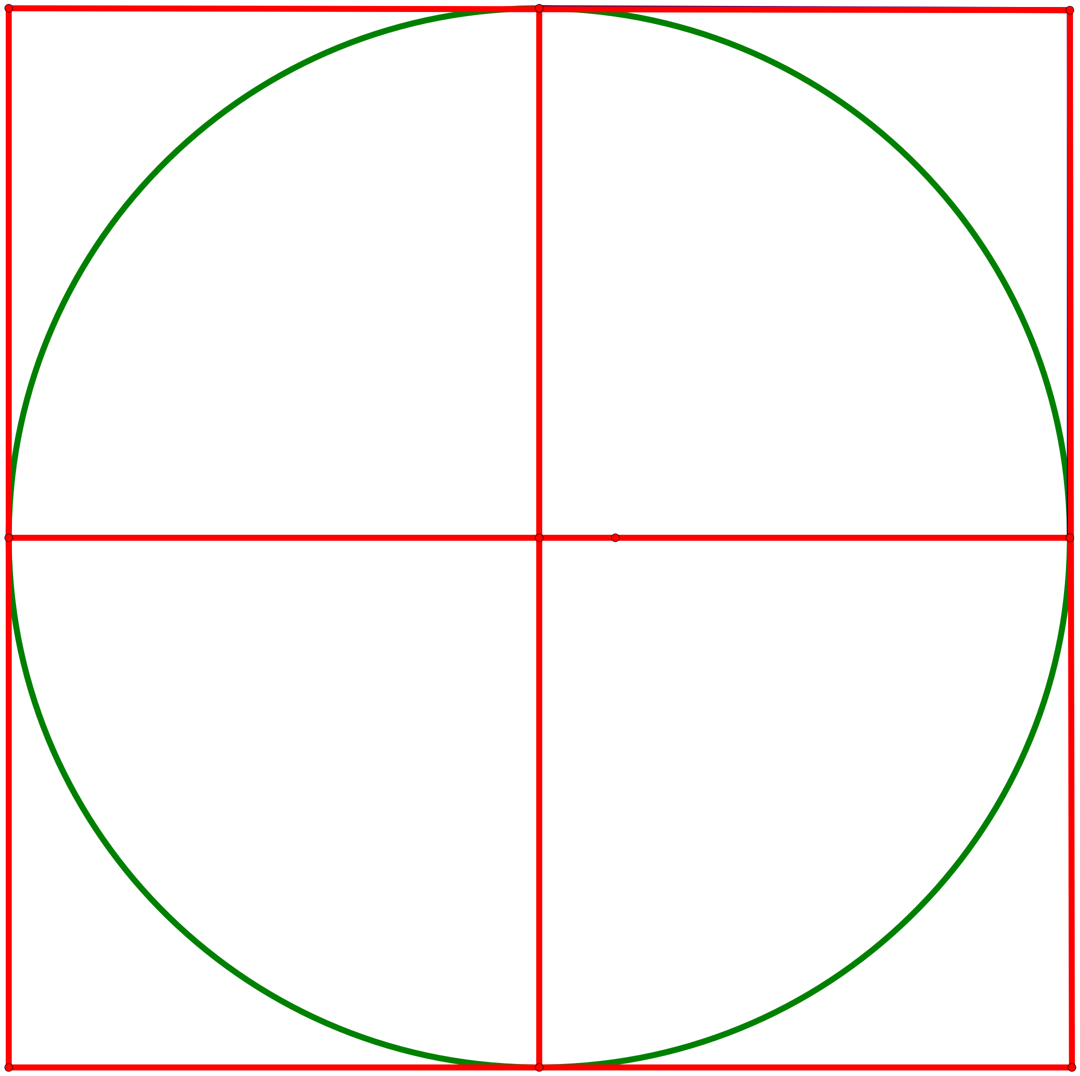
Cut away the three shaded areas covered by the squares which are **not** part of the inside of the circle. Fit the cut out shaded area pieces onto the remaining quarter of the circle. The pieces may need to be cut into smaller pieces to fit inside the circle.

Compare the area of 3 squares to the area of the circle.
Do the shaded pieces completely cover the remaining part of the circle?

Jason said: The area of a circle is just a little more than three times its radius squared.
(Area of a circle) > $3r^2$

Confirm or deny his statement based on your exploration.

18.3: Estimating the Area of a Circle (continued)



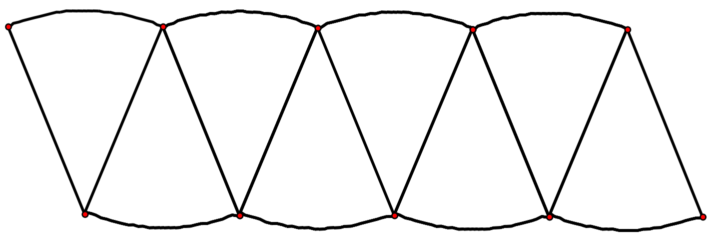
18.3: Estimating the Area of a Circle (continued)

Activity 2: Circles and Parallelograms

<u>Materials:</u>
Fraction circles

Instructions:

Arrange the eight, one-eighth fraction pieces to form a "curvy" parallelogram.
Arrange the 12, one-twelfth fraction pieces to form a different "curvy" parallelogram.



Jessie recalls that the area of a parallelogram is (base x height).

She considers the arrangement of fraction pieces to be a parallelogram and she reasons that the height of the parallelogram is the radius (r) of the full circle.

She further reasons that the base of the parallelogram is half the circumference of the circle, i.e. ($2\pi r \div 2$), which is πr. With all of this information, Jessie concludes that the area of the "parallelogram" is πr^2.

Jessie concludes that the area of a full circle is πr^2.

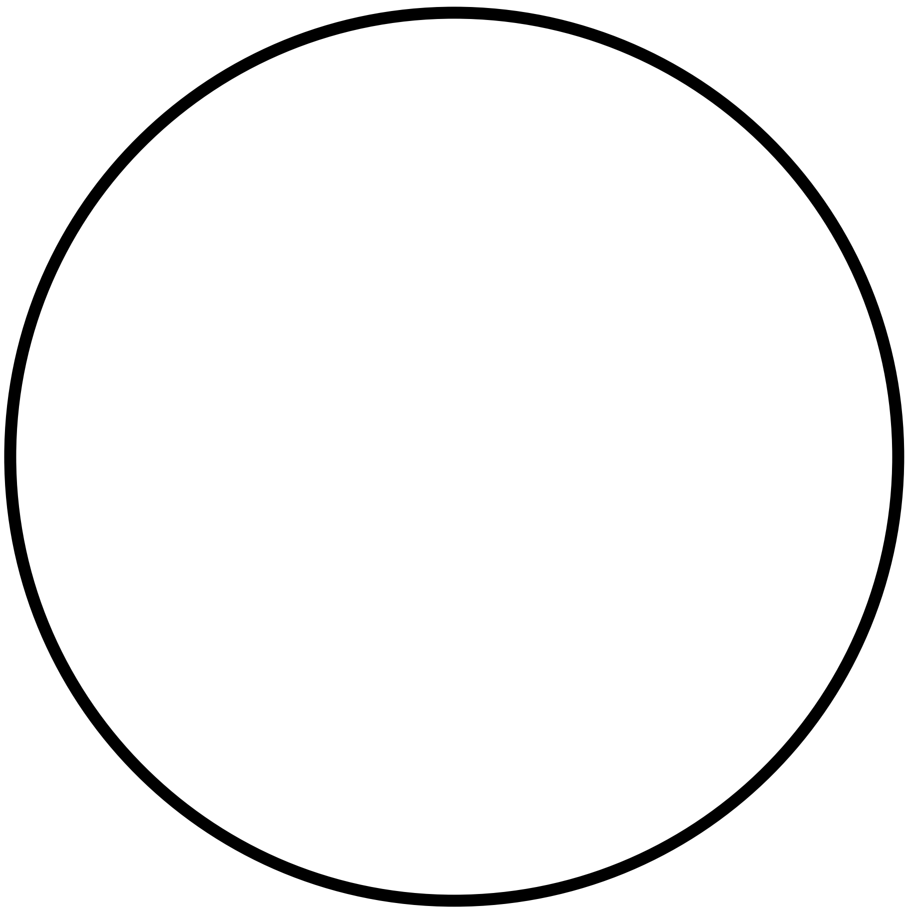
Do you agree or disagree with Jessie's reasoning?

Give reasons for your answer.

18.3: Estimating the Area of a Circle (continued)
Activity 3: Circles on Grid Paper

Trace the circle below onto centimeter grid paper.
Colour the squares it completely covers.
Estimate the area of the circle, remembering to think about the partially covered squares.

How could you use a similar method to get a more accurate answer?



18.3: Estimating the Area of a Circle (continued)

Activity 4: Triangles from Squares on Circles

Diagram 1:
The length of the side of the square is the same as the length of the radius of the circle.

Diagram 2:
Kim cut the square into 4 congruent triangles and placed the squares onto the circle as shown in the second diagram.

Questions:
How many squares will Kim need to completely cover the circle?
(Notice that part of each triangle extends outside the circle).
If the side of the square is 5.0 cm long, then
a) what is the area of one square?
b) what is the area of one circle?
Give reasons for your answer.

Diagram 1

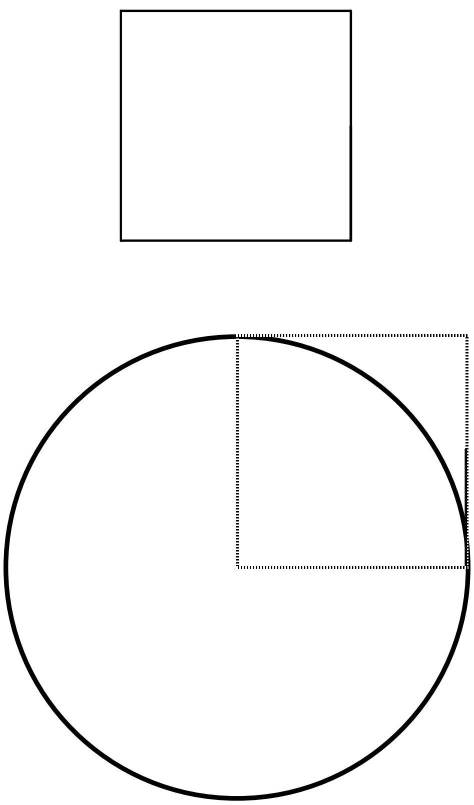
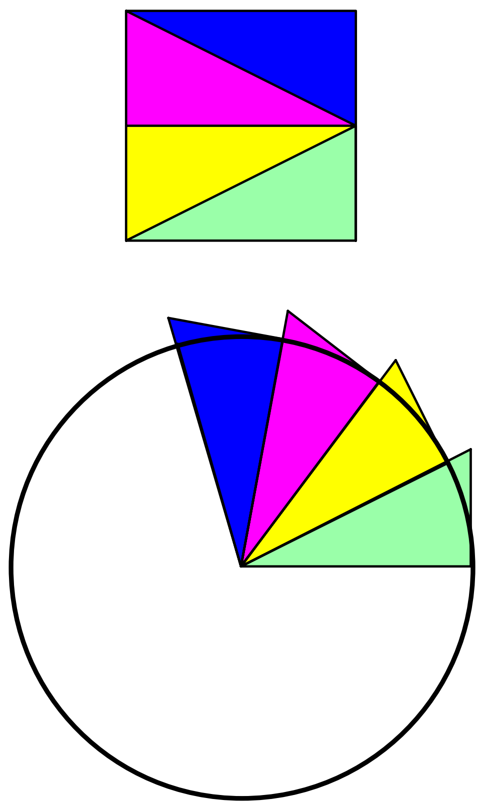
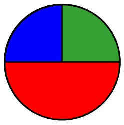


Diagram 2





Description

- Apply formulas for circumference and area of circles in problem solving situations.

Materials

- fraction circles
- chart paper, marker
- index cards

Assessment Opportunities

Minds On ...

Individual → Reflection

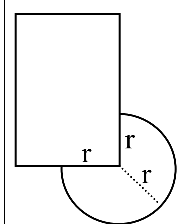
Distribute one index card to each student. Students write one or two interesting points about π that they learned as part of their Day 17 Home Activity. Collect the assignments (students keep the index cards).

Consider completing a few index cards in advance. These can be used by students who did not complete the assignment but who can still benefit from this day's activity.

Whole Class → Give One-Get One

Students share one interesting point from their card with a partner then the partner shares one interesting point from his/her card. Students find new partners and repeat the process.

Draw the "dog on a chain" problem on the board, attaching the chain to a corner of a rectangular doghouse. Illustrate a length of chain that is less than either the width and length of the doghouse. Students describe how to determine the area that the dog has access to. Facilitate a discussion that results in the calculation of the area. Students identify a question that requires the perimeter for an answer.



Action!

Pairs → Practice

Students determine the surface area of a penny, dime and quarter (or surfaces of objects in the classroom).

Curriculum Expectations/Observations/Mental Note: Circulate to assess use of the area formula and use of calculator for order of operations.

Stress good communication.

Small Groups → Discussion

Give each group one piece of chart paper, a marker and one fraction piece from a set of fraction circles. Groups find the perimeter and area of their fraction piece. Prompt students to add the radii lengths when calculating the perimeter of fraction pieces, as necessary.

Calculator keystrokes may need to be reviewed.

Consolidate Debrief

Small Groups → Presentations

Each group presents its solutions on chart paper.

Application
Differentiated

Home Activity or Further Classroom Consolidation

Research: Call two pizza restaurants or use the Internet to find and record the size of large pizzas (diameter), the number of slices, and the prices for pepperoni and cheese pizzas. Make sure you ask whether the measurements are given in centimeters or inches.

Extend the doghouse problem by making the chain length longer than the side length of the doghouse. For example: the doghouse is 2m x 3m and the chain is 4m long. What is the maximum distance that the dog can walk if the chain is always kept tight? What is the area of grass the dog can walk on?

Extending Question…can be offered as a challenge to some students

Description

- Apply formulas for circumference and area of circles in problem solving situations.

Materials

- overhead of BLM 20.1
- BLM 20.2

Assessment Opportunities

Minds On ...

Whole Class → Data Collection

Some students record their data from Day 19 on an overhead of BLM 20.1.

Model several examples of how to convert from inches to cm.

Examples:

18 inches x 2.54 cm / 1 inch = 45.72 cm

55 cm x 1 inch / 2.54 cm = 21.65 inches

Add some "imaginary" pizza prices, if necessary.

Individual → Quiz

Students complete quiz (BLM 20.2).

Curriculum Expectations/Quiz/Marking Key: Circulate to determine differentiated needs for home activity.

To consolidate conversions between fractions, decimals, and percents, students could use geometry skills to find the sector angle of each piece, express size of the piece as a fraction of the whole and then convert the fraction to a decimal and/or percent.

Action!

Small Groups → Activity

Students complete BLM 20.1 on Pizza Prices using the information gathered from the previous day.

Groups identify the most and least expensive pizzas based on surface area.

Consolidate Debrief

Whole Group → Discuss

Facilitate a discussion on the pizza problem. Discuss different factors when determining "best buys," e.g., thickness of pizza, quality of cheese, inclusion of dips.

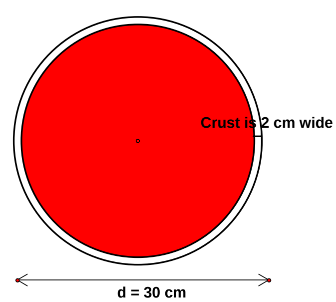
Individual → Response Journal

Students choose a company from which they would purchase their pizza and justify their choices.

Application
Differentiated

Home Activity or Further Classroom Consolidation

1. Pepperoni covers a fraction of the surface of a pepperoni and cheese pizza. Describe how you would determine the percentage of surface area covered by pepperoni.
2. The crust of a pizza is like a rim around the centre part. If this "rim" is 2 cm wide and if the whole pizza has a diameter of 30 cm then how would you find the surface area of the "rim" part? Describe the steps you would take in your solution.



20.1: Pizza Prices

Item Number	Size of Large Pizza (diameter)	Number of Slices	Cost	Area of Pizza	Cost per 1000 cm^2	Rank for Lowest Price per Unit Area

Write a concluding statement about marketing strategies and the best value for your money.

20.2: Developing Proficiency

Grade 8

(from TIPS Section 2)

Name:
Date:

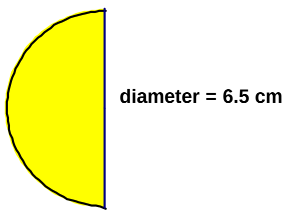
Proficiency

Target Met.
[Practise occasionally to maintain or improve your proficiency level.] ☐

Still Developing
[Search out extra help and practise until you are ready for another opportunity to demonstrate proficiency.] ☐

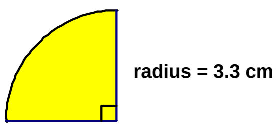
1. Determine the **area** of each shape. *Show your work.*

a)



Answer: ________________

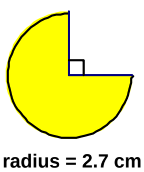
b)



Answer: ________________

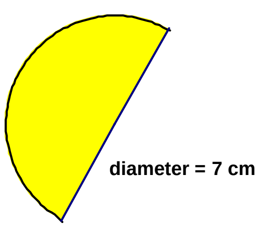
2. Determine the **perimeter** of each shape. *Show your work.*

a)

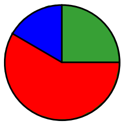


Answer: ________________

b)



Answer: ________________



Description

- Apply formulas for circumference and area of circles in problem solving situations involving composite shapes.

Materials

- BLM 21.1, 21.2, 21.3, 21.4
- graph paper

Assessment Opportunities

Minds On ...

Individual → Questioning

Students write one or two questions they still have about circles. Students create a circular airplane using the piece of paper (see BLM 21.1). Students fly their airplane, possibly during an outdoor activity break, and deconstruct the plane that lands closest to them. Students read the hidden question and think about a response. Volunteers share questions and responses to the questions. Asking students to estimate the distance between them and the location the plane landed. Then ask students to calculate the possible "landing area" if the estimated distance is the maximum distance their plane could fly.

Think Literacy: Cross-Curricular Approaches – Mathematics for the reading strategy Following Instructions.

Whole Class → Brainstorm

Brainstorm a list of shapes that have circular parts giving examples from everyday life, e.g. Lead a discussion about why one might need to know the area or perimeter of the shape.

The level of difficulty of the questions in the pairs activity allows for differentiating by readiness.

Action!

Pairs → Creating Measuring Problems

Cut BLM 21.2 into activity cards. Give each pair a card with directions to verify the area or perimeter and then to create a problem for the answer using a familiar context, e.g., doghouse. Students record their solutions individually. When a pair finishes a problem distribute a new card to the pair. Continue as time allows.

Curriculum Expectations/Observation/Mental Note: Circulate to assess conceptual and procedural knowledge.

GSP 21.1 Skateboard Park. gsp can be used to create similar composite shapes.

Whole Class → Measuring Area

In preparation for the next day's activity, visit the school playground, field or parking area. Tell students that the selected area is now going to become a mini-track for jogging and races. Measure the dimensions of the area using a trundle wheel.

Consolidate Debrief

Whole Class → Discussion

Discuss the connection between the circumference of a trundle wheel and using a trundle wheel to measure distances.

Lead a discussion on the decomposition of the figure in preparation for the home activity (BLM 21.4).

Home Activity or Further Classroom Consolidation

Application
Concept Practice
Differentiated

Determine the area of the model of the skateboard park on worksheet 21.4.

Create a model of the area for the track.

Optional: Create a crossword puzzle with terms from this unit of study.

21.1 How to Build a Circular Paper Airplane

Instructions

1. Fold the paper in half horizontally then re-open the paper.
   Refer to the fold you created as the half-line.

2. Fold the bottom half of the paper in half horizontally to the half line.
   Refer to the new fold line as the quarter-line. **Do not** unfold this time.

3. Fold the top of the new “flap” to the bottom of the paper to the quarter line.
   Refer to the new fold line as the eighth-line.

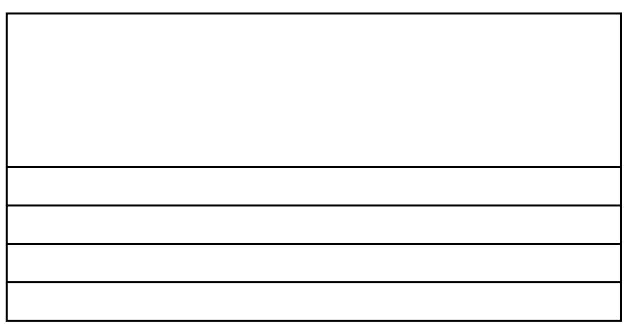
4. Hold the three layers of paper at each end. Flip the whole sheet horizontally so that the folds are now farthest away from you.

5. Fold the new top edge to the half-line. (See side view below.)
   Then re-fold the half line.
   You should now be looking at a paper with four folds in it, and should see only half of the sheet.

6. Now roll the paper lengthwise into a loop. Then, slide the folded ends into each other and overlap approximately 5 cm.

Your circular paper airplane is thrown like a football with the folds forward.

Top “unfolded” view of sheet and folds



Half line

Quarter line

Side “folded” view of sheet and folds

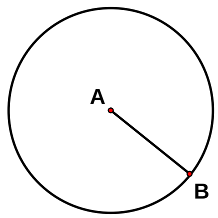
Quarter line

Half line

21.2: Composite Figures

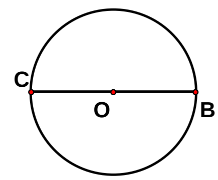
1) Area (A) Circumference (C)

AB = 3.0 cm
C = 18.8 cm
A = 28.0 cm^2



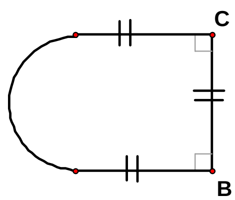
2) Area (A) Circumference (C)

CB = 4.8 cm
C = 15.1 cm
A = 18.1 cm^2



3) Area (A) Perimeter (P)

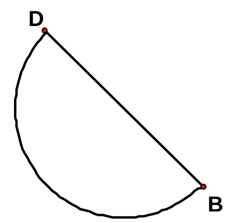
CB = 4.0 cm
P = 18.4 cm
A = 22.5 cm^2



4) Area (A) Perimeter (P)

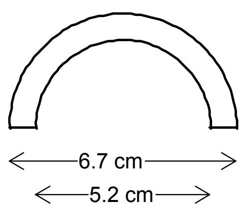
DB = 6.5 cm

A = 16.8 cm^2
P = 16.8 cm



5)

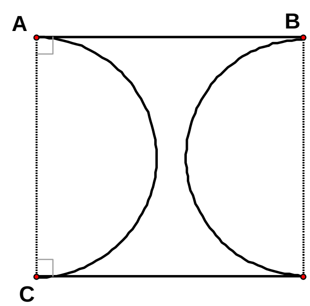
Area of "rim" = 7.2 cm^2



6)

AB = 7.8 cm
AC = 7.0 cm
Area = 16.2 cm^2

The broken lines are diameters.

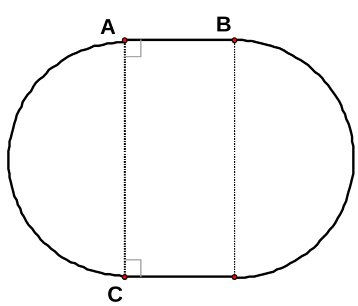


7) Area (A) Perimeter (P)

AB = 3.3 cm
AC = 7.0 cm

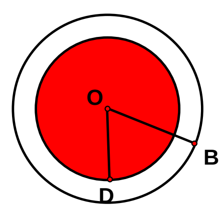
P = 28.5 cm
A = 61.1 cm^2

The broken lines are diameters.



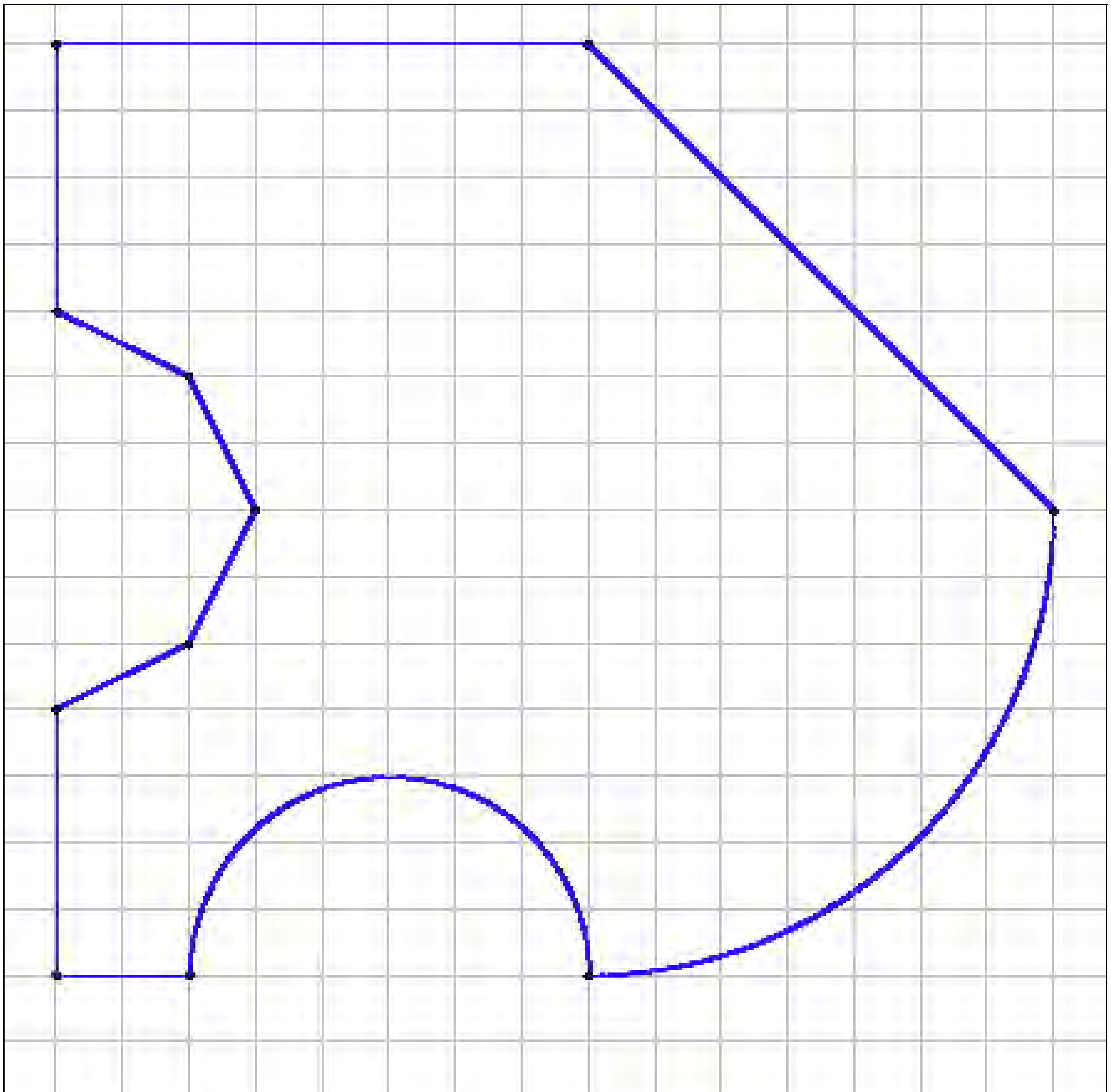
8)

OB = 2.7 cm
OD = 2.1 cm



Unshaded area = 9.9 cm^2

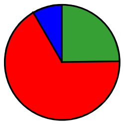
21.4: Skateboard Park – A Composite Shape



Each Square is 1cm by 1cm
When calculating, round all values to the nearest tenth

1. Calculate the perimeter of this figure (show all of your work) **Hint:** label all line segments and arcs and don't forget to use the Pythagorean Theorem

2. Calculate the area of this figure (show all of your work) Hint: try to find shapes that you already know, then add or subtract the areas as required



Description
- Performance task

Materials
- BLM 22.1

Assessment Opportunities

Minds On ...

Pairs → Discussion

Students share solutions to the home activity. Students display their models for the race track area.

Debrief the home activity as a whole class.

Curriculum Expectations/Model/Rubric: Circulate to assess models for the track area.

Ensure that each student has a model that will allow them to complete the performance task.

Action!

Individual → Performance Task

Students individually do BLM 22.1.

Curriculum Expectations/Performance Task/Rubric: Evaluate students for accuracy of computation, appropriateness of mathematical models formed, reasoning and clarity of communication.

GSP 22.1. RaceTrack.gsp can be used to check student responses or to debrief the performance task.

Consolidate Debrief

Whole Class → Discussion

Begin preparing students for the next unit of study.

Application

Home Activity or Further Classroom Consolidation

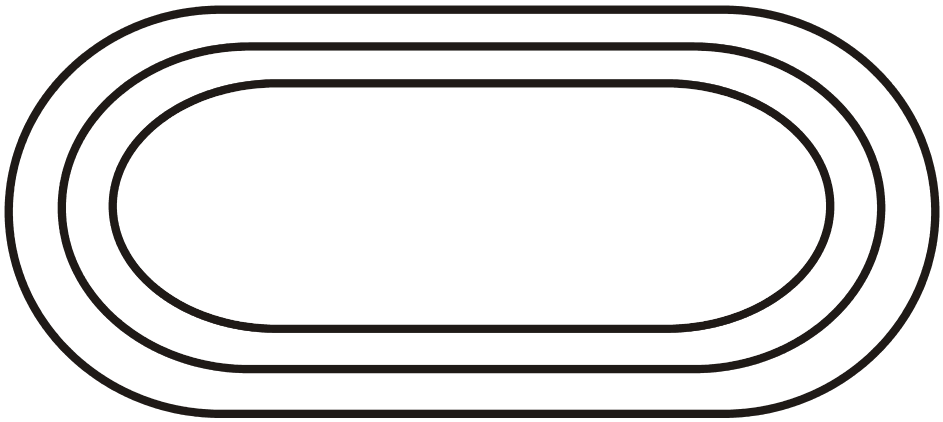
When returning graded work to students, consider photocopying samples of Level 3 and 4 responses with student names removed. Select and discuss, with the class, samples that show a variety of strategies.

22.1: Track and Field

Your school is going to use part of its school grounds to build a two-lane track.

1. Determine the dimensions of the space available for the track.
   Sketch a model of the area and record dimensions on the model.

2. Add a two-lane track to the model of your school's available area.
   Determine appropriate dimensions for the two-lane track.
   Record the dimensions on your model.
   Record enough information so you can calculate perimeter and area.



3. Compare the distances for one complete lap for a runner in each lane.
   Show all work and state any assumptions you make, e.g., the runner uses the centre part of the lane, or the runner uses the innermost part of the lane.

4. Explain how to find the number of laps a racer would have to complete for a 2-km race. Do **not** show calculations.

5. Your school is going to place grass sod in the middle of the track. Determine the size of the area that needs to be covered with sod.

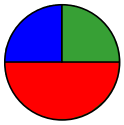
6. Create and solve a problem based on your two-lane track.

BIG PICTURE

Students will:
- demonstrate understanding that operations with fractions have the same conceptual foundation as operations with whole numbers;
- use problem solving strategies like “make a simpler problem,” and “draw a picture”;
- demonstrate understanding that the multiplication and division of fractions can be modeled with area;
- use patterning to determine the “invert and multiply” rule for division by a fraction;
- demonstrate proficiency in operations with fractions;
- represent composite numbers as products of prime factors;
- demonstrate proficiency in using a calculator to determine calculations with up to two fractions;
- solve and explain multi-step problems involving fractions;
- use concrete representations in problem solving situations.

Day	Lesson Title	Description	Expectations
26	Pizza and Cake	• Activate and assess prior knowledge of fractions.	8m7, 8m13, 8m14, 8m15, 8m18, 8m30, 8m32, 8m33 CGE 2c, 3c, 5a
27	Fraction Frenzy	• Assess for prior learning of fractions.	8m9, 8m14 CGE 4f, 5a
28	Parts Problems	• Use manipulatives and symbols to represent the multiplication of a whole number by a fractional quantity. • Calculate the product of a whole number and a fractional quantity.	8m2, 8m14, 8m15, 8m18, 8m31 CGE 3b, 5a, 5e
29	Multiplying – Zero to One and Beyond!	• Compare the result of multiplying a number by a fraction between 0 and 1 with the result of multiplying a number by a number greater than 1.	8m2, 8m9, 8m15, 8m18, 8m28, 8m28, 8m30 CGE 2b, 3c, 3e
30	Modelling with Area	• Represent the multiplication of two fractions where both fractions are between 0 and 1.	8m2, 8m9, 8m15, 8m18 CGE 3c, 4b, 5e
31	Simply Using Symbols	• Multiply two fractions where both fractions are between 0 and 1 using symbols.	8m2, 8m15, 8m18, 8m30 CGE 3b, 4e
32	Mixed Models	• Multiply mixed fractions.	8m2, 8m15, 8m18, 8m7, 8m9 CGE 2c, 5a, 5g

Day	Lesson Title	Description	Expectations
33	Let's Share	• Represent the division of two fractions.	8m2, 8m15, 8m18, 8m31 CGE 2c, 5a
34	Just One Please	• Use patterning to develop the 'invert-and-multiply' rule for dividing two fractions. • Practise dividing fractions using unit rate questions.	8m2, 8m6, 8m7, 8m15, 8m18 CGE 3b, 4e, 5a
35	TIPS Resources	• Use resources from TIPS continuum on fractions to consolidate understanding.	8m2, 8m15, 8m18, 8m29, 8m9 CGE 4e, 5a, 5g
36	Connecting to Composites	• Express composite numbers as products of prime numbers to find lowest common multiples. • Solve problems that require the lowest common multiple.	8m13, 8m9, 8m7 CGE 3b, 3c
37	Summative Assessment	• Administer a paper-and-pencil summative assessment.	



Description

- Activate and assess prior knowledge of fractions.

Materials

- geoboards
- fraction circles
- chart paper
- markers
- BLM 26.1, 26.2

Assessment Opportunities

Minds On.

Small Groups → Ordering Fractions

As math class time begins distribute fraction cards from BLM 26.1. Students find other students who have cards of the same colour and arrange their group's fractions in order and discuss their reasoning.

Two groups with the same colour set of fractions form a larger group to discuss the strategies they used. Each of the two large groups plan a presentation of two of their strategies for the other group.

Groups → Presentation

Each group presents a summary of their strategies.

Curriculum Expectations/Observation/Anecdotal Notes: Assess prior knowledge.

Students may use a calculator to change each fraction to a decimal.

Action!

Small Groups → Modelling Fractions

Set up multiple stations of the two activities. (BLM 26.2).

Students work in small groups at one of the stations for half the time then switch stations. Students prepare their solutions for whole-class discussion on chart paper.

Curriculum Expectations/Observation/Anecdotal Notes: Assess prior knowledge.

Students use their knowledge of multiples to determine common denominators.

Consolidate Debrief

Whole Class → Discussion

Use the chart paper solutions to consolidate understanding:

1) Equal fraction pieces (same area) can have different shapes.
2) Equal fractions can be expressed in different ways.
3 Fractions can be expressed with common denominators for addition.
4) $\frac{n}{n} = 1$
5) Fractions can be reduced when numerator and denominator share a common factor greater than 1.

Refer to the Minds On activity to discuss how to use common denominators and benchmarks (0, $\frac{1}{2}$, and 1) when comparing fractions.

Students to review the factors of composite numbers, as they reduce fractions.

Reflection

Home Activity or Further Classroom Consolidation

To prepare for next day's assessment task, make a graffiti sheet of things you remember about fractions. Consider including:

a) terminology (e.g., proper, improper)
b) how to add and subtract fractions using symbols
c) how to represent fractions on a number line

26.1: Fraction Cards

Cut into vertical strips. This produces sufficient cards for four classes.

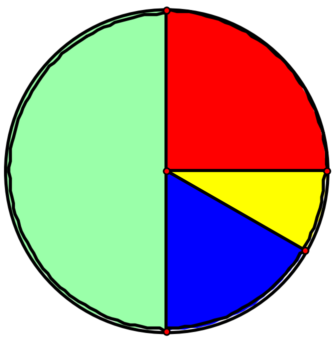
$\frac{7}{16}$	$\frac{7}{16}$	$\frac{5}{12}$	$\frac{5}{12}$
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
$\frac{13}{21}$	$\frac{13}{21}$	$\frac{5}{8}$	$\frac{5}{8}$
$\frac{2}{3}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{3}{4}$
$\frac{7}{6}$	$\frac{7}{6}$	$\frac{23}{22}$	$\frac{23}{22}$
$\frac{6}{5}$	$\frac{6}{5}$	$\frac{22}{21}$	$\frac{22}{21}$
$\frac{7}{5}$	$\frac{7}{5}$	$\frac{23}{21}$	$\frac{23}{21}$

26.2: Fraction Stations

Pizza Pieces

1. Use circular fraction pieces to create a model for a pizza that has been cut into pieces. On chart paper:
   a) Draw the model.
   b) Write an equation for the model.
   c) Show that the equation is true.

Example:

Draw the model.	Write the equation.	Show that the equation is true.
	$\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+\frac{1}{12}=1$	$\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+\frac{1}{12}$ $=\frac{6}{12}+\frac{3}{12}+\frac{2}{12}+\frac{1}{12}$ $=\frac{6+3+2+1}{12}$ $=\frac{12}{12}$ $=1$

2. Create different models using the same procedure.

Different models might have:

a) a small number of fraction pieces
b) a large number of fraction pieces
c) all fraction pieces the same size
d) some fraction pieces the same size and some pieces of different size

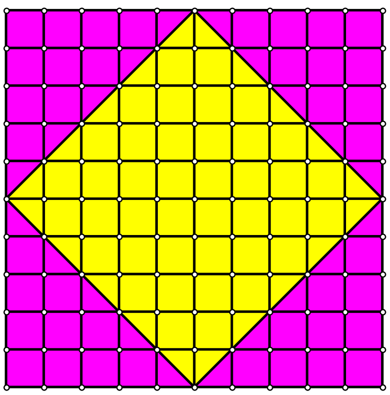
26.2: Fraction Stations

Pieces of Cake

1. Create a geoboard model for one whole cake that has been cut into pieces. On chart paper:

   a) draw the model;
   b) write an equation for the model;
   c) show that the equation is true.

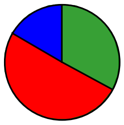
Example:

Draw the model.	Write the equation.	Show that the equation is true.
	$\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{2}=1$	$\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{2}$ $=\frac{4}{8}+\frac{1}{2}$ $=\frac{1}{2}+\frac{1}{2}$ $=1$

2. Create different models using the same procedure.

Different models might have:

   a) a small number of pieces
   b) a large number of pieces
   c) the same size for all pieces
   d) some pieces of the same size and some pieces of different size



Description

- Assess prior learning

Materials

- manipulatives
- chart paper
- markers
- calculators
- BLM 27.1

Assessment Opportunities

Minds On ...

Small Groups → Pass It On!

Post Graffiti sheets with the following titles:

1) Show different ways to find $2\frac{2}{3}+1\frac{1}{2}$
2) Show different ways to find $2\frac{2}{3}-1\frac{1}{2}$
3) List fraction words and meanings.
4) Show some fractions on a number line.

At posted sheets, students work in small groups to share prior knowledge of fractions. Groups move to different sheets. Circulate to answer/pose questions. Leave sheets posted during assessment for prior learning.

Think Literacy: Cross-Curricular Approaches, Grades 7-12, p.104
On the fourth sheet draw a number line.

Manipulatives available at each station: pattern blocks, fraction circles, geoboards, rulers.

Check that sheets contain correct information.

Action!

Individual → Assessment

Review instructions with students (BLM 27.1). Students complete the worksheet.

Curriculum Expectations/Paper-pencil Assessment/Rubric: Assess students' prior learning.

Consolidate Debrief

Individual → Reflection

Students reflect on their answers to questions 8.

Reflection
Application

Home Activity or Further Classroom Consolidation

Think of different contexts for questions that can also be answered using the expression: $6\times\frac{2}{3}$.

27.1: Fraction Frenzy: Pre-Unit Assessment

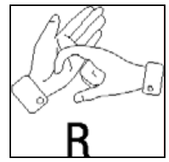
Name:
Date:

Instructions:

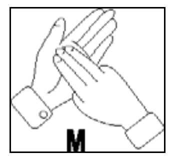
1. Answer all of the questions as completely as possible.
2. Your answers will be assessed for understanding, communication, and correctness.
3. If you think you can explain your reasoning much better by talking or by showing something to me, draw a speaker symbol beside the question.

Speaker symbol:
<)))

4. If you need me to read something to you, show me the sign language symbol for R.



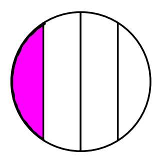
5. If you need manipulatives, show me the sign language symbol for M.



6. You may use a calculator for any part of this assessment.

---

1. A circle is divided into four parts as shown in the diagram. One of the parts is shaded.

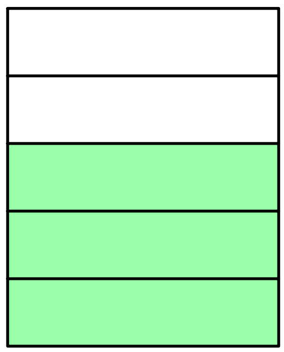


   Which fraction of the whole circle is shaded?
   Give reasons for your answer.
   a) one-quarter   b) less than one-quarter   c) more than one-quarter

27.1: Fraction Frenzy: Pre-Unit Assessment (continued)

2. a) Use the diagram to convince Robyn that

$\frac{3}{5}$ of a chocolate cake is equivalent to $\frac{12}{20}$ of the cake.



b) Could you convince Robyn that $\frac{3}{5}$ is equivalent to $\frac{12}{20}$ without using a diagram?

Justify your answer.

3. a) Put a checkmark (✓) in the one column that best describes the given number.

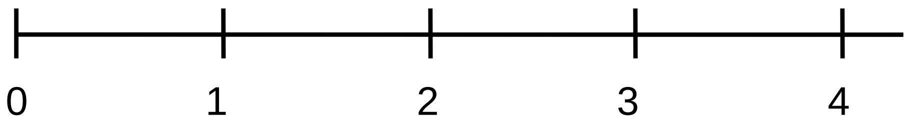
		Between 0 and $\frac{1}{2}$	**Between $\frac{1}{2}$ and 1**	**Greater than 1**
i)	$\frac{6}{11}$			
ii)	$\frac{62}{61}$			
iii)	$\frac{42}{83}$			

b) *Explain* your answer for $\frac{42}{83}$.

4. Sue, Lila, and Mei Ling jog on a track every morning.

   Sue jogs $\frac{7}{8}$ km, Lila jogs $2\frac{5}{6}$ km and Mei Ling jogs $3\frac{1}{2}$km.

   a) Use the number line to represent the distance that each person jogs.



   b) How much farther does Mei Ling jog every morning than Sue? *Show your work.*

   c) How much farther does Mei Ling jog than Lila in one week? *Show your work.*

5. Calculate. *Show your work.*

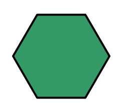
   a) $2\frac{1}{2}+3\frac{2}{3}$

   b) $4\frac{1}{3}-2\frac{3}{4}$

6. Both Jay and Ali save a fraction of their weekly allowances.
Compare the fractions to determine who saves the largest fraction of their allowance each week.

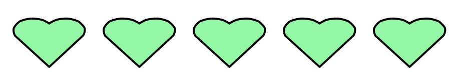
	Jay's Fraction	Ali's Fraction	Explanation
Week 1	$\frac{4}{11}$	$\frac{3}{11}$	_______ saves the largest fraction. I know this because …
Week 2	$\frac{3}{7}$	$\frac{3}{8}$	_______ saves the largest fraction. I know this because …
Week 3	$\frac{10}{11}$	$\frac{9}{10}$	_______ saves the largest fraction. I know this because …

7. a) Deb drew this picture to represent one whole:



Draw a picture to represent $\frac{7}{6}$ of Deb's whole:

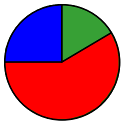
b) Chi drew these five hearts to represent one whole:



Draw a picture to represent $1\frac{1}{2}$ of Chi's whole:

8. Answer one of the following questions clearly using mathematics vocabulary.

a) Describe how fractions are used in the kitchen.
b) I am most like a fraction when I …
c) I am not yet comfortable with fractions because …
d) I always enjoy working with fractions because …



Description

- Use manipulatives and symbols to represent the multiplication of a whole number by a fractional quantity.
- Calculate the product of a whole number and a fractional quantity.

Materials

- fraction circles, pattern blocks, cube links, graph paper
- chart paper
- markers
- BLM 28.1

Assessment Opportunities

Minds On ...

Pair/Share → Multiple Responses

Students share responses to the previous day's Home Activity: Record and post samples of their responses. Discuss whether each problem is well connected to the computation.

Example Responses:

1) 6 bottles are $\frac{2}{3}$ filled with water. How many full bottles of water are there in total?

2) Jay walked $\frac{2}{3}$ of a km. Keri walked 6 times as far. How far did Keri walk?

Action!

Small Groups → Modelling using manipulatives

Students choose one of the posted problems and represent its solution using manipulatives. Students explain how to find the answer using manipulatives. Challenge students to represent the problem using a different manipulative. Students report to the whole group.

Curriculum Expectations/Observation/Anecdotal Notes: Circulate, asking reflective questions at each group. Determine if each student can state the representation for one whole – every other representation depends on this.

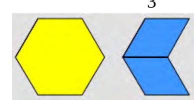
Whole Class → Demonstration

Help students understand that multiplication problems like this are like whole number computations. Demonstrate 6 × 4 by putting 4 identical objects in each of 6 bags. The total is 24 objects.

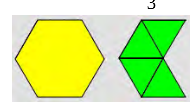
Model $6\frac{2}{3}$. Represent one whole with one hexagonal pattern block piece, and represent $\frac{2}{3}$ with 4 triangle pieces. Put 4 triangles into each of six bags. Write the symbols for the solution and discuss why the answer is $\frac{24}{6}$. Further discuss why this is simplified to 4.

Sample Representations:

1 whole $\frac{2}{3}$



1 whole $\frac{2}{3}$



Pairs → Practice

Students works in pairs on BLM 28.1.

Consolidate Debrief

Whole Class → Discussion

Compare these questions: 5 × 3, 5 × $\frac{3}{8}$, 5 × 3 cm. What is the same and what is different when you calculate answers to these questions using just the symbols? Summarize student discoveries on multiplying a whole number by a fractional part. Include observations on reducing fractions and changing forms (proper to improper and vice versa).

Application

Home Activity or Further Classroom Consolidation

Create and solve five questions that involve a whole number multiplied by a fractional part.

28.1: Parts Problems

1. Determine a solution to each of the following problems. Show a manipulative representation as well as a symbolic solution.

Problem	Manipulative Solution	Symbolic Solution
a) Dave ate $\frac{2}{5}$ of a mini pizza. John ate 3 times as much. How many mini pizzas did John eat?	One whole is represented by: Here's my solution:	
b) Farrell spent $\frac{3}{4}$ of an hour doing homework every night for 8 nights in a row. How many hours did he spend on homework?	One whole is represented by: Here's my solution:	

2. Calculate:

a) $7 \times \frac{5}{6}$ b) $7 \times \frac{5}{14}$ c) $10 \times \frac{3}{25}$

Description

- Compare the result of multiplying a number by a fraction between 0 and 1 with the result of multiplying a number by a number greater than 1.

Materials

- Alice in Wonderland story
- BLM 29.1

Assessment Opportunities

Minds On ...

Whole Class → Discussion

Read an excerpt from "Alice in Wonderland" or discuss the parts in the story where Alice changes size. Tell students that the author Lewis Carroll was a mathematician named Charles L. Dodgson then explain that you are going to use mathematics to show what happened to Alice.

Reference: *Making Sense of Fractions, Ratios, and Proportions*, 2002 NCTM Yearbook.

Action!

Pairs → "A" answer to "B"

Display the first row of BLM 29.1 on an overhead. Ask Partner A to answer the following question to Partner B:

If Alice was 120 cm tall and drank from a magic bottle that will make her 1/2 of her height, what will her new height be?

Ask any partner B to explain and justify the answer to the whole class. Continue with each row of the chart and record student responses on the overhead.

Learning Skills/Observation/Mental Note: Assess Staying on Task during pair discussions.

Prepare BLM 29.1 as an overhead transparency.

Whole Class → Discussion

Discuss what mathematical operation could go between Alice's *before* height and the magic bottle fraction to get Alice's *after* height. (answer: multiplication) Students answer additional questions in which a whole number is multiplied by mixed number. Students estimate an answer before calculating it.

Help students equate division by 2 and multiplication by $\frac{1}{2}$; division by 3 and multiplication by $\frac{1}{3}$, etc.

Curriculum Expectations/Observation/Mental Note: Assess student understanding that multiplying a number by a number larger than one results in a product that is larger than the original number.

If students think that the operation is division, ask:

- Would Alice be taller or shorter after drinking this potion?
- Do I get a shorter height by multiplying or by dividing by this fraction?

Consolidate Debrief

Pairs → Reflection

Students individually answer the question on the poster – *Who is correct?* In pairs, share responses. Discuss to form a whole group response with justification.

Sample Response:

Multiplying by a number greater than 1 results in a product that is larger than the starting number. Multiplying by a number that is between 0 and 1 results in a product that is smaller than the starting number.

Classroom poster TIPS Section 5 – Who is Correct

Application
Concept Practice

Home Activity or Further Classroom Consolidation

For each of the expressions determine if the answer will be larger or smaller than the first number. Create a word problem for each of the expressions. Calculate the answer. Show your work.

a) $4 \times 2\frac{3}{4}$ b) $2 \times \frac{3}{8}$ c) $6 \times \frac{15}{12}$

Consider giving some students fraction circles or strips to use at home.

29.1: Alice Grows and Shrinks

Alice's Height (cm) Before	Magic Bottle	Alice's Height (cm) After
120 cm	$\frac{1}{2}$	
120 cm	$\frac{1}{3}$	
120 cm	$\frac{1}{6}$	
120 cm	$\frac{5}{6}$	
120 cm	$\frac{11}{6}$	
120 cm	$1\frac{2}{3}$	

Description

- Represent the multiplication of two fractions where both fractions are between 0 and 1.

Materials

- data projector
- internet access
- BLM 30.1

Assessment Opportunities

Minds On ...

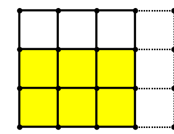
Pairs → Creating Models

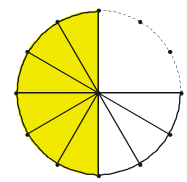
Display a question:

Three-quarters of a cake was left over from the Mad Hatter's Tea Party. Alice ate $\frac{2}{3}$ of the left-over cake. How much of the whole cake did she eat?

Working in pairs, students create a pictorial model that could be used to solve the problem.

Sample Models:

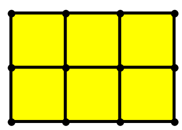




Action!

Whole Class → Discussion

Students share responses for the Minds On activity. Discuss which types of models were easiest to use. Would any of the models be easy to use if you had to find an answer to a question like: $\frac{2}{17} \times \frac{14}{15}$? Make the problem simpler as a model for multiplying fractions. Draw this model:



Students explain how this rectangular model can be used to show that 2 × 3 = 6. Discussion should lead to understanding that the multiplication of 2 numbers can be represented by the length and width of a rectangle and the product represents the area of this rectangle.

Further explore the rectangular model for multiplication of whole numbers with a virtual manipulative: http://matti.usu.edu/nlvm/nav/vlibrary.html → Choose Index → Choose Rectangle Multiplication—Numbers & Operations (3-5).

Explain that you are now going to use an area model for the multiplication of two fractions.

The length of the rectangle is 3 units and the width is 2 units, so the area is 3 x 2 which equals 6 square units.

Whole Class → Presentation

Use a computer projection unit to access the site:

http://matti.usu.edu/nlvm/nav/vlibrary.html → Choose Index → Choose *Fractions - Rectangle Multiplication - Numbers & Operations (*3-5). Model several examples, with students directing your actions.

The presentation can be modelled using an overhead and a marker but using the Internet allows time for many more examples.

Consolidate Debrief

Pairs → Investigation Using Computers

In pairs, students work with virtual manipulatives for fractions of rectangles, (http://matti.usu.edu/nlvm/nav/vlibrary.html). One partner is the "driver," one partner is the "recorder." Partners exchange roles for each question. Students record their solutions and submit them for assessment at the end of the activity. Circulate, asking questions to further develop understanding of the area model.

Learning Skills/Staying on Task/Checklist: Assess students for staying on task and working with other students to interpret instructions.

Students can use this interactive site for further practice.

Concept Practice

Home Activity or Further Classroom Consolidation

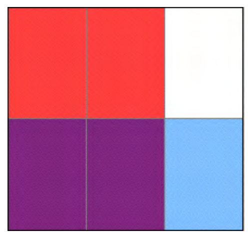
Show this website to someone and explain a solution. If Internet access is not available at home consider visiting the local library or share a pictorial model of multiplying fractions using the rectangle model.

30.1: What's My Share?

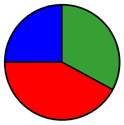
There is part of one rectangular pizza left over from Friday night's supper.

The *first* column in the chart shows the fraction of the left-over pizza that you get to eat. The *second* column shows the fraction of one pizza that is left over.

Complete the chart. (The first row is already complete.)

Your share of the left overs	Fraction of pizza that is left over	Picture solution	Your share as a fraction of the whole pizza	Do you get to eat more or less than one-half of a full pizza?
$\frac{2}{3}$	$\frac{1}{2}$		$\frac{2}{6}$ or $\frac{1}{3}$	Less
$\frac{4}{5}$	$\frac{1}{3}$			
$\frac{3}{4}$	$\frac{2}{5}$			

Create more questions. Record solutions.



Description

- Multiply two fractions where both fractions are between 0 and 1 using symbols.

Materials

- one set of number cards for every four students
- BLM 31.1

Assessment Opportunities

Minds On ...

Small Groups (4) → Game

Students play Fraction Action. (BLM 31.1)

Curriculum Expectations/Observation/Mental Note: Circulate and assess for understanding of use of benchmarks and common denominators for comparing fractions.

Students may need instructions on using calculators to compare fractions.

Action!

Individual → Model making

Ask: Which fraction is larger, $\frac{3}{4}$ or $\frac{2}{3}$?

Will the product of these fractions will be: larger or smaller than $\frac{3}{4}$; larger or smaller than $\frac{2}{3}$? Students individually determine an answer to the problem.

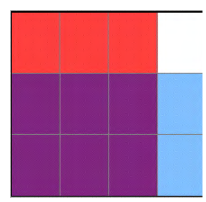
Curriculum Expectation/Quiz/Rubric: Collect and assess for understanding.

Whole Class → Discussion

Some students may have done a symbolic solution. Connect to their symbolic responses to the rectangular model.

$$\frac{3}{4} \times \frac{2}{3} = \frac{6}{12} = \frac{1}{2}$$

Discuss each number in the symbolic solution.
Where do we see "6" in the area model?
Where do we see "12" in the area model?
etc.



Whole Class → Brainstorm

Advise students that they will be working in small groups on some problems. Brainstorm a list of things to do when problem solving and when working with fractions, e.g., ask yourself – What is one whole in this question and how can I model it?

See *Think Literacy: Cross-Curricular Approaches*, Reading Strategies for problem solving.

Small Groups → Problem Solving

Students solve teacher-prepared problems that involve the product of two fractions (both less than one). Example: Robyn's recipe for salad dressing requires 2/5 cup of vinegar. Eila's thinks she needs only 1/3 of Robyn's recipe for her salad. How much vinegar will Eila need?

Consolidate Debrief

Whole Class → Class Journal

Guide the development of a class journal entry for someone who is absent, explaining how to find fraction products symbolically.

Concept Practice

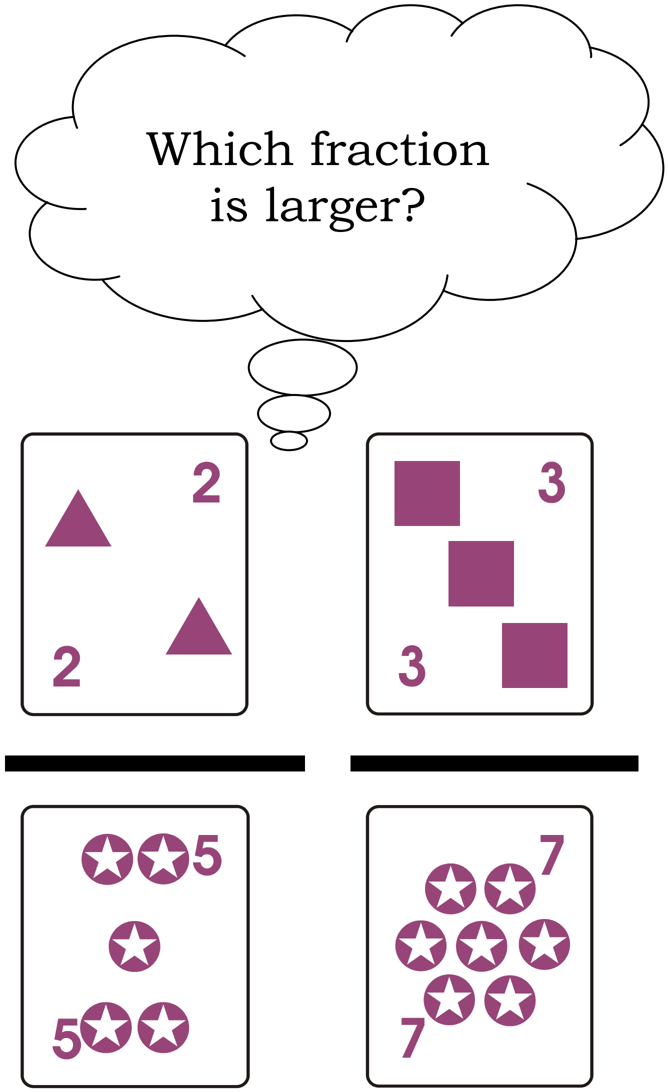
Home Activity or Further Classroom Consolidation

Create and solve fraction questions that involve the product of two fractions that are less than one whole. Record any questions you still have about the multiplication of fractions. Share your response.

31.1: Fraction Action!

Instructions:

1. Two pairs of students form a group of four.
2. Each group of four students needs one set of cards.
3. One player deals the entire set into two equal stacks of cards, numbers down. Each pair gets one of the stacks.
4. Each person turns over one card from their team's stack.
5. Each pair forms a fraction by using the smaller card as the numerator and the larger card as the denominator.
6. The pair with the largest fraction claims all of the cards and puts them on the bottom of their stack. The other pair may challenge the claim and check using a calculator, but if the pair loses the challenge, two cards are given to the other team.
7. If a tie occurs, the tie cards are put back into the middle of each pair's stack.
8. Play can continue until one team has all of the cards or until time is called.



Description

- Multiply mixed fractions.

Materials

- manipulatives (variety)
- BLM 32.1

Assessment Opportunities

Minds On ...

Small Groups → Equivalent Fractions

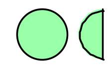
Distribute one fraction card to each student (BLM 32.1). Students find three other students who have equivalent fractions. Small groups discuss how they could convince someone else that their fractions are equivalent. Have a variety of manipulatives available.

Curriculum Expectations/Observation/Mental Note: Circulate and assess students' understanding in preparation for a whole group debrief.

Whole Group → Discussion

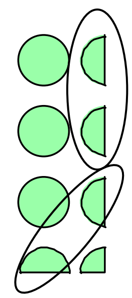
Discuss equivalent fractions based on your observations from small group work.

Sample Solution with fraction circles:
A representation of $1\frac{1}{2}$ using circles is:



therefore

$3\frac{1}{2}$ of these can be represented by:



In total there are $5\frac{1}{4}$ circles.

Action!

Small Groups → Solving Measuring Problems

Students work in small groups to solve the problem:

Hal's recipe for bread calls for

$1\frac{1}{2}$ cups of flour. Hal wants to make $3\frac{1}{2}$ batches of bread. How much flour should he use?

Challenge groups who determine a correct solution to find the answer using different manipulatives and/or pictorial solutions, e.g., measuring cups, pictures of whole and fractional circles or squares.

Whole Class → Discussion

Discuss the limitations of manipulatives, e.g., some denominators are difficult to work with, large whole number parts require too many manipulatives. Establish a need for alternative methods for calculating products of fraction. Tell students that today they will extend the area model for multiplication so they can discover how to symbolically multiply mixed numbers.

Small Groups → Concept Development

Students work in small groups at the website:

http://matti.usu.edu/nlvm/nav/vlibrary.html → choose Index → choose Fractions (Improper) - Rectangle Multiplication, Numbers & Operations (3-5) Give each group a different product question (use fractions from initial activity). Each group uses the rectangle model to illustrate their product and represents its concrete model symbolically.

Consolidate Debrief

Whole Class → Presentations and Discussions

One group shares its symbolic multiplication and discusses how it connects to the rectangle model. Debrief each of the small group questions as a whole class. Model the multiplication of two fractions where the fractions are larger than two.

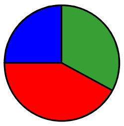
Concept Practice
Application

Home Activity or Further Classroom Consolidation

Tanya is 16 years old and just got her G1 driver's license. For every hour she spends on math at home, her parents will give her $1\frac{2}{3}$ hours of practice driving time. This week Tanya spent $2\frac{2}{5}$ hours on math. Your challenge is to see how many *different* ways you can show that Tanya can get $3\frac{1}{5}$ hours of practice driving time.

32.1: Equivalent Fractions

$1\frac{2}{3}$	$\frac{5}{3}$	$\frac{10}{6}$	$\frac{15}{9}$
$1\frac{3}{4}$	$\frac{7}{4}$	$\frac{14}{8}$	$\frac{21}{12}$
$1\frac{5}{6}$	$\frac{11}{6}$	$\frac{22}{12}$	$\frac{33}{18}$
$1\frac{2}{5}$	$\frac{7}{5}$	$\frac{14}{10}$	$\frac{21}{15}$
$1\frac{4}{5}$	$\frac{9}{5}$	$\frac{18}{10}$	$\frac{27}{15}$
$1\frac{3}{8}$	$\frac{11}{8}$	$\frac{22}{16}$	$\frac{33}{24}$
$1\frac{5}{8}$	$\frac{13}{8}$	$\frac{26}{16}$	$\frac{39}{24}$



Description

- Represent the division of two fractions.

Materials

- manipulatives
- BLM 33.1

Assessment Opportunities

Minds On ...

Small Groups → Sharing Solutions

Students form small groups to share solutions from the Home Activity. Assign one type of solution e.g., symbolic, rectangle model, other manipulative, using decimals, using calculator, to each group for presentation.

Groups reflect on the different types of problems that require the division of two numbers.

Examples:
- finding the number of groups
- finding the size of a group
- finding the length or width of a rectangle given the area
- finding a unit rate

Action!

Small Groups → Problem Solving with Manipulatives

Pose the first question on BLM 33.1. Students should have a variety of manipulatives available to help them solve the problem. Students can use calculators to check solutions.

Debrief, then pose the second question. Continue with questions 3 and 4. (Multiple solutions will be presented using a variety of manipulatives.)

Curriculum Expectations/Observation/Anecdotal: As you circulate, ask specific students which operation (division) could replace the triangle in question 5.

Pair/Share → Reflection

Students highlight words in the four problems that help identify this problem as one that can be solved using division, and share their responses with another pair.

Read the "Finding Signal Words" *Think Literacy: Cross-Curricular Approaches*.

Consider making an overhead of this BLM and reveal the questions one at a time.

Consolidate Debrief

Whole Class → Discussion

Discuss each of the four problems as representative of a type of division problem. Discuss how making a problem simpler is a strategy that they might find useful when fractions are involved, e.g., the first question could become: *A group of friends buys 12 pizzas to share equally. Each friend receives 2 pizzas.* If a student knows that $12 \div 2$ can be used to find the answer to this question, then this knowledge can be transferred to conclude that $3 \div \frac{3}{5}$ can be used to solve the given question.)

Have samples of other "division" problems to share with students during the Consolidate/ Debrief discussion.

Exploration

Home Activity or Further Classroom Consolidation

Mystery Number Problems: Find a number to replace the question mark. Record strategies that you used.

$$3 \times ? = 5$$

$$\frac{1}{4} \times ? = \frac{1}{12}$$

$$5 \times ? = 7\frac{1}{2}$$

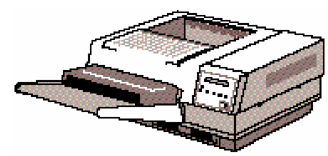
$$2\frac{5}{8} \times ? = 1\frac{1}{2}$$

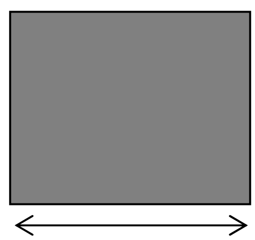
33.1: Let's Share!

1. A group of friends buys 3 pizzas to share equally. Each friend receives $\frac{3}{5}$ of a pizza.
   Your challenge is to show different ways to find the total number of friends in the group.

2. Amy, Sue, and Alex bought $\frac{1}{4}$ kg of trail mix to share equally.
   Your challenge is to show different ways to determine how much trail mix each person will receive.

3. Sandra's printer can print 5 pages in $\frac{2}{3}$ of a minute.
   Your challenge is to show different ways to determine how many pages Sandra's printer can print in one minute.





$1\frac{3}{4}$ units

4. The area of a rectangle is $2\frac{5}{8}$ square units. The length of the rectangle is $1\frac{3}{4}$ units.
   Your challenge is to show different ways to determine the width of the rectangle.

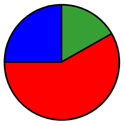
5. Replace the triangle in each equation below to create a true statement.

a) $3 \quad \Delta \quad \frac{3}{5} = 5$

b) $\frac{1}{4} \quad \Delta \quad 3 = \frac{1}{12}$

c) $5 \quad \Delta \quad \frac{2}{3} = 7\frac{1}{2}$

d) $2\frac{5}{8} \quad \Delta \quad 1\frac{3}{4} = 1\frac{1}{2}$



Description

- Use patterning to develop the 'invert-and-multiply' rule for dividing two fractions.
- Practise dividing fractions using unit rate questions.

Materials

- chart paper
- markers
- BLM 34.1

Minds On ...

Whole Class → Discussion

Facilitate a whole class discussion around the concept of unit rate: What is it? Why is it important? What are some examples? (See question 3 on BLM 33.1.) Consider using *Smart Ideas* software to organize a concept map of unit rate.

Action!

Small Groups → Problem Posing

Each small group creates 1 or 2 unit rate problems that involve fractions, writes problems on chart paper, and exchanges with another group to peer review and edit. Circulate, questioning groups, to help them refine the problems.

Each small group problem must require the calculation of a unit rate that involves the division of two fractions. Ask: Is it easy to mentally find an exact answer to any of the questions posed? Tell them the next activity will help them discover a method for making the division of two fractions easier to mentally compute. Before beginning the activity, verify that students know how to divide two fractions using their calculators.

In pairs, students complete BLM 34.1.

Assessment Opportunities

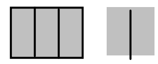
A unit rate is the quantity associated with a single unit of another quantity, e.g., \$6 per shirt, 50 km per litre, 6 km per hour.

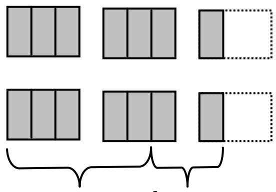
If students are not used to peer reviewing/editing, consider having a whole group discussion on the process.

Consolidate Debrief

Whole Class → Discussion

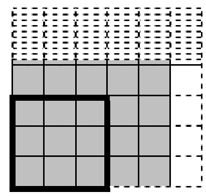
Use the fractions in the last question on BLM 34.1 and the context from one of the student-created unit rate problems to pose a fraction problem. Model several solutions for the problem, including pictorial solutions. Students must understand that dividing by a number produces the same result as multiplying by the reciprocal of the number. Sample solutions:

a) $2\frac{1}{3} \div 1\frac{2}{3}$ asks us to determine the number of $1\frac{2}{3}$-sized pieces there are in a $2\frac{1}{3}$-size piece. Pictorially, how many  groups there are in



1 group + 2/5 of a group

b) $2\frac{1}{3} \div 1\frac{2}{3}$ asks us to find the width of a rectangle having area $2\frac{1}{3}\text{cm}^2$ if its length is $1\frac{2}{3}$ cm. Pictorially,

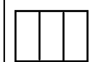


c) Students may agree that the solutions above are not easy ways to determine an answer and that a computational solution is needed. From the worksheet, they saw that division by a fraction results in the same answer as multiplication by the reciprocal.

Therefore, $2\frac{1}{3} \div 1\frac{2}{3}$ has the same answer as $2\frac{1}{3} \times \frac{3}{5}$. The answer can then be determined symbolically or using an area model.

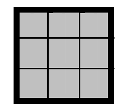
Curriculum Expectations/Quiz/Rubric: Assess responses from a paper-and-pencil quiz on the division of two fractions.

Add "reciprocal" to the class Word Wall.



represents a whole in this solution.

In the 2nd solution 1 cm² is represented by



Application

Home Activity or Further Classroom Consolidation

Solve the unit rate problems created by your peers.

Provide student-created problems from the activity.

34.1: What's the Pattern?

Determine the answer to each question, using a calculator.

a)	$4 \div 2 =$	$4 \times \frac{1}{2} =$
b)	$6 \div 3 =$	$6 \times \frac{1}{3} =$
c)	$8 \div 4 =$	$8 \times \frac{1}{4} =$
d)	$4 \div \frac{2}{3} =$	$4 \times \frac{3}{2} =$
e)	$\frac{1}{4} \div 2 =$	$\frac{1}{4} \times \frac{1}{2} =$
f)	$\frac{1}{4} \div \frac{1}{2} =$	$\frac{1}{4} \times 2 =$
g)	$1\frac{3}{5} \div 4 =$	$1\frac{3}{5} \times \frac{1}{4} =$
h)	$2\frac{1}{3} \div 1\frac{2}{3} =$	$2\frac{1}{3} \times \frac{3}{5} =$

What do you notice about the answers in each row?

Show how you could find an answer to $2\frac{1}{3} \div 1\frac{2}{3}$ without using a calculator.

Description
- Use resources from TIPS continuum on fractions to consolidate understanding.

Materials
- BLM 35.1
- data projector

Assessment Opportunities

Minds On ...

Whole Class → Discussion
Each student receives a fractional piece and draws a scale picture of one whole. Compare the "wholes." Talk about why each whole has the same area but not necessarily the same shape. Discuss how the sizes of any two shapes can be compared e.g. 12 is four times the size of 3; 12 is nine more than 3.

Roaming Pairs → Problem Solving
Pairs of students compare their fractions in two ways. Students record the comparison then find another partner and repeat.

Curriculum Expectations/Assignment/Marking Scheme: Collect and mark individual responses.

Action!

Groups → Learning Centres
Structure different stations of activities according to the needs and interests of your students. Consider using materials from TIPS Section 2 – Fractions for some students.

- Developing Mathematical Processes – page 6
- Developing Proficiency – page 9
- Extend Your Thinking – page 14
- Is This Always True? – page 22

Consolidate Debrief

Whole Class → Concept Map
Summarize the big ideas of this unit using a concept map. Consider using Smart Ideas software for this process.

Familiarize students with the recipe and process for making "Lasagne" (See Day 37: Summative Assessment).

Concept Practice

Home Activity or Further Classroom Consolidation
Make a review booklet as you study for the assessment.

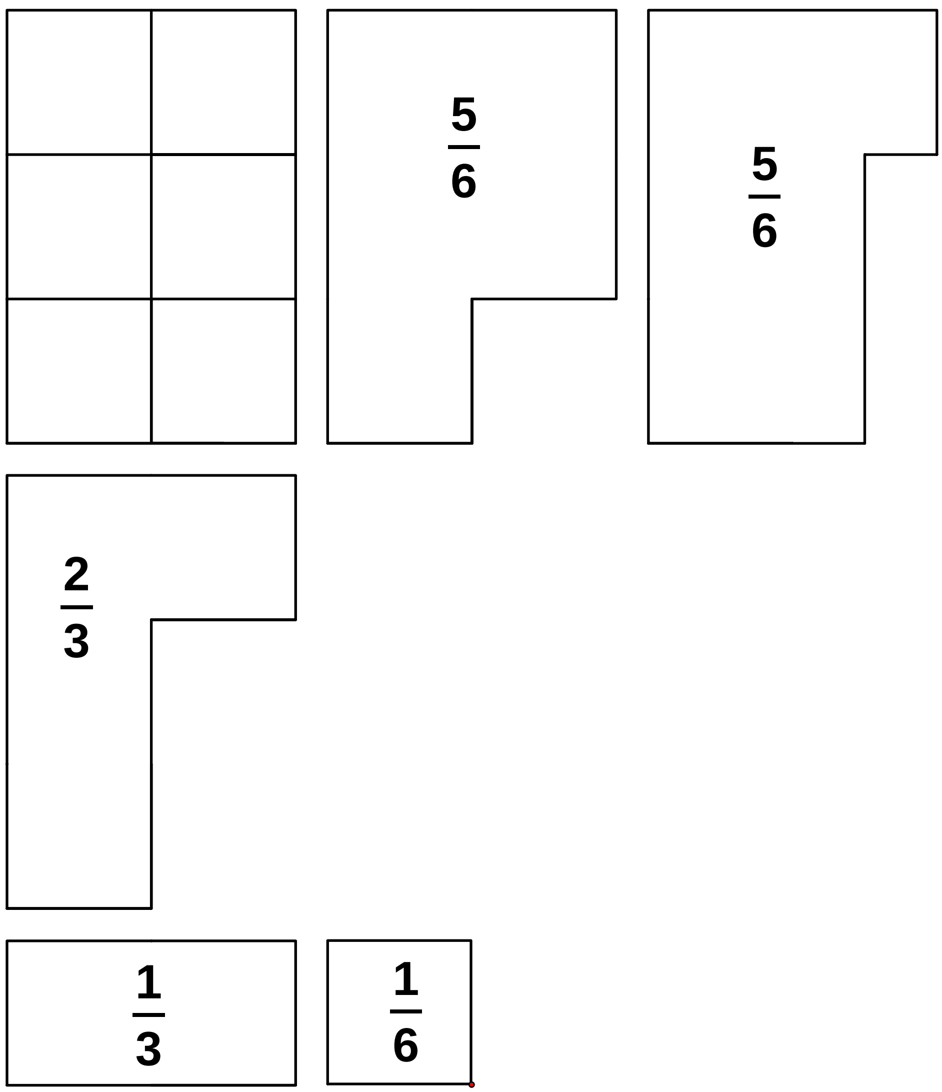
35.1: Cracker Fractions (Teacher)

Each piece is a fraction of the whole cracker. All pieces are drawn to scale.

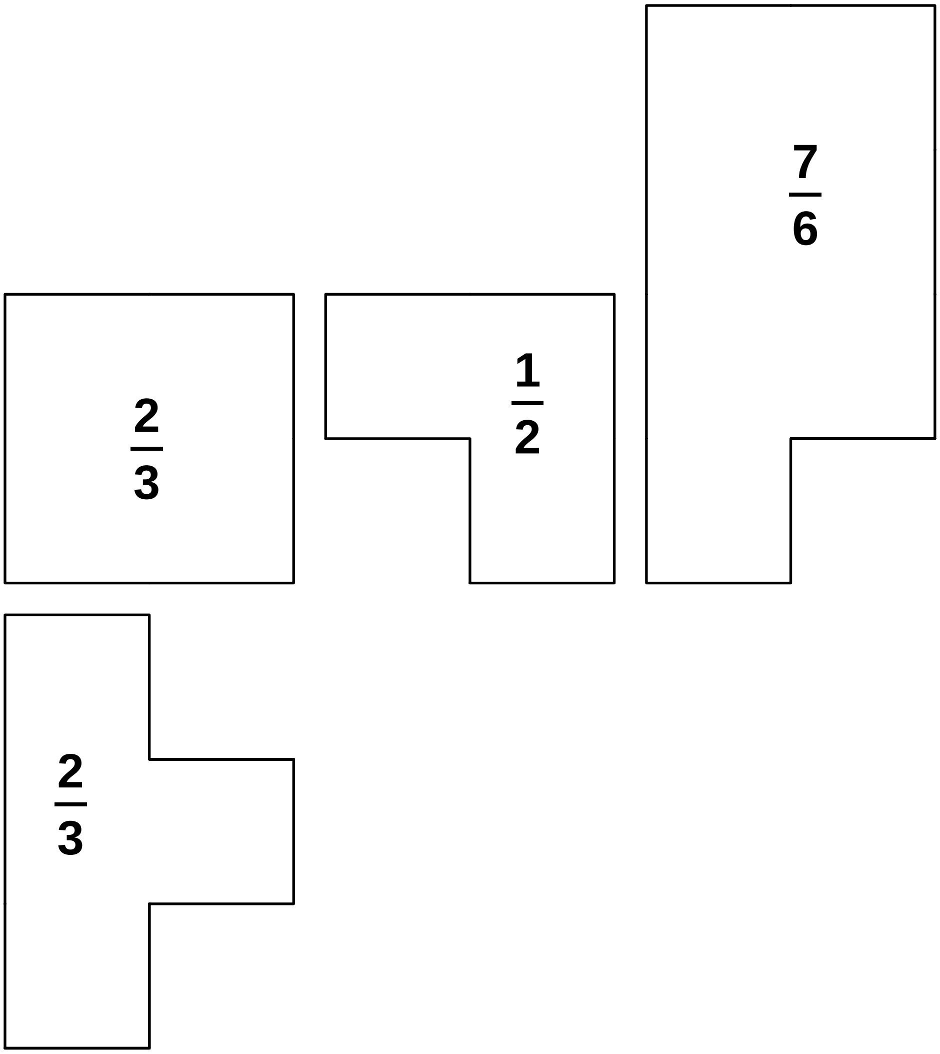
Distribute the fractional pieces and ask students to determine what one whole looks like. The "whole" answers may not look the same but they should all have the same area.
Ask students to make two comparative statements about their pieces. For example:

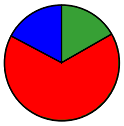
The $\frac{2}{3}$ piece is 4 times the size of the $\frac{1}{6}$ piece. $\frac{2}{3} \div \frac{1}{6} = 4$

The $\frac{2}{3}$ piece is $\frac{1}{2}$ more than the $\frac{1}{6}$ piece . $\frac{2}{3} - \frac{1}{6} = \frac{1}{2}$



35.1: Cracker Fractions (Teacher) (continued)





Description

- Express composite numbers as products of prime numbers to find lowest common multiples.
- Solve problems that require the lowest common multiple.

Materials

- data projector

Assessment Opportunities

Minds On ...

Pairs → Multiple Solutions

Students work in pairs to solve this problem:

Jerry spends $\frac{1}{9}$ of his free time playing computer games. Harry spends $\frac{1}{8}$ of his free time playing computer games. Lui spends more time than Jerry but less time than Harry. Find different possibilities for the fraction of his free time that Lui spends playing computer games.

$\frac{1}{9} = \frac{8}{72} = \frac{16}{144}$

$\frac{1}{8} = \frac{9}{72} = \frac{18}{144}$

So, $\frac{1}{9} < \frac{17}{144} < \frac{1}{8}$

$\frac{1}{9} = 0.11111...$

$\frac{1}{8} = 0.125$

So, $\frac{1}{9} < 0.15 < \frac{1}{8}$

Action!

Whole Class → Discussion

Facilitate a discussion on multiple solutions to the initial activity. Discuss a solution that involves finding a common denominator. Discuss other types of problems that require the lowest common multiple of two numbers.

The planet Zerk has two moons. Tonight the two moons are directly in line with each other and with the planet's sun. Moon 1 can be viewed in this same location every 10 days. Moon 2 can be viewed in this same location every 15 days. When is the next time that both moons will be in this same location?

In pairs student find a solution to the Zerk problem. [The answer is 30, i.e., the lowest common multiple.]

Demonstrate for the class a process for finding the lowest common multiple using the website:

http://matti.usu.edu/nlvm/nav/vlibrary.html → Choose Index → Choose *Factor Tree* → Choose *Numbers & Operations (9-12)*

Students work in pairs with the virtual activity. One partner is the driver and one partner is the recorder. Students try several examples and regularly exchange roles.

Curriculum Expectations/Recorded Solutions/Rubric: Assess student solution.

Consolidate Debrief

Whole Class → Discussion

Summarize the process used in the virtual activity. Write composite numbers as products of primes, using exponents when needed. Discuss how the ability to find a LCM (lowest common multiple) was very important when adding fractions before calculators became common mathematical tools. Illustrate this with an example like: $\frac{37}{210} + \frac{53}{90}$. Students use calculators to check the answer.

Discuss why the skill is still important.

$210 = 2 \times 3 \times 5 \times 7$
$90 = 2 \times 3^2 \times 5$

The lowest common multiple is: $2 \times 3^2 \times 5 \times 7$ which is 630.

Scenario: two cyclical events, one occurring every 210 minutes, the other every 90 minutes. How often will the events occur at the same time?
Answer: every 630 minutes (the LCM).

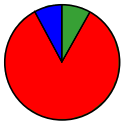
Application
Reflection

Home Activity or Further Classroom Consolidation

Create and solve one "easy" question and one "not-so-easy" question that requires finding either a lowest common multiple or a greatest common factor.

If Internet access is available, show someone the website you used in class and discuss how calculators can be used for fraction arithmetic.

Continue working on your review booklet.



Description
- Administer a paper-and-pencil summative assessment.

Materials
- summative assessment
- manipulatives
- calculators

Assessment Opportunities

Minds On ...

Whole Class → Preparing for Assessment
Do a whole class relaxation/calming activity prior to the assessment.

Distribute the assessment. Students scan the assessment for unfamiliar words/instructions. Remind students that the class word wall might be useful during the assessment.

Clarify instructions.

Review the scoring criteria.

Action!

Individual → Summative Assessment
Students complete the assessment using manipulatives and calculators as requested.

Consolidate Debrief

Whole Class → Discussion
Assessments are opportunities for reflection and planning next steps. Discuss how the results of this assessment can help them, you (their teacher), and their parents plan next steps.

Application

Home Activity or Further Classroom Consolidation
Complete the challenge:

Julia's mother is a math teacher. She looked at the geoboard model of the "Lasagne" and immediately saw all kinds of fraction problems. Create and solve a fraction problem based on the geoboard model. You will be assessed for creativity, communication, complexity and correctness … the four Cs of cooking with Math!

Students who complete the summative assessment task early can begin this assignment

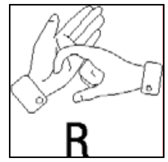
Summative Assessment – Fractions

Name:
Date:

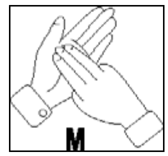
Instructions:
1. Answer all of the questions as completely as possible.
2. Your answers will be assessed for understanding, communication, and correctness.
3. If you think you can explain your reasoning much better by talking to me or by showing something to me, then draw a speaker symbol beside the question.

Speaker symbol: <)))

4. If you need me to read something to you, show me the sign language symbol for R.



5. If you need manipulatives, show me the sign language symbol for M.



6. You may use a calculator for any part of this assessment.

Assessment Checkbric

Criteria	Level 1	Level 2	Level 3	Level 4
Computing and carrying out procedures				
Making convincing arguments, explanations and justifications				
Integrating narrative and mathematical forms				
Representing a situation mathematically				
Selecting and applying problem-solving strategies				

Assessment Feedback

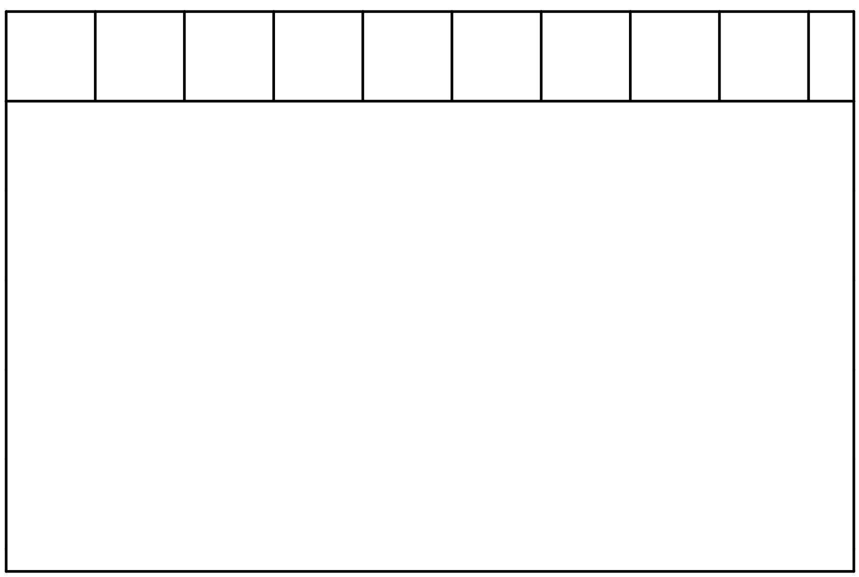
Strengths
Areas for Improvement
Next Steps

Quick Lasagne

Small square pieces of pasta
1 can spaghetti sauce
Grated mozzarella cheese

1. Preheat oven to 325°F.
2. Grease a rectangular pan.
3. Pre-cook the pasta.
4. Line the bottom of the pan with the square pasta pieces.
5. Spread spaghetti sauce evenly over the top of the pasta.
6. Sprinkle mozzarella cheese over the sauce.
7. Bake for 30 minutes.

Julia is making "Quick Lasagne." She starts to cover her greased pan with the cooked pasta. Julia discovers that she needs to use part of a pasta piece to complete the row. The picture below is a scale model of the sheet with first row of pasta pieces.



1. Circle the answer that best represents the number of pasta pieces in the first row.

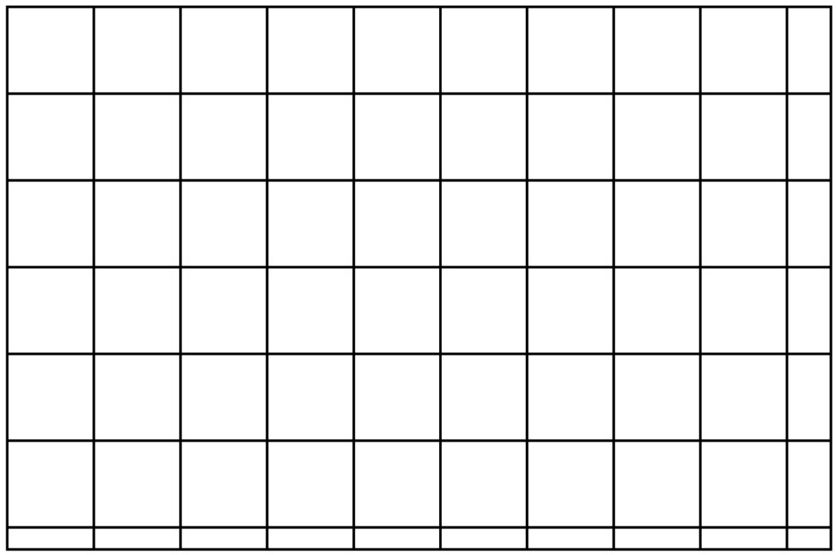
a) 9 b) $9\frac{1}{3}$ c) $9\frac{1}{2}$ d) $9\frac{3}{4}$

Unit Assessment – Fractions (continued)

2. The picture below is a scale model of the sheet covered with pasta pieces.
   Determine the number of pasta pieces on the sheet. *Show your work.*

$9\frac{1}{2}$ pasta pieces across the top

$6\frac{1}{4}$ pasta pieces down the side



3. Forty pasta pieces cover two-thirds of a rectangular pan.
   Circle the expression that gives the number of pasta pieces that will cover all of the pan.

a) $40 \times \frac{2}{3}$

b) $\frac{2}{3} + 40$

c) $\frac{2}{3} \div 40$

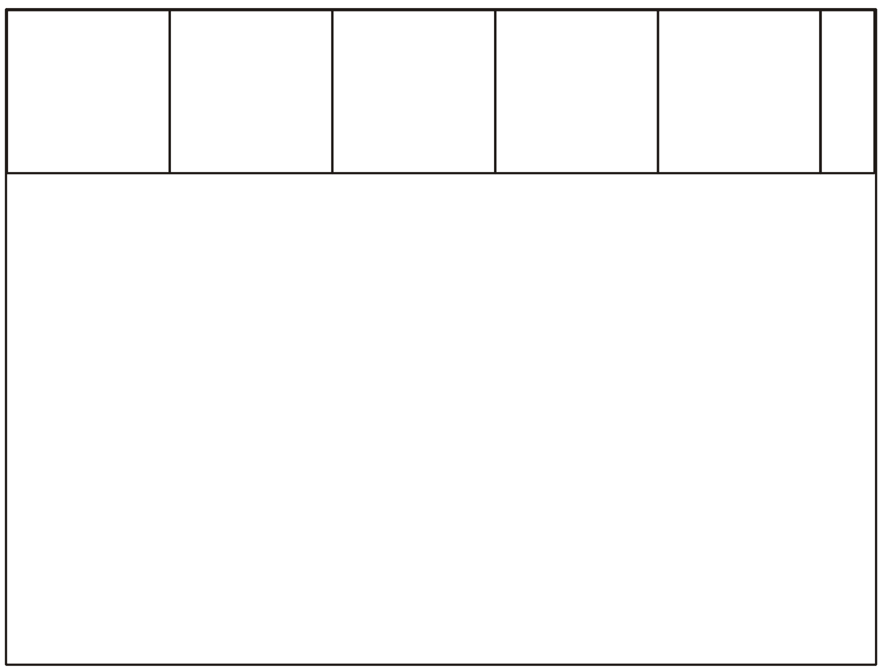
d) $40 \div \frac{2}{3}$

Unit Assessment – Fractions (continued)

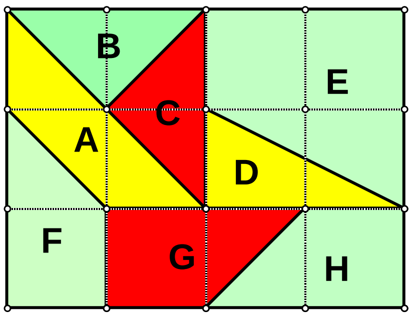
4. When Jamel makes the "Quick Lasagne" recipe he uses a rectangular pan that holds $59\frac{1}{9}$ pasta pieces. Each pasta piece has an area of 36 cm^2.
Circle the expression that gives the total area of the rectangular pan in cm^2.

a) $36 \times 59\frac{1}{9}$ b) $36 \div 59\frac{1}{9}$ c) $59\frac{1}{9} \div 36$ d) $36 + 59\frac{1}{9}$

5. Miguel has a very small rectangular pan on which he can place $5\frac{1}{3}$ pasta pieces across the top and $3\frac{1}{2}$ pasta pieces down the side. He has a package of 36 pasta pieces and wants to know if he has enough pasta pieces to make two pans of "Quick Lasagne." *Justify your reasoning.*



6. Kaye cuts her "Quick Lasagne" into pieces of different shapes and sizes.
   The diagram below is a geoboard model of Kaye's pan of lasagne.
   The lasagne was made with 12 pasta pieces but it is cut into 8 pieces for serving – A, B, C, D, E, F, G, and H.



a) Which piece(s) is/are $\frac{1}{12}$ of the pan of lasagne?
Justify your reasoning.

Hint:

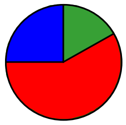
The dashed lines show the 12 pasta pieces.

7. If Julia eats a piece that is $\frac{1}{8}$ of the pan and another piece that is $\frac{1}{12}$ of the pan, then what fraction of the whole pan of lasagne does she eat?
   Justify your reasoning.

Assessment Checkbric – “Look Fors”

Record observations from student work.

Criteria	Level 1	Level 2	Level 3	Level 4
Computing and carrying out procedures				
Making convincing arguments, explanations and justifications				
Integrating narrative and mathematical forms				
Representing a situation mathematically				
Selecting and applying problem solving strategies				



Description

- Develop pre-requisite knowledge and skills needed to be successful in working with fractions to:
  - model equivalent fractions
  - create equivalent fractions symbolically
  - represent fractions on a number line
  - subtract fractions

Materials

- chart paper
- markers
- fraction circles
- rulers
- BLMs 1.1 to 1.5

Assessment Opportunities

Minds On ...

Small Groups → Discussion

Students create and record a group response to Question 1 (see BLM 1.1) on the prior learning assessment, explaining why "less than one-quarter" is the correct response. Advise students that any group member should be prepared to share the group response with the whole class.

Select one or two groups to present responses.

Curriculum Expectations/Observation/Mental Note: Circulate to observe if the activity is "closing the gap" for students who did not previously demonstrate understanding. Intervene with appropriate questioning, as necessary.

Use the Prior Learning Assessment responses to pair students with mixed readiness levels.

The diagram in question 1 illustrates one out of four pieces; however, one-quarter of a whole is one of four *equal* parts of the whole. This is a very subtle difference.

Action!

Pairs → Carousel

Distribute one response sheet (BLM 1.2) for each pair.

Explain that each station has four student exemplars for each of the questions on their response sheet. Their task is to discuss the exemplars at the station and jointly create a response that is better than any of the exemplars. When they are finished at one of the four stations they move to another station.

After completing their task at each of the four stations they submit their pair response and begin their individual task.

Curriculum Expectations/Observation/Mental Note: Circulate to observe if the activity is "closing the gap" for students who did not previously demonstrate understanding. Intervene with appropriate questioning, as necessary.

Set up two sets of 4 stations as described in BLM 1.5.

Students need to know that they will be creating responses to similar questions. Each student in the pair needs to take responsibility for their own understanding.

Individual → Practice

Students choose whether they work on BLM 1.3 or BLM 1.4. Encourage students who feel they need more practice with the concepts to work on BLM 1.3. Students who are confident in their knowledge and skills in working with fractions might choose to be challenged with the questions on BLM 1.4.

Curriculum Expectations/Observation/Mental Note: Circulate to observe if the activity is "closing the gap" for students who did not previously demonstrate understanding. Intervene with appropriate questioning, as necessary.

Consolidate Debrief

Whole Class → Reflection

Guide a discussion about revising and editing solutions. Discuss students' observations about the completeness and organization of the exemplar solutions.

Reflection

Home Activity or Further Classroom Consolidation

Reflect on the learning goals at each station.
Identify concepts you still don't understand and/or concepts that you need to practise.

Refer to *Think Literacy - Mathematics*: Revising and Editing for considerations.

Additional Observations and Plans for Differentiation (Teacher)

1. Throughout this unit, ask students to compare fractions.

   a) Build "benchmark" skills. When a fraction response is given, ask students whether the fraction is closer to 0, one-half or one.
   Adjust benchmarks to reflect the two closest whole numbers that the fraction is between. For example, for a response of $3\frac{1}{8}$, ask if the fraction is closer to 3, $3\frac{1}{2}$, or 4.

   b) Build opportunities to further develop comparison strategies:
      - comparing two fractions with the same denominator
      - comparing two fractions with the same numerator
      - comparing fractions when both the numerator and denominator differ by one

2. During the Minds On section of some lessons, do further work to develop conceptual understanding (and then proficiency) on converting between improper and mixed fraction forms.
   Fraction circle pieces or pattern blocks could be used. For example, give a student or group of students 13, one-sixth fraction circle pieces so they can demonstrate both $\frac{13}{6}$ and $2\frac{1}{6}$. Calculators could also be used to practise and self-check conversions.

   Suggested Questions for Home Activity or Minds On:

   a) How many different ways can you explain how to change $3\frac{5}{8}$ to an improper fraction?

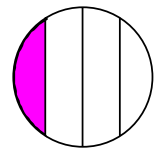
   b) How many different ways can you explain how to change $\frac{17}{3}$ to a mixed number?

3. Continue to ask students to represent fractions with pictures and a variety of concrete models.

4. Ask questions to determine that students know how one whole is defined for each fraction question.

1.1: Minds On

1. A circle is divided into four parts as shown in the diagram. One of the parts is shaded.



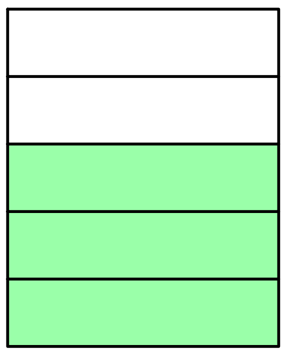
   Which fraction of the whole circle is shaded?
   Give reasons for your answer.

   a) one-quarter   b) less than one-quarter   c) more than one-quarter

1.2: Pair Response Sheet

2. a) Use the given diagram to convince Robyn that

$\frac{3}{5}$ of a chocolate cake is equivalent to $\frac{12}{20}$ of the cake.

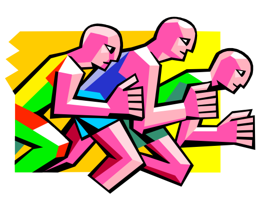


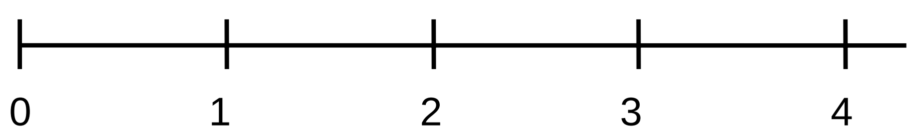
b) Convince Robyn that $\frac{3}{5}$ is equivalent to $\frac{12}{20}$ without using a diagram.

Justify your answer.

3. Alan, Barry and Cusam jog on a track every morning.
Alan jogs $\frac{7}{8}$ km, Barry jogs $2\frac{5}{6}$ km and Cusam jogs $3\frac{1}{2}$ km.

a) Use the number line to represent the distance that each person jogs.



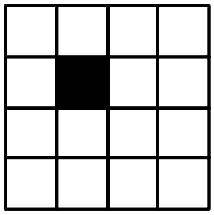


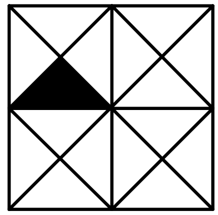
b) How much farther does Cusam jog every morning than Alan? *Show your work.*

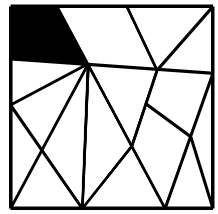
1.3: Individual Response Sheet

1. In diagrams a, b, and c, the same square is divided into pieces of different shapes.

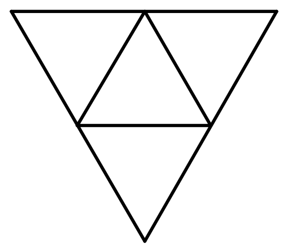
   For *each* of the diagrams below, state whether the shaded portion is one-sixteenth of the whole square. *Give reasons for your answer.*

   a) 

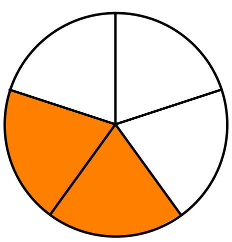
   b) 

   c) 

2. The diagram shows the top of a piece of cheese that is divided into 4 equal pieces. Shade three-eights of the surface.



3. a) Use the diagram to convince Kelly that $\frac{2}{5}$ of a pizza is equivalent to $\frac{6}{15}$ of a pizza.



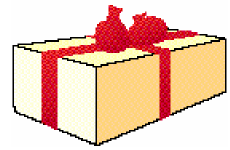
   b) Convince Kelly that $\frac{2}{5}$ is equivalent to $\frac{6}{15}$ without using a diagram.

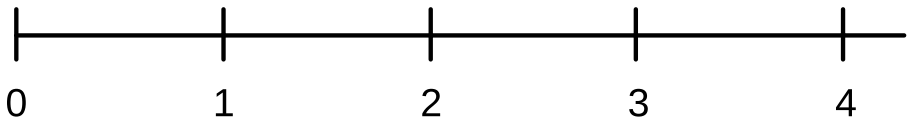
   Justify your answer.

1.3: Individual Response Sheet (continued)

4. George, Nicky and Tula need ribbon for wrapping gifts.
   George needs $\frac{3}{4}$ m, Nicky needs $1\frac{7}{8}$ m, and Tula needs $2\frac{2}{3}$ m of ribbon.

   a) Use the number line to *accurately* represent the amount of ribbon that each person needs.





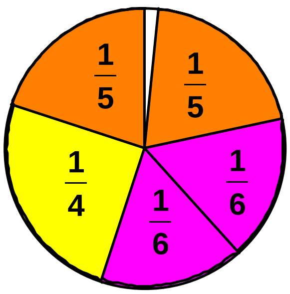
   b) How much more ribbon does Tula need than Nicky?
   Show your work.

1.4: Fraction Challenges

Challenge 1

Sam is using the fraction circle pieces to create one whole circle. In his first attempt, he notices that a small section of the circle is not covered. None of his fraction pieces will fit into the uncovered section. Sam wonders how he can determine what fraction of the circle is not covered.

Find at least two different ways to show that the fraction of the circle that is **not** covered is $\frac{1}{60}$ of the circle. *Show your work.*



Challenge 2

After a class pizza party, it is Jody's job to determine how to equally share the remaining pizza from four groups among eight students who are still hungry.

The four groups report that they have the following amounts of pizza left over:

Group 1	Group 2	Group 3	Group 4
$\frac{1}{2}$ pizza	$1\frac{1}{8}$ pizza	$\frac{5}{8}$ pizza	$\frac{1}{4}$ pizza

Show at least two different ways to determine how much pizza each of the eight students will receive. *Show your work.*

1.5: Student Exemplars (Teacher)

Instructions/Suggestions

1. Make two sets of each of the student exemplars on the next pages. Consider mounting the two identical sets, Set A and Set B, on two different colours of paper.

2. Set up 8 stations – four for Set A and four for Set B. Place one set of exemplars for one question at each station as well as any manipulatives or tools the students may find useful, e.g., rulers for question 4a.

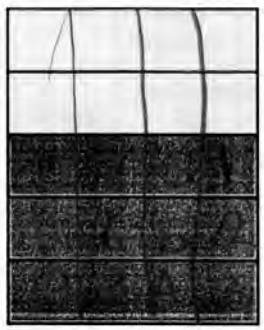
3. Pair a student who demonstrated stronger responses in the prior learning assessment with a student who demonstrated weaker responses.

4. Instruct each pair of students to visit each of the four stations in Set A **or** each of the four stations in Set B. At each station, students discuss the four exemplars, then create and record a correct solution that integrates words and symbols. Their goal is to create a solution that is better than any of the exemplars. Instruct students that each partner needs to be able to explain the solutions they submit.

Station 1 — Question 2a Example 1

Did she really prove it?

2. a) Use the given diagram to convince Robyn that $\frac{3}{5}$ of a chocolate cake is equivalent to $\frac{12}{20}$ of the cake.

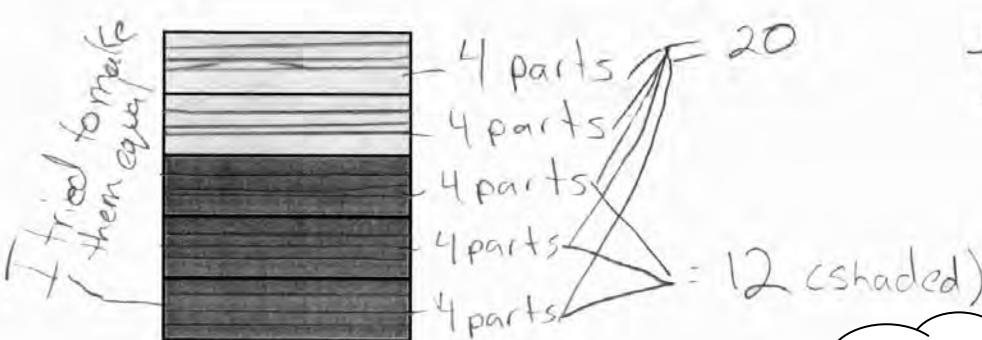


I cut the cake into twentyeths and shaded 12 peices and proved it was the same as when it was in fifths

Learning Goal – to model equivalent fractions

Station 1 — Question 2a Example 2

2. a) Use the given diagram to convince Robyn that $\frac{3}{5}$ of a chocolate cake is equivalent to $\frac{12}{20}$ of the cake.



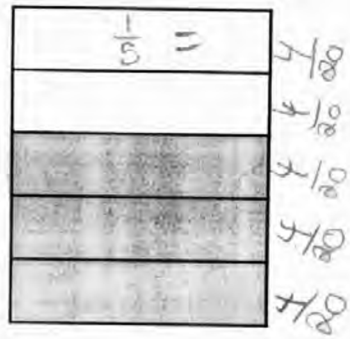
$\frac{3}{5} = \frac{12}{20}$

Is this solution easy for you to follow?

Learning Goal – to model equivalent fractions

Station 1 — Question 2a Example 3

2. a) Use the given diagram to convince Robyn that $\frac{3}{5}$ of a chocolate cake is equivalent to $\frac{12}{20}$ of the cake.



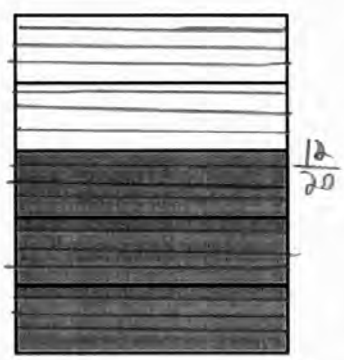
$$\frac{3 \times 4}{5 \times 4} = \frac{12}{20}$$

$$\frac{1}{5} = \frac{4}{20}$$

Learning Goal – to model equivalent fractions

Station 1 — Question 2a Example 4

2. a) Use the given diagram to convince Robyn that $\frac{3}{5}$ of a chocolate cake is equivalent to $\frac{12}{20}$ of the cake.



Learning Goal – to model equivalent fractions

Station 2 **Question 2b Example 1**

b) Could you convince Robyn that $\frac{3}{5}$ is equivalent to $\frac{12}{20}$ without using a diagram?

Justify your answer.

Yes I could because $\frac{12}{20}$ Simplafied is $\frac{6}{10}$ and $\frac{6}{10}$ simplafied is $\frac{3}{5}$

Learning Goal – to create equivalent fractions symbolically

Station 2 **Question 2b Example 2**

b) Could you convince Robyn that $\frac{3}{5}$ is equivalent to $\frac{12}{20}$ without using a diagram?

Justify your answer.

$\frac{3}{5} \times \frac{4}{4} = \frac{12}{20}$ Yes because you cold take $\frac{3}{5}$ and multiply it by something to get it to equal $\frac{12}{20}$

Learning Goal – to create equivalent fractions symbolically

Station 2 **Question 2b Example 3**

b) Could you convince Robyn that $\frac{3}{5}$ is equivalent to $\frac{12}{20}$ without using a diagram?

Justify your answer.

Yes you could because you Just Multiply

$\frac{3 \times 4 = 12}{5 \times 4 = 20}$

Learning Goal – to create equivalent fractions symbolically

Station 2 **Question 2b Example 4**

b) Could you convince Robyn that $\frac{3}{5}$ is equivalent to $\frac{12}{20}$ without using a diagram?

Justify your answer.

without a diagram you could still get $\frac{12}{20}$ from $\frac{3}{5}$ all you do is multiply it by 4 and you get $\frac{12}{20}$

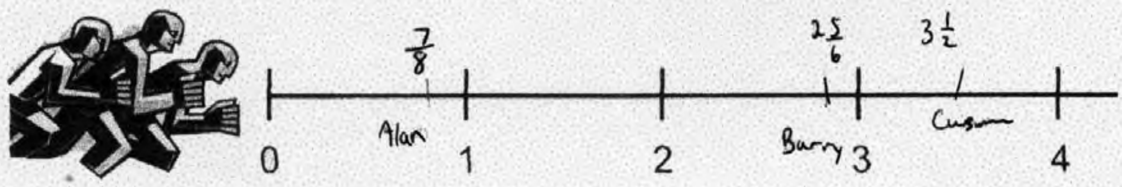
What does "it" refer to?

Learning Goal – to create equivalent fractions symbolically

Station 3 — Question 4a Example 1

4. Alan, Barry and Cusam jog on a track every morning.
Alan jogs $\frac{7}{8}$ km, Barry jogs $2\frac{5}{6}$ km and Cusam jogs $3\frac{1}{2}$km.

a) Use the number line to represent the distance that each person jogs.

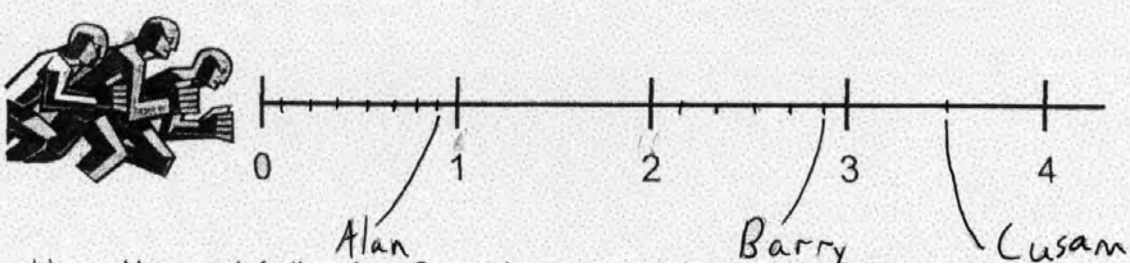


Learning Goal – to represent fractions on a number line

Station 3 — Question 4a Example 2

4. Alan, Barry and Cusam jog on a track every morning.
Alan jogs $\frac{7}{8}$ km, Barry jogs $2\frac{5}{6}$ km and Cusam jogs $3\frac{1}{2}$km.

a) Use the number line to represent the distance that each person jogs.

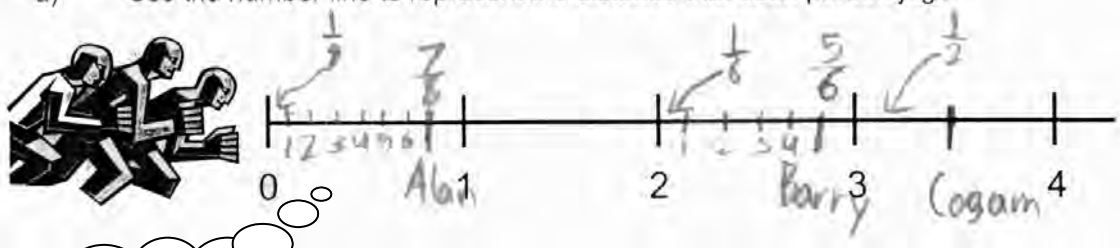


Learning Goal – to represent fractions on a number line

Station 3 — Question 4a Example 3

4. Alan, Barry and Cusam jog on a track every morning.
Alan jogs $\frac{7}{8}$ km, Barry jogs $2\frac{5}{6}$ km and Cusam jogs $3\frac{1}{2}$ km.

a) Use the number line to represent the distance that each person jogs.



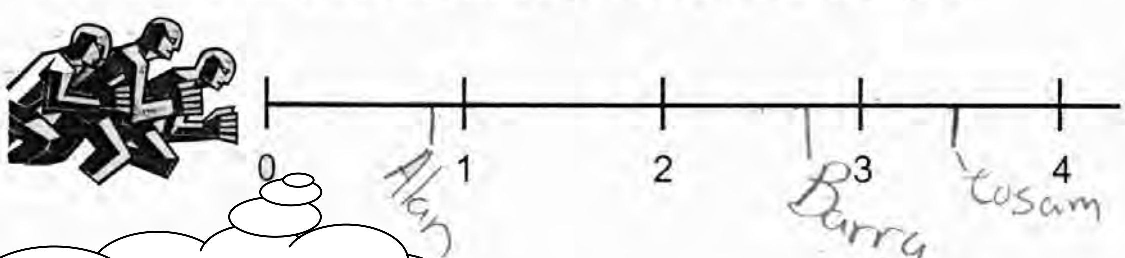
Should all of tick marks have the same distance between them?

Learning Goal – to represent fractions on a number line

Station 3 — Question 4a Example 4

4. Alan, Barry and Cusam jog on a track every morning.
Alan jogs $\frac{7}{8}$ km, Barry jogs $2\frac{5}{6}$ km and Cusam jogs $3\frac{1}{2}$ km.

a) Use the number line to represent the distance that each person jogs.



Could anything be added to this solution to make it better and/or more accurate?

Learning Goal – to represent fractions on a number line

Station 4 — Question 4b Example 1

b) How much further does Cusam jog every morning that Alan? *Show your work.*

I Know have to convert $3\frac{1}{2}$ into eighths.

$3\times8=\frac{24}{8}$ $\quad \frac{1}{2}=\frac{4}{8}$ $\quad 4+24=\frac{28}{8}-\frac{7}{8}=\frac{21}{8}$

Cusam runs $\frac{21}{8}$ or $2\frac{5}{8}$ km more then Alan runs I know this beacuse I converted Cusams distance into eighths and subtracted Alans distance from it.

Do the words help you understand this solution?

Learning Goal – to subtract fractions

Station 4 — Question 4b Example 2

b) How much further does Cusam jog every morning that Alan? *Show your work.*

$2\frac{5}{8}$

$\frac{7}{2}$ $\quad \frac{7}{8}$ $\quad \frac{28}{8}-\frac{7}{8}=\frac{21}{8}-2\frac{5}{8}$

Cusam Jogs $2\frac{5}{8}$ farther than Alan

Can you see places where equal signs might be used?

Learning Goal – to subtract fractions

1.5: Student Exemplars (Teacher) (continued)

Station 4 — Question 4b Example 3

b) How much further does Cusam jog every morning that Alan? *Show your work.*

He will jog $3\frac{3}{8}$ more than Alan

$3\frac{4}{8} - \frac{7}{8} = 3\frac{3}{8}$

$\frac{1}{2} \times 4 = \frac{4}{8}$ 4

$3\frac{3}{8}$

Can you see the error in this solution?

Learning Goal – to subtract fractions

Station 4 — Question 4b Example 4

b) How much further does Cusam jog every morning that Alan? *Show your work.*

$3\frac{1\times 8}{2\times 8} - \frac{7\times 2}{8\times 2}$

$= 3^{2}\frac{8^{24}}{16} - \frac{14}{16}$

$= 2\frac{10}{16}$

Cusam would jog $2\frac{10}{16}$ Km more than Alan.

Can you figure out why 24 is written beside 8?

Learning Goal – to subtract fractions

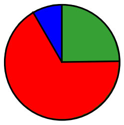
Lesson Outline: Days 23 – 33

Grade 8

BIG PICTURE

Students will:
- connect prior knowledge of surface area and volume of rectangular prisms to surface area and volume of triangular prisms;
- develop and apply formulas for surface area and volume of triangular prisms in problem solving situations;
- identify, sketch, design and build 3-D figures;
- connect 3-D figures to their various skeletons, nets and views;
- use the Pythagorean relationship as a tool to determine unknown measurements;
- develop appropriate communication and inquiry skills.

Day	Lesson Title	Description	Expectations
23	Activation	• Activate and assess prior knowledge.	CGE 4f, 5a
24	Mysterious Figures	• Identify and sketch 3-D figures. • Explore 3-D figures from their top, side and front views. • Investigate and describe geometric concepts related to 3-D figures.	8m56, 8m61, 8m60 CGE 3c, 5a
25	Sketchy Skeletons	• Sketch and build 3-D figures. • Research and report on uses of measurement involving 3-D objects in the community.	8m62, 8m41, 8m36 CGE 3c, 4e, 5e
26	Nifty Nets	• Sketch and create nets. • Use mathematical terminology.	8m62, 8m43 CGE 3b, 3c, 5e
27	Surface Area of Triangular Prisms	• Develop a formula for surface area of a triangular prism. • Apply the formula to solve problems.	8m50, 8m39, 8m43 CGE 5a, 5e, 5g
28	Wrap It Up	• Apply surface area formula to problems involving triangular prisms using nets. • Demonstrate estimation strategies in solving problems.	8m38, 8m35, 8m43, 8m42, 8m53 CGE 2c, 4e
29	Explore Prism Relationships	• Develop a formula for finding the volume of a triangular prism. • Apply volume formula in problem solving.	8m38, 8m35, 8m43, 8m42, 8m51, 8m53 CGE 3b, 4f, 5a
30	Volume of Rectangular Prism	• Apply volume and area formulas to explore the relationship between triangular prisms with the same surface area but different volumes. • Estimate volumes.	8m60, 8m40, 8m43, 8m42, 8m52, 8m53 CGE 2c, 3e, 5a
31	Volume of a Triangular Prism	• Sketch triangular prisms having a given volume. • Demonstrate an understanding of relationship between volume and varying dimensions.	8m39, 8m43, 8m42, 8m52, 8m53, 8m54 CGE 2d, 4e, 5g
32	Class Catalogue	• Assess students' knowledge and understanding of surface area, volume and triangular prisms.	CGE 3c, 4e, 4f
33	Tent Text	• Assess students' knowledge and understanding of surface area, volume and triangular prisms.	CGE 2b, 2c, 4f



Description

- Activate and assess prior knowledge

Materials
- geosolids
- BLM 23.1

Assessment Opportunities

Minds On ...

Whole Class → Discussion

Show a set of Geosolids and ask students to name a solid that has the same type of shape, e.g., the moon is a *sphere*, a book is a *rectangular prism*, the roof of a house can be a *triangular prism*. Use the “A answer B” oral communication strategy for responses.

Discuss units of measure that would be used for lengths, surface areas, and volumes of the Geosolids. Ask students if the same units would be used to measure examples from everyday life, e.g., would cm^3 be used to measure the volume of both the Geosolid sphere and the moon?

Small Groups → Activity

Students sort and classify the solids in many different ways. Each group shares one classification it used.

Students share strategies for finding the area of one of the triangular faces on one of the prisms.

Curriculum Expectations/Observation/Checklist: Circulate to assess communication (correct use of vocabulary) and understanding of finding the area of a triangle.

Whole Group → Demonstration

Use one large triangle to review the three base/height combinations that can be used to find the area of the triangle. Briefly remind students that sometimes the Pythagorean theorem needs to be used to find a missing measure.

Anticipation Guide
Think Literacy: Cross-Curricular Approaches Grades 7-12

Solids can be sorted by height, colour, base shape, capacity, surface area, curved or non-curved faces, number of faces, etc.

If the Minds On section takes longer than 15 minutes, you may wish to distribute just part of the test on this day and administer the remaining part on the next day or assign the remaining questions as an independent home activity.

Action!

Individual → Assessment

Distribute tests.

Curriculum Expectations/Paper-Pencil Test/Marking Key

Consolidate Debrief

Individual → Reflection

Students write a journal entry to reflect on their experience of writing the test. They should identify what they need/want to know more abut.

Application

Home Activity or Further Classroom Consolidation

Collect five different types of 3-D shapes that can be displayed in our class during this unit.

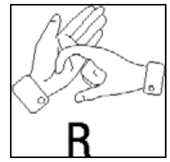
23.1: Measurement Assessment

Instructions

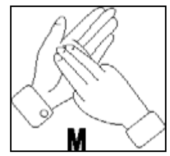
1. Answer all of the questions as completely as possible.
2. Your answers will be assessed for understanding, communication, and correctness.
3. If you think you can explain your reasoning much better by talking to me or by showing me something, then draw a speaker symbol beside the question.

Speaker symbol:
<)))

4. If you need me to read something to you, show me the sign language symbol for R.



5. If you want manipulatives, show me the sign language symbol for M.



6. You may use a calculator for any part of this assessment.

---

1. Circle the most appropriate unit of measurement for each of the following:

   a) the area a postage stamp
   cm cm^2 cm^3 m m^2 m^3

   b) the length of this room
   cm cm^2 cm^3 m m^2 m^3

   c) the volume of one sugar cube
   cm cm^2 cm^3 m m^2 m^3

   d) the amount of air space in the gym
   cm cm^2 cm^3 m m^2 m^3

   e) the distance between this room and the entrance to the school
   cm cm^2 cm^3 m m^2 m^3

   f) the size of a bedroom floor
   cm cm^2 cm^3 m m^2 m^3

23.1: Measurement Assessment (continued)

2. Circle the word that best describes the shape.

a) 

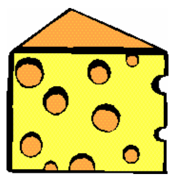
prism
pyramid
neither

b) 

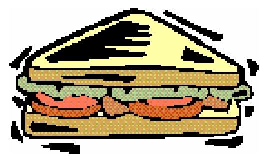
prism
pyramid
neither

c) 

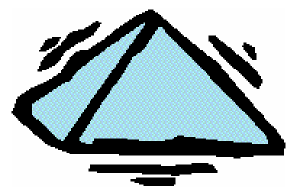
prism
pyramid
neither

d) 

prism
pyramid
neither

e) 

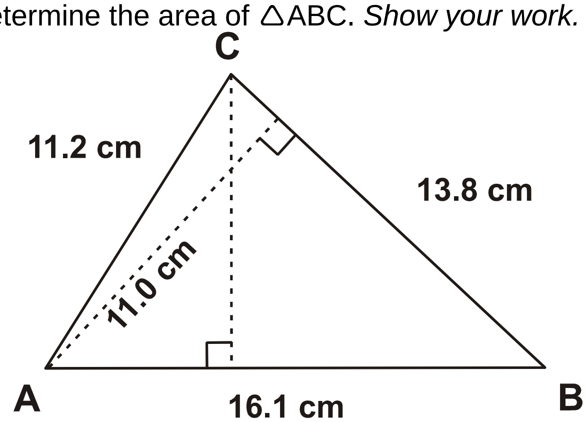
prism
pyramid
neither

f) 

prism
pyramid
neither

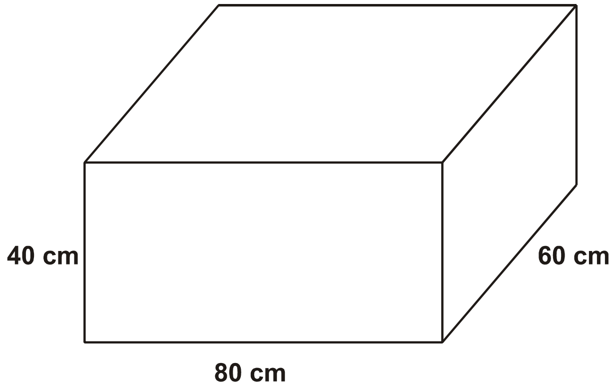
3. Give reasons for your choice for question 2 part (d).

4. Determine the area of $\triangle$ABC. *Show your work.*

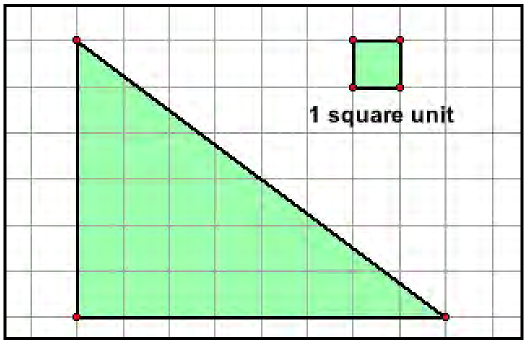


23.1: Measurement Assessment (continued)

5. Determine the amount of paper needed to cover the box shown below.
   The length is 80 cm. The width is 60 cm. The height is 40 cm.
   Show your work.



6. The diagram shows a triangular piece of land.
   The owner of this land needs to know both the perimeter and the area of the piece of land.



a) Determine the area.

b) Determine the perimeter.

c) Give one example of why the owner might need to know the area.

d) Give one example of why the owner might need to know the perimeter.

e) The area can be found in more than one way. Show different ways to find the area. (Use the back of this sheet.)

Description

- Identify and sketch 3-D figures.
- Explore 3-D figures from their top, side and front views.
- Investigate and describe geometric concepts related to 3-D figures.

Materials

- calculators
- chart paper
- Geosolids
- bags of solids
- BLM 24.1, 24.2, 24.4.

Assessment Opportunities

Minds On ...

Whole Class → Discussion

Pose questions to help the students identify Geosolids by name. Use the Geosolids to discuss definitions for side, face, vertex, edge, line of symmetry, prism, pyramid, base, etc. Add vocabulary to the word wall. Demonstrate strategies for drawing solids (see BLM 24.1).

Note and share "Active Listening" social skills Anchor Chart from Section 4 –TIPS for Teacher, p. 2

Action!

Small Groups → Investigation

Small groups use chart paper and Geosolids to create a table:

3-D no.	Shape name	No. of faces	No. of vertices	No. of edges	Shape of base	Other?
1						

Students calculate the percent of the solids that are cubes, rectangular prisms, triangular prisms, cylinders, pyramids, etc.

Curriculum Expectations/Observation/Mental Note: Circulate to assess understanding of terminology.

Prepare the numbered bags in advance and tape them shut. Put a 3-D solid into each bag.

Pairs of Groups → Discussion

Pairs or small groups compare charts and verify calculations.

Independent →Investigation

Students identify the type of hidden 3-D solid in one numbered bag, using only touch. Students sketch the 3-D solid on isometric dot paper. (BLM 24.4.) Students select a different bag and repeat the process.

Connect to art concepts by sharing the GSP 24.1 Cabin.gsp file. This sketch features a dynamic model of a cabin, drawn with a one-point perspective.

Consolidate Debrief

Whole Class→ Guided Exploration

Open the bags and place the shapes by their numbers so the students can see them. Identify the shapes and their characteristics. Students check that they drew each figure.

GSP 24.2 Drawing.gsp file was used to create TIP 24.1. Teachers may find this file useful for demonstration purposes or for creating other blackline masters.

Reflection
Application

Home Activity or Further Classroom Consolidation

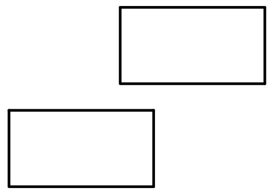
1. List 3-D objects of different shapes. Your list will be used for a game.
2. Complete Worksheet 24.2 for answers.
3. Research the Internet for tips on drawing 3-D figures. Try searching "perspective drawing." (Sample site: http://www.mste.uiuc.edu/m2t2/geometry/perspective/)

See worksheet 24.3 answers.

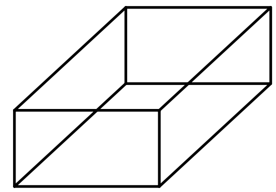
24.1: Drawing 3-D Solids (Teacher)

Rectangular Prism

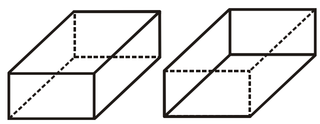
Step 1: Draw two congruent rectangles.



Step 2: Join corresponding vertices



Step 3: Consider using broken lines for edges that can't be seen.

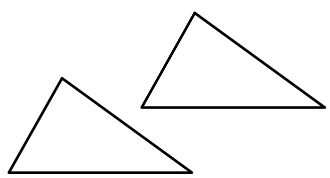


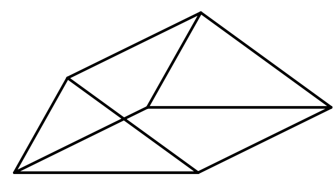
Triangular Prism

Step 1: Draw two congruent triangles.

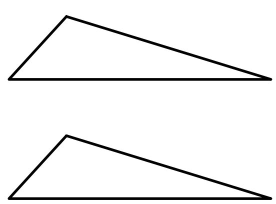
Step 2: Join corresponding vertices.

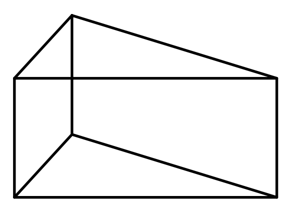
Example 1



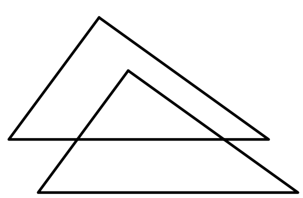


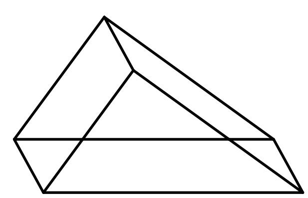
Example 2



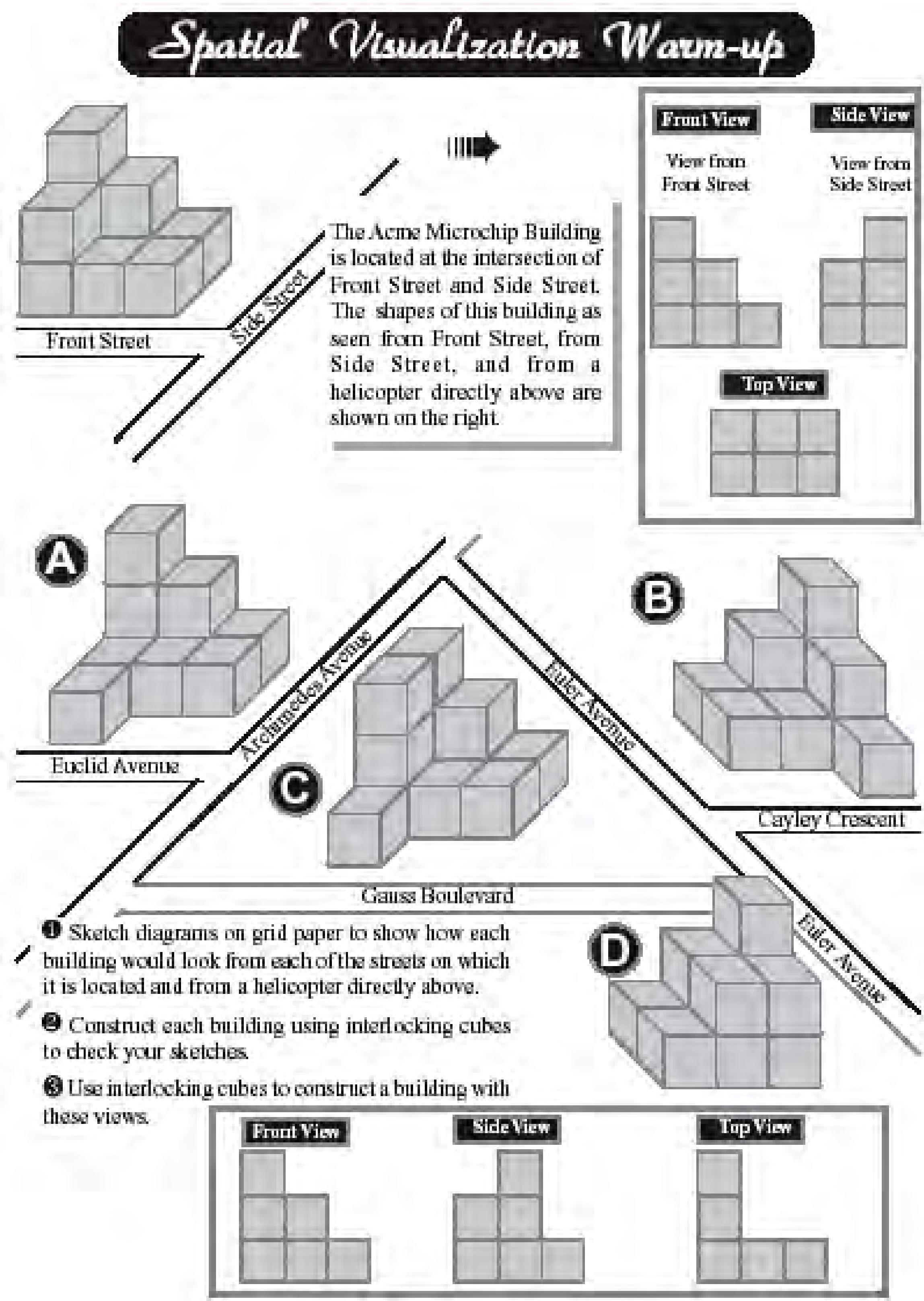


Example 3





24.2 (from *Impact Math – Geometry and Spatial Sense*, pg. 54)



Spatial Visualization Warm-up

The Acme Microchip Building is located at the intersection of Front Street and Side Street. The shapes of this building as seen from Front Street, from Side Street, and from a helicopter directly above are shown on the right.

❶ Sketch diagrams on grid paper to show how each building would look from each of the streets on which it is located and from a helicopter directly above.

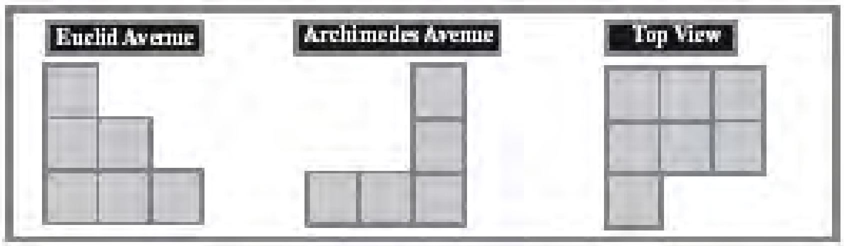
❷ Construct each building using interlocking cubes to check your sketches.

❸ Use interlocking cubes to construct a building with these views.

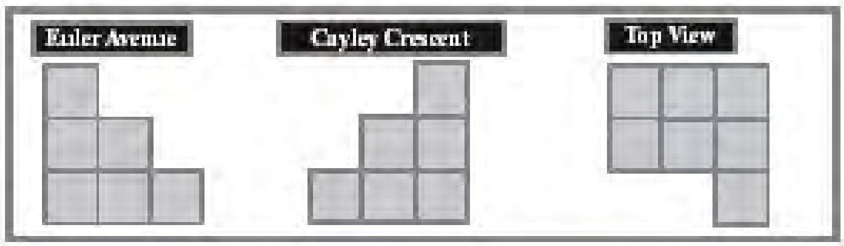
24.3 (from *Impact Math – Geometry and Spatial Sense*, pg. 54)

Answers to Spatial Visualization Warm-up

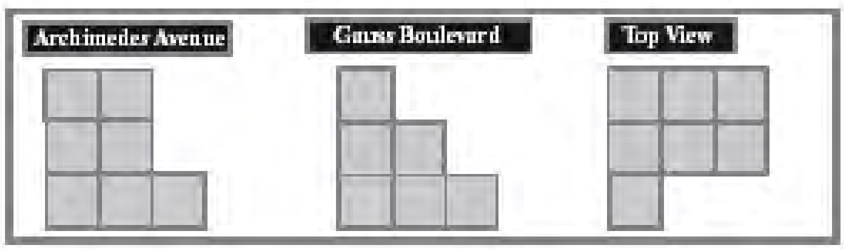
A



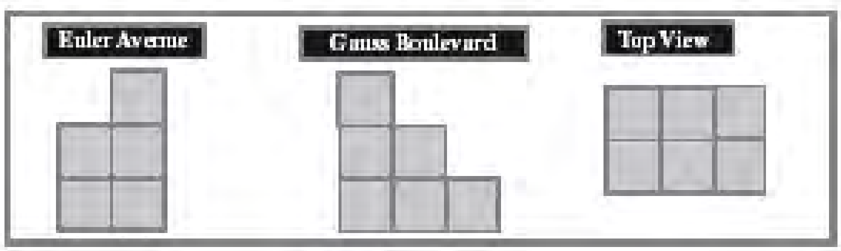
B



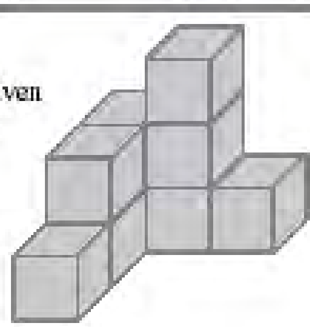
C



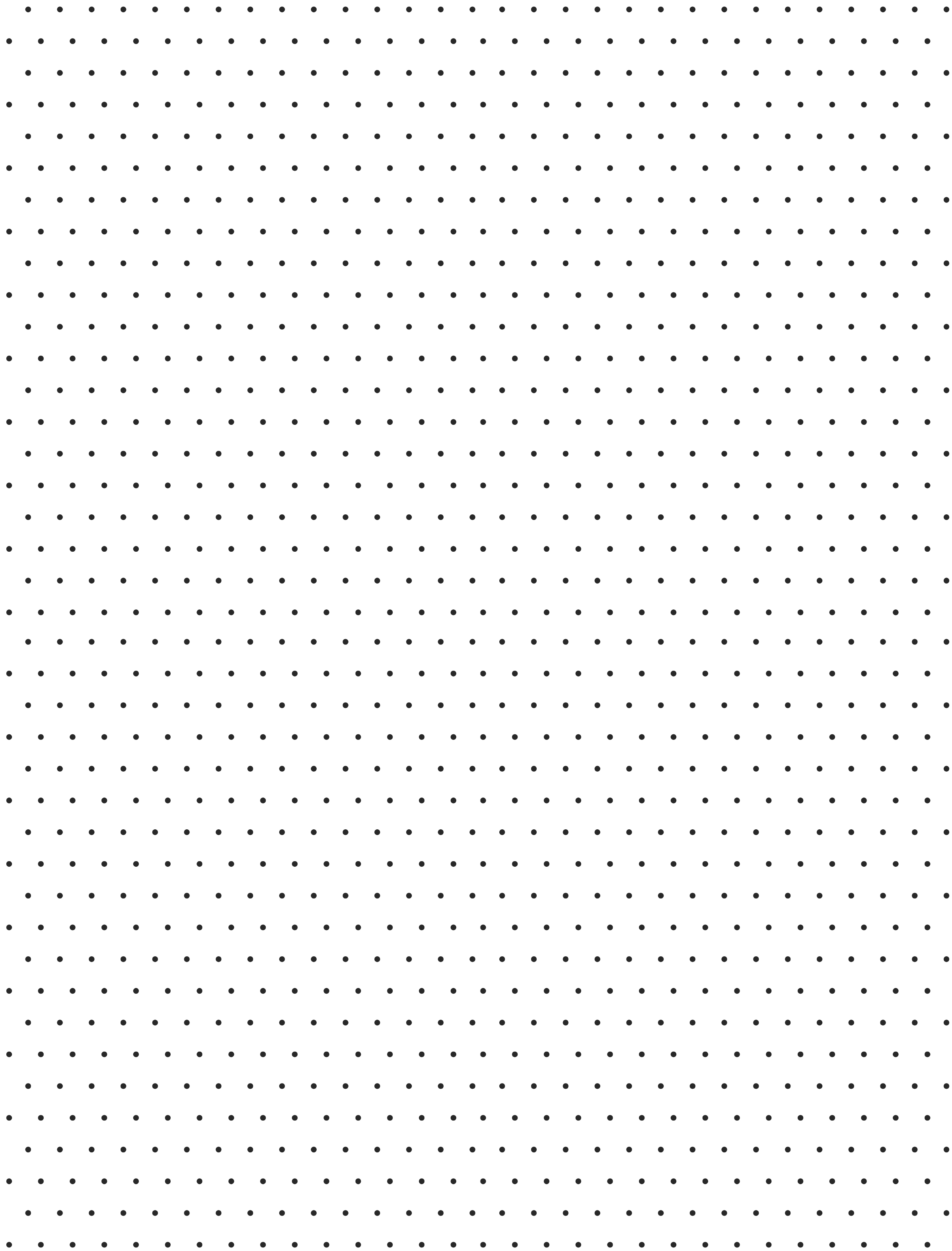
D



❸ One building with the given profiles looks like this.



24.4: Isometric Dot Paper



Day 25: Sketchy Skeletons — Grade 8

Description
- Sketch and build 3-D figures.
- Research and report on uses of measurement of 3-D objects in the community.

Materials
- BLM 24.2, 24.3
- straws
- pipe cleaners
- scissors

Assessment Opportunities

Minds On ...

Individual → Self-Assessment

Post answers (BLM 24.3). Students check their solutions to BLM 24.2.

Whole Class → Game

Students share their lists of 3-D items (Home Activity, Day 24). One person begins sharing his or her list. If more than one person has listed it the item is crossed off everyone's lists. Students continue sharing items until only unique items remain on each student's list.

Note and share "Taking Turns" social skills Anchor Chart from Section 4 – TIPS for Teacher, p. 2

Action!

Pairs→ Model Making

Students create a 3-D solid skeleton of a triangular prism using straws and small pieces of pipe cleaner. Students draw the shape on isometric dot paper to scale. Keep these shapes for the next day.

Curriculum Expectations/Observation/Mental Note: Observe students as they work through the process of constructing and drawing 3-D figures. Look for difficulties with producing scaled drawings.

Straws and pipe cleaners could be replaced by gum drops and toothpicks.

Consolidate Debrief

Whole Class→ Brainstorming

Invite students to create a brainstorming web about where 3-D design and construction is used in the community, e.g., computer graphics, architecture, artwork, model replicas, sculptures, film. Extend the conversation by having students brainstorm about careers that would use 3-D designs.

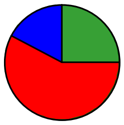
Differentiated

Home Activity or Further Classroom Consolidation

Choose a career that was brainstormed during the class discussion. Describe, in your math journal, or on a poster, how this career needs skills for drawing or building 3-D shapes. Consider researching the career on the web or at the library.
Create a concept circle with pictures of triangular prisms.

As an extension, students could create a geodesic dome – see *Impact Math, Geometry and Spatial Sense,* p. 48.

See *Think Literacy: Cross-Curricular Approaches – Mathematics* for information on Concept Circles.



Description

- Sketch and create nets.
- Use mathematical terminology.

Materials

- Connecting plastic shapes
- 3-D solids
- BLM 26.1, 26.2

Assessment Opportunities

Minds On ...

Inside Outside Circle→ Sharing Research

Students share home activity. (See TIPS - Section 4, p. 14 Inside Outside Circle strategy).

Whole Class→ Vocabulary Development

Ask students to describe the prism skeletons built on the previous day. Encourage the use of appropriate geometric terminology. Add words to the word wall. Create definition charts for some or all of the following words: prism, pyramid, face, volume, surface area. Discuss the purpose of "math" adjectives like *rectangular* prism, *triangular* prism.

Action!

Whole Class→ Guided Instruction

Show students a cube and its net. Activate prior knowledge about the relationship between the cube and its net, and the measurements for each. Discuss different nets for the same cube.

Pairs→ Model Making

Students construct nets for a variety of 3-D solids. Challenge students to create different nets for the same solid.

Curriculum Expectations/Observation/Mental Note: Circulate to assess students understanding of nets.

Pair/Share → Activity

Distribute 6 square plastic shapes of equal size to each pair of students. Challenge them to make all possible nets for a cube. After four minutes, pairs of students to share and complete this task, looking for more possibilities. Students share by drawing the various ways of making a net for a cube. Explain to students that some nets may be transformations of nets already found and are not considered to be different.

Distribute 2 equilateral triangular plastic shapes of equal size to each pair of students so that they can create one triangular prism. Pairs share, showing the net for their prism.

Circulate to prompt students with their investigations. Pose inquiry questions such as:

- What if …?
- What do you think would happen if …?
- What generalizations can you make about …?
- Can you confirm …?

Consolidate Debrief

Whole Class→ Reflection

Students draw conclusions about making nets and about the different ways to make nets (e.g., Is there always more than one way to create a net for a solid?).

Application
Concept Practice

Home Activity or Further Classroom Consolidation

1. Sketch all possible nets of a triangular prism.
2. Create two mystery nets for next class.
3. Cut out the triangular prism net on worksheet 26.1. Describe how you would find the surface area of the net. Create the prism with the description of how to calculate surface area for the prism on the inside.

Consider using the skeleton 3-D prism models to make a class mobile.

See *Think Literacy: Cross Curricular Approaches – Mathematics* for a variety of definition charts, e.g., Frayer Model, Verbal Visual Word Association.

Key Terms:
skeletons, 3-D shapes (names), prism, pyramid, views, vertices, edges, faces, etc.

Students can use isometric paper.

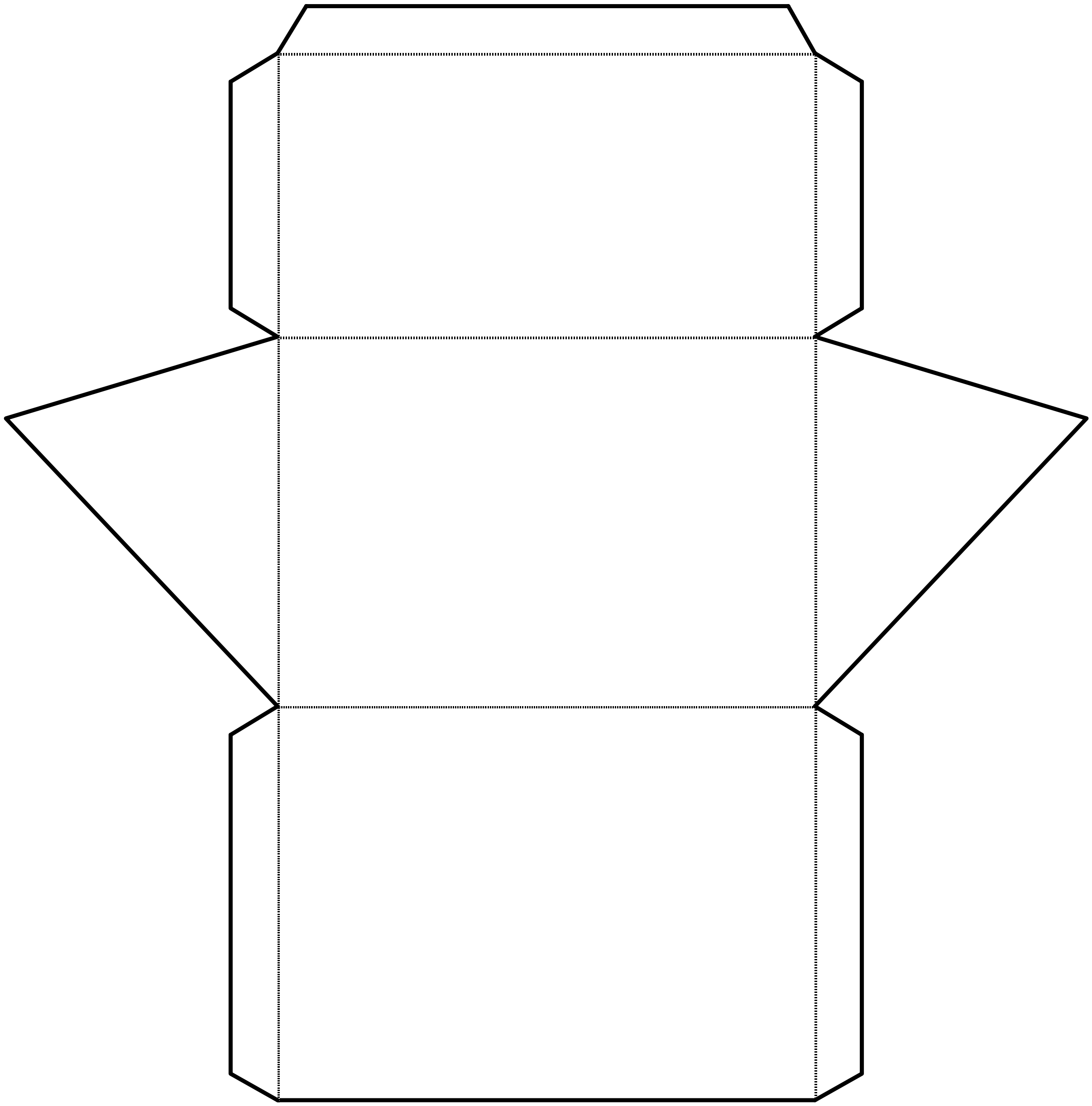
[This is a review from Grade 7 so not too much time should be given to this task.]

See TIP 26.1 for cube net solutions.

Some students may be ready to create dynamic nets using *The Geometer's Sketchpad.*

The file GSP 26.1 Nets.gsp contains adjustable nets for rectangular and triangular prisms.

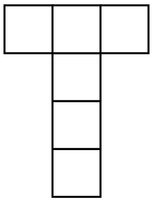
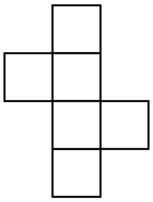
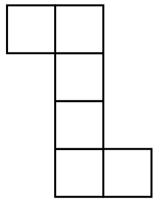
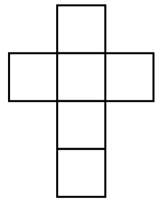
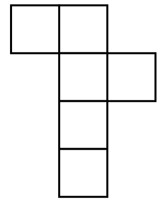
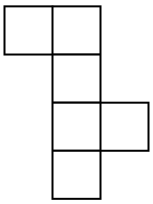
26.1: Triangular Prism Net



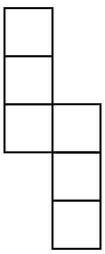
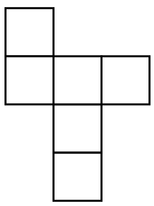
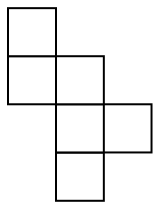
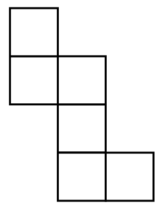
26.2: Nifty Nets

(Solutions for Cube Nets)

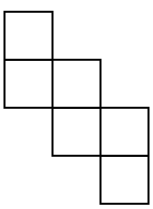
Possible nets where the net uses four linking squares:

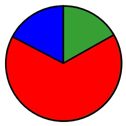
Possible nets where the net uses three linking squares:

Possible nets where the net uses two linking squares:



* Please note that the number of nets increases when you include reflections and rotations of these different configurations.



Description

- Develop the formula for surface area of a triangular prism.
- Apply the formula to solve problems.

Materials

- TIPS, Section 2, p. 13
- polydrons
- chart paper
- BLM 27.1
- data projector
- GSP 26.1

Assessment Opportunities

Minds On ...

Think/Pair → Activity

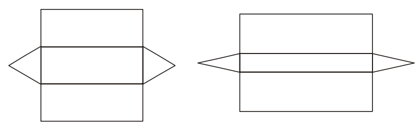
Students gently toss their prisms with descriptions of how to compute the surface area to a partner of their choice. Each partner does a peer review of the description.

(See *Think Literacy: Cross Curricular Approaches – Mathematics* for information on the Peer Review strategy).

Action!

Small Groups → Conferencing

Students use their descriptions for calculating surface area to develop a formula for the surface area of a triangular prism. Remind them to consider triangular prisms where the triangles are not equilateral. Give each group a set of polydrons to help them visualize during discussions.

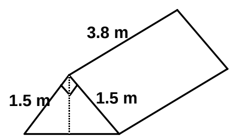


Surface Area = 2 × (area of one triangle) + (areas of 3 rectangles)

Make certain students can identify the three base and height pairs for a triangle. To reinforce this concept, demonstrate using a drafting square.

Small Groups → Application

Challenge groups to use their formula to find out how much material is required for the illustrated tent. **(Note: Peak of tent makes a right angle.)** Record solutions for whole class presentation.



Curriculum Expectations/Observation/Checklist: Circulate to assess students' prior knowledge on Pythagorean Theorem.

Encourage students to represent their method using either words, variables or numbers or a combination.

Use *TABS+* software to further create and investigate nets.

Consolidate Debrief

Whole Class → Discussion

Discuss the small group formulas and tent question. Ask students how the formula changes if the prism has no top or bottom, i.e., the tent is open on one or both ends. Ask students how the formula can be simplified if the prism has congruent faces (like the tent example).

Use GSP 26.1 Nets.gsp file to make observations about how the net changes when dimensions are changed.

Differentiated

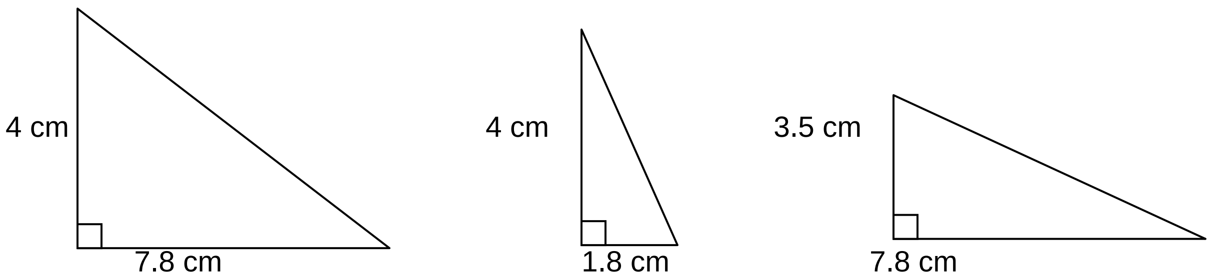
Home Activity or Further Classroom Consolidation

Construct a net for a triangular prism that has two equilateral triangle faces. Create the prism and show calculations for the surface area.

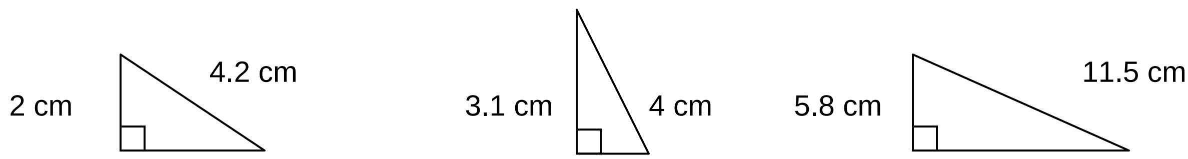
OR Practise using the Pythagorean Theorem by completing worksheet 27.1.

27.1: Using the Pythagorean Theorem

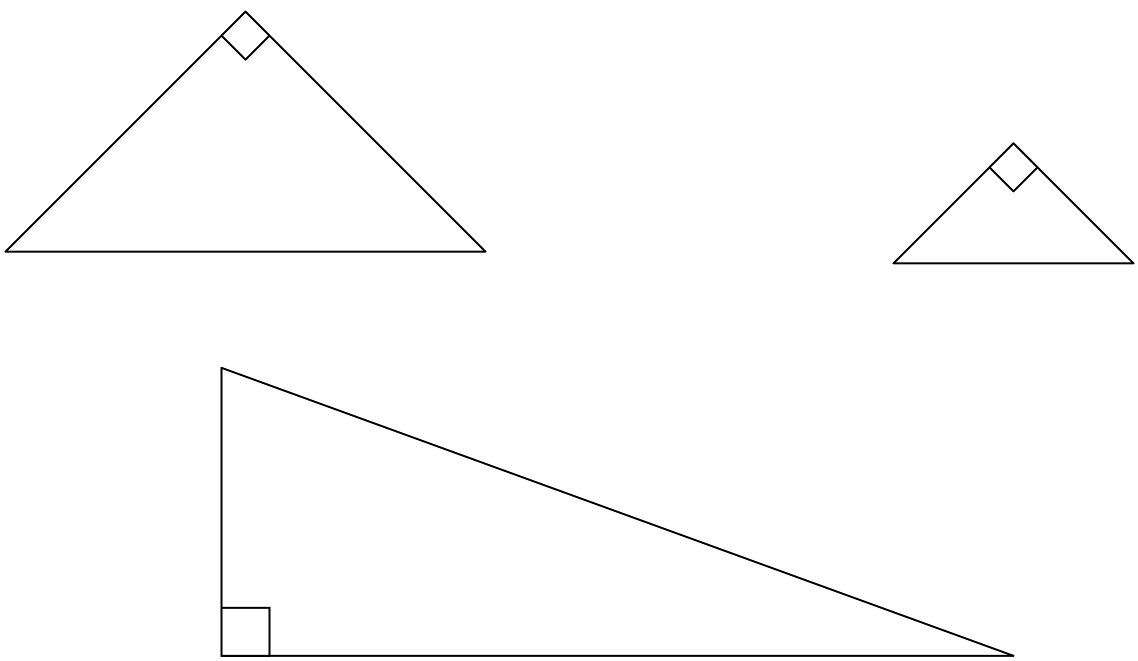
Determine the length of the hypotenuses.



Determine the length of the unknown leg.



Use a ruler to measure the lengths of the sides of the triangles. Find the area of each triangle in two different ways.



Day 28: Wrap It Up

Grade 8

Description

- Apply surface area formula to problems involving triangular prisms using nets.
- Demonstrate estimation strategies in solving problems.

Materials

- a variety of triangular prisms
- a box to hold a triangular prism
- BLM 28.1, 28.2

Assessment Opportunities

Minds On ...

Pairs → Game

Give students instructions for "Root Magnet" game (see BLM 28.1). Pairs play the game for a few minutes to ensure that they understand the rules.

Action!

Whole Class→ Guided Exploration

Explain that you have a gift for a friend that needs to be wrapped. It is in the shape of a triangular prism. You are debating putting it in a box, because you think it would be easier to wrap than as it is. Ask the students what they think you should do. Show the class a triangular prism and a box that could hold the triangular prism. Discuss how to estimate- the amount of wrapping paper needed for each scenario. Discuss how estimation can be used to help solve problems.

Pairs → Practice

Distribute BLM 28.2. Create and use Anticipation Guide questions for BLM 28.2: Before students complete BLM 28.2, use questions to clarify their understanding of instructions and problems.

Anticipation Guide *Think Literacy: Cross-Curricular Approaches Grades 7-12* p. 20.

Consolidate Debrief

Pairs → Presentation

Students present their solutions to first question on BLM 28.2. Stress good communication and mathematical notation.

Curriculum Expectations/Observation/Rubric: Assess communication.

Reflection
Application

Home Activity or Further Classroom Consolidation

1. Describe situations where the Pythagorean Theorem is needed to find surface area of a prism.
2. Complete worksheet 28.2.
3. Play "Root Magnet."

28.1: Root Magnet

Names:
Date:

Getting Ready

Work in pairs. You need a calculator for this activity.

How to Play

Each player selects 2 pairs of numbers from 1 to 10 to represent side measures for a right triangle. The numbers are entered on their opponent's score sheet. Each player then estimates the length of the hypotenuse of the triangle without using a calculator. The actual length is determined using the Pythagorean theorem and a calculator. Enter the length on the score sheet. To score, enter the difference between the estimated value and the actual value. The score is the total of all the differences. The player with the lowest score is declared the Root Magnet!

Example:

Lengths	Estimate	Actual	Score
5, 10	12	11.2	0.8
2, 8	8.8	8.2	0.6
Player A		Total	1.4

Let's Play

Round One

Lengths	Estimate	Actual	Score
Player:		Total	

Target	Estimate	Actual	Score
Player:		Total	

Round Two

Lengths	Estimate	Actual	Score
Player:		Total	

Target	Estimate	Actual	Score
Player:		Total	

Round Three

Lengths	Estimate	Actual	Score
Player:		Total	

Target	Estimate	Actual	Score
Player:		Total	

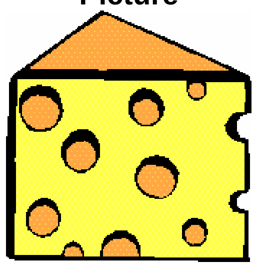
(adapted from Grade 8 BLM Lesson 10.1)

28.2: Surface Area of Triangular Prisms

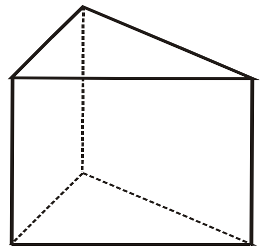
Show your work in good form and be prepared to tell how you solved the problem.

1. Determine the minimum amount of plastic wrap needed to cover the cheese by finding the surface area of the prism. Why might you need more wrap?

Picture

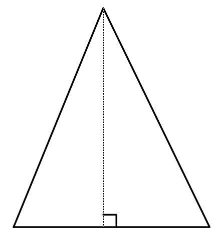


Skeleton



height of prism = 5.0 cm

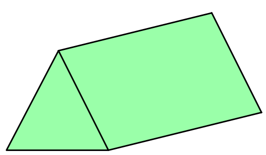
Base



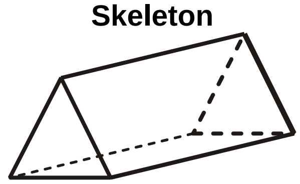
height of triangle = 6.0 cm
base of triangle = 4.0 cm

2. Determine the surface area of the nutrition bar.

Picture



Skeleton



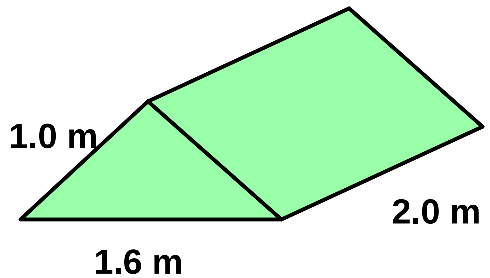
height of prism = 5.0 cm

Base

height of triangle = 3.0 cm
base of triangle = 3.5 cm

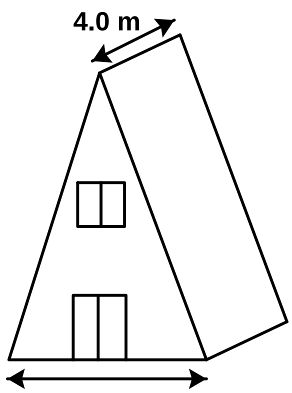
3. Determine the surface area of the tent.
The front of the tent has the shape of an isosceles triangle.
(Hint: You will need to use the Pythagorean Theorem to find the height of the triangle.)

Create a problem based on the surface area.



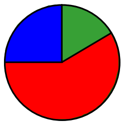
4. This "A-Frame" ski chalet needs to have the roof shingled. Determine the surface area of the roof.

Hint:
Think about whether the height of the chalet is the same as the height of the prism.



Extension: If the shingles were 35 cm long and 72 cm wide, how many would you need to cover the roof?

Day 29: Volume of a Rectangular Prism — Grade 8



Description

- Develop a formula for finding the volume of a triangular prism.
- Apply volume formula in problem solving.

Materials

- Open-topped triangular prisms
- BLM 29.1
- Graduated cylinders
- rulers

Assessment Opportunities

Minds On ...

Pairs→ Discussion

Write the formula for the volume of a rectangular prism ($V = A_{base} \times height$) on the board. In pairs, students discuss how to design an experiment that would confirm or refute the hypothesis that the formula will also work for a triangular prism.

Whole Class → Experiment

Fill an open-topped waterproof triangular prism with water. Pour the water into a graduated cylinder to measure the volume. Calculate the volume of the prism using the formula, then compare the two volume measures.
Discuss experimental errors. Discuss whether one example is sufficient to confirm the hypothesis.

Action!

Pairs→ Collecting Data

Give each pair an open-topped triangular prism and measuring instruments so they can determine the volume of the prism, using the formula, and filling then measuring.
Learning Skills/Observation/Rubric: Circulate to assess students in teamwork and staying on task.

Whole Class→ Making Conclusions

Create a two-column chart to collect class data for the two measures of volume for each triangular prism. Discuss experimental errors and any outliers (unusual) entries. Lead a discussion which concludes with consensus that the formula will work for any prism. Students might make the observation that the formula would also work for a cylinder and other solids that have two identical bases and vertical sides.

Consolidate Debrief

Whole Class→ Guided Instruction

Distribute BLM 29.1 and discuss some of the challenges when moving from the physical model of a triangular prism to a pictorial or word model (see sidebar notes). Do question 1 together. Discuss when you might have to use the Pythagorean Theorem.

Concept Practice

Home Activity or Further Classroom Consolidation

Complete worksheet 29.1.

When calculating volume of a rectangular prism, any of its faces can be thought of as the "base". However, when calculating the volume of a triangular prism, its "base" is one of the triangles, not one of the rectangles.

Note that the height of a triangle is perpendicular to its base and the height of the prism is perpendicular to the triangular base.

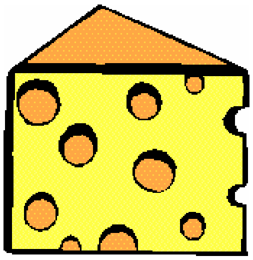
Some students may need to physically or mentally re-orient triangular prisms in order to find the volume.

29.1: Volume of Triangular Prisms

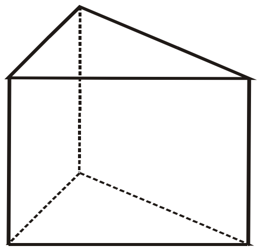
Show your work and use good communication.

1. Determine the volume of the piece of cheese (ignore the holes!).
   Create a problem based on the volume.

Picture

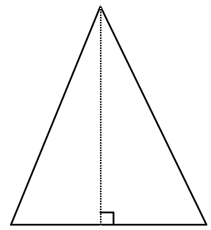


Skeleton



height of prism = 5.0 cm

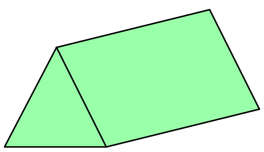
Base



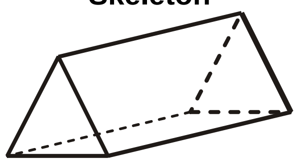
height of triangle = 6.0 cm
base of triangle = 4.0 cm

2. Determine the volume of the nutrition bar.
   Create a problem based on the volume.

Picture



Skeleton



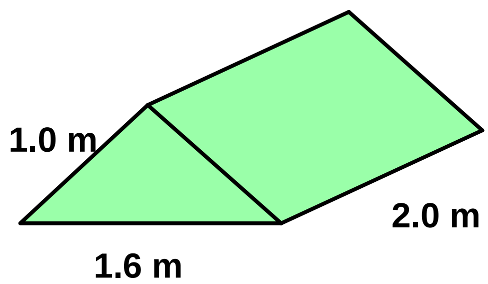
height of prism = 5.0 cm

Base

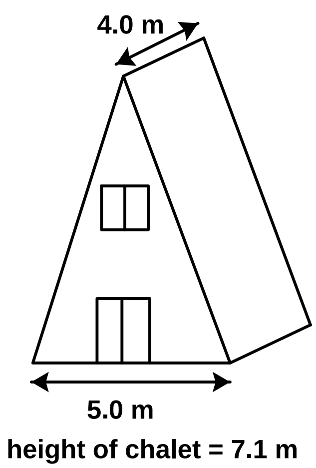
height of triangle = 3.0 cm
base of triangle = 3.5 cm

3. Determine the volume of air space in the tent.
   The front of the tent has the shape of an isosceles triangle.
   (*Hint:* You will need to use the Pythagorean Theorem to find the height of the triangle.)

   Create a problem based on the volume.



4. If you could only have 1 person per 15 m^3 to meet fire safety standards, how many people could stay in this ski chalet?



Hint:
Think about whether the height of the chalet is the same as the height of the prism.

How much longer would the chalet need to be to meet the safety requirements to accommodate 16 people?

Description

- Apply volume and area formulas to explore the relationship between triangular prisms with the same surface area but different volumes.
- Estimate volumes.

Materials

- rectangular tarp (rope – see sidebar)
- connecting cubes
- BLM 30.1

Assessment Opportunities

Minds On ...

Small Groups → Discussion

Students share solutions for BLM 29.1. Assign each group one solution for a whole class presentation.

Whole Class → Instruction

This activity might be done outside or in a gymnasium. Place a large tarp on the floor/ground. Invite six students to become vertices of a triangular prism tent. Instruct four of the students that their task is to keep their vertices on the ground. The remaining two students stand on opposite sides of the tarp, equidistant from the ends, to become the fifth and sixth vertices. These two vertices gradually raise the tarp until a tent is formed. Note that two of the "ground" vertices will have to move. Invite two or three other students to become campers. Their task is to verbalize observations about the tent's capacity as the tent's height is increased and decreased. Ask: Does it feel like there is more or less room?

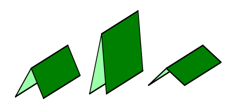
Consider using a rope to hold the peak of the tent in place.

Students might investigate changes when the "fold" is moved from lengthwise to widthwise.

Action!

Pairs → Model Making

Students simulate the tent experiment using a sheet of paper and connecting cubes. Data may be collected in a two-column chart – height of "the tent" vs. number of connecting cubes that will fit inside the "tent" without bulging the sides.



Learning Skills/Observation/Checklist: Assessing students' ability to stay on task.

Think/Pair/Share → Discussion

In pairs, students respond to the question: Is the following statement sometimes, always, or never true?

Two triangular prisms with the same surface area also have the same volume.

While circulating, ask probing questions to ensure that students realize that investigation of this statement differs from the 'tent' investigation since the 'floor' was ignored in the tent scenario, but cannot be ignored in this question. Ask students if their conclusion would be the same for closed and open-ended prisms.

From the model making activity, students will have a sense that the statement is not always true. Since the areas of the triangular ends of the tent prisms were not investigated, encourage students to question the importance of these measurements when considering the statement.

Consolidate Debrief

Whole Class → Discussion

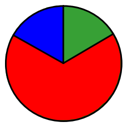
Discuss how an experiment might be designed to help students confirm or deny hypotheses about the relationship between surface area and volume.

Skill Practice

Home Activity or Further Classroom Consolidation

Use BLM 30.1. Make two folds using the two solid lines. Form a triangular prism. Imagine that it also has paper on the two triangular ends. Sketch the prism and its net. Take the measurements needed to calculate the surface area (including the two triangular ends) and volume. Label the diagrams with the measurements. Calculate the surface area and volume. Repeat the process for the prism formed used the two broken line.

30.1: Triangular Prisms



Description
- Sketch triangular prisms having a given volume.
- Demonstrate an understanding of relationship between volume and varying dimensions.

Materials
- data projector/ computer
- BLM 31.1

Assessment Opportunities

Minds On ...

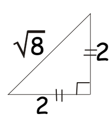
Whole Class → Discussion
Use GSP 31.1 to debrief the Home Activity from Day 30. The dynamic model can be used to quickly check student responses and investigate additional scenarios. See BLM 31.1 for screen shots of the three GSP pages in the file.

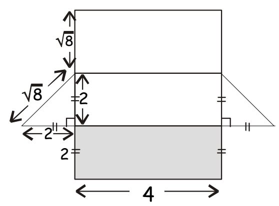
Whole Class → Brainstorm
Discuss when and why a company might be concerned about the size of the packaging for a product.

For students who benefit from visual and kinesthetic learning, consider preparing cut-out nets of the triangular prisms so that students can better visualize the surface area and the different elongated shapes that are created.

Action!

Small Groups → Investigation
Introduce the problem using BLM 31.2 Wrapping Gifts.
Students investigate:
a) For prisms with the same volume, is the surface area also the same? *(No)*
b) What shape of rectangular/triangular prism has the largest surface area for a given volume?





Note: Some students will require a review of the Pythagorean Theorem in order to use the hypotenuse length to calculate the dimensions of the rectangles.

Formula Sheet is found in Section 4 – TIPS for Teachers, p. 21. Students might wish to refer to the formulas for the rectangular prism and the isosceles triangular prism.

Curriculum Expectations/Observation/Rubric: Observe students as they work on the investigation. Assess inquiry skills.

Individual → Written Report
Individually, each student prepares a written report of their findings.

Consolidate Debrief

Whole class → Student Presentations
Students present their findings. The teacher may choose to conduct this debriefing with the class after the assessments are returned.

Apply the mathematics learned in today's activity to answer this question: *Why would a Husky dog curl up in the winter to protect himself from the cold winds when he is sleeping outdoors?*

If the dog remains "long and skinny" he has greater surface area exposed to the cold. If he curls up, he has less surface area exposed to the cold, and will lose much less body heat. Although his volume stays the same, his surface area decreases as he becomes more "cube-ish" or spherical.

Concept Practice

Home Activity or Further Classroom Consolidation
Complete the following surface area and volume problems.

In preparation for the test, prepare area and volume for students to solve.

31.1 The Geometer's Sketchpad® Dynamic Model

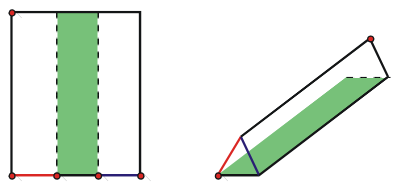
Paper Folding to Investigate Triangular Prisms

If two triangular prisms have the same surface area will they have the same volume?

The dynamic model below shows a piece of paper with two fold lines. If possible, the folded sections are joined so that the paper becomes an open-ended triangular prism.

Change the prism by changing the location of the folds.

Any red point in the dynamic model can be dragged.



Next Page: Measurements

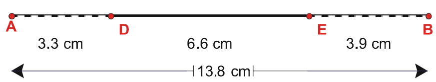
Paper Triangular Prisms

Fold Paper

Unfold Paper

Show Area of Triangle

Imagine you are looking at the edge of the paper.
The paper is folded at point D and point E.



Next Page: Volume

Paper Folding to Investigate Triangular Prisms

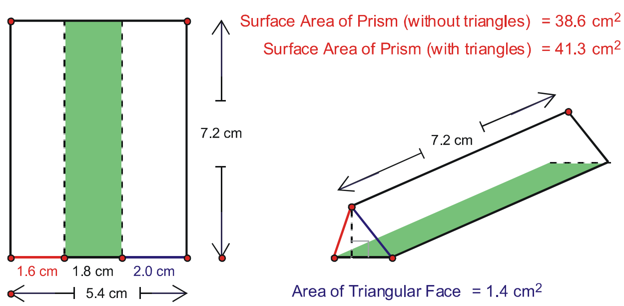
1) What paper size are you using?
2) Where do you want to place your folds?
3) If two triangular prisms have the same surface area do they have the same volume?

Hide Text

Volume of Prism = 9.8 cm^3

Surface Area of Prism (without triangles) = 38.6 cm^2

Surface Area of Prism (with triangles) = 41.3 cm^2



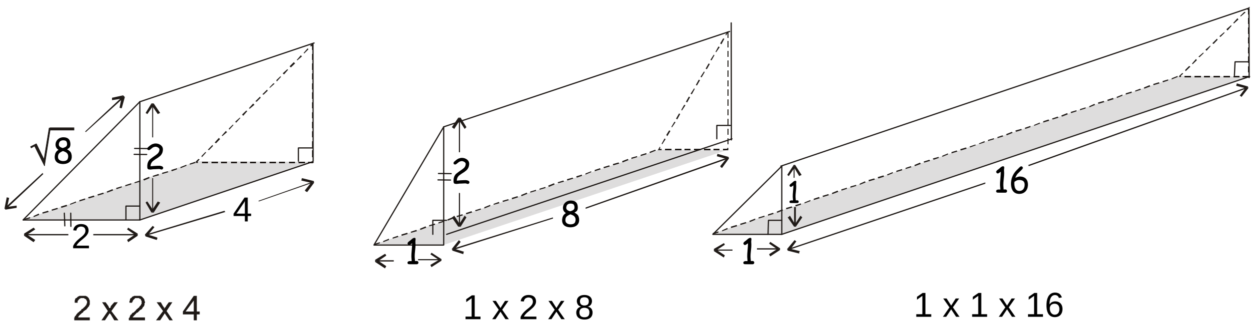
Area of Triangular Face = 1.4 cm^2

Return to page 1

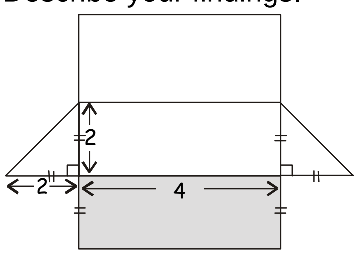
31.2: Wrapping Gifts

Let's investigate!

Three different triangular prism gift boxes each have a volume of 8 cubic units.
Does each box require the same amount of paper to wrap?



1. a) Verify that each triangular prism illustrated above has a volume of 8 cubic units.
   b) Draw to scale the net for each triangular prism box.
   c) Determine the amount of paper required by calculating the surface area. (Ignore the overlapping pieces of paper you would need.) The first one has been started for you.
   d) Describe your findings.



2. a) How many different triangular prism boxes can be designed to have a volume of 24 cubic units?
   b) Draw several of the boxes, labelling the dimensions.
   c) How much paper is required to wrap each box?
   d) Describe your findings.

3. Investigate wrapping triangular prism boxes with a volume of 36 cubic units. Determine the dimensions of the triangular prism with the greatest surface area.

4. Write a report of your findings. Include the following information, justifying your statements.
   - Describe how surface area and volume are related, when the volume remains the same.
   - What shape of triangular prism box uses the most paper?
   - What shape of triangular prism box uses the least paper?

Description

- Assess students' knowledge and understanding of surface area and volume of triangular prisms.

Materials

- recipe cards
- tent catalogues
- BLM 32.1

Assessment Opportunities

Minds On ...

Whole Class → Instructions

Ask students to read the description of the final product on BLM 32.1. Explain that each student will create a different triangular prism tent design for a specified purpose, e.g., a tent for a doll, a 2-person tent for camping, a tent to cover a boat for the winter.

Students can highlight to mark the corresponding instructions on BLM 32.1 as you describe the assessment.

Action!

Small Groups → Brainstorm

Students brainstorm ideas for their tent. Have available sample tent catalogues – print or electronic. Allow internet use to find tent ideas.

Individual → Design

Students work individually on their design.

Curriculum Expectations/Observation/Anecdotal Notes: Circulate and help students as needed, recording notes for which parts the students needed extra help.

Give students an opportunity to clarify instructions. Ask if they have a clear picture of what the end product will be.

Consolidate Debrief

Whole Class→ Discussion

Describe activities for next day. Students will do a peer review of another student's recipe card and answer questions based on a new catalogue item. Show students the picture of the tent which is two prisms put together. Use Geosolids to demonstrate how two prisms can be joined to form one solid. Discuss how surface area and volume would be calculated for a composition of more than one solid.

Familiar compositions of 3-D solids:

- ice cream cones
- silos
- houses
- sculptures

Home Activity or Further Classroom Consolidation

Reflection

Write a journal entry about this unit. Reflect on what you found the hardest, the easiest, the most interesting, the least interesting. Write a question that you still have about the surface area and/or volume of prisms.

Describe basic house designs in terms of prisms, e.g., a house with a flat roof might be described as a rectangular prism.

Think about how you might find the surface area and volume of a solid that has a rectangular prism topped with a triangular prism.

32.1: Class Catalogue

You work for a company that produces tents. Part of your job is to produce the annual catalogue used to advertise your company's products.

Each item in the catalogue includes:

a) a picture of the item
b) a description of the item to convince the reader to buy this item
c) the price
d) footprint dimensions and/or area
e) the capacity (as related to the tent's purpose, e.g., 2-person tent)

Each item in the catalogue is displayed in an area the same size as one recipe card.

More detailed information about the item is stored in a collection of item reports. Your item report includes:

a) sketch of the item
b) sketch of the skeleton of the item
c) sketch of a net for the item
d) calculation of surface area
e) calculation of volume
f) calculation of footprint area
g) other relevant information

All sketches include dimensions.
The report also includes comments about why this particular design was chosen for this particular purpose.

Examples:
The height of the tent was kept small because …
One wall of the tent is perpendicular to the ground because …
The surface area was kept to a minimum because …
A large volume was important because …

You must submit a completed recipe card and a report.

Description

- Assess students' knowledge and understanding of surface area and volume of triangular prisms.

Materials

- geosolids
- BLM 33.1

Assessment Opportunities

Minds On ...

Small Groups → Brainstorm

Students discuss the decomposition of complex solids. Make Geosolids available as a visualization aid. Use examples from Home Activity Day 32, e.g., triangular prism roof sitting on a rectangular prism base.

Action!

Individual → Assessment

Discuss the instructions on BLM 33.1. Students complete the task.

Curriculum Expectations/Observation/Anecdotal Notes: Circulate and help students as needed. Note strengths, area for improvement and next steps to give oral feedback. Collect student work and score using the checkbric.

Small Groups → Math Trail

Take a walk around the school or school grounds. In small groups, students brainstorm possible locations and questions for measurement math trail questions.

Consider creating a math trail for a school event. Use a digital camera to include pictures with the trail questions.

Consolidate Debrief

Whole Class→ Reflection

Share possible math trail questions and brainstorm ideas for use of a math trail in this class or another grade level.

School playgrounds and buildings have lots of hidden math trail questions.

Consider questions on snow removal, landscaping, roofing, repairs, etc.

Differentiated

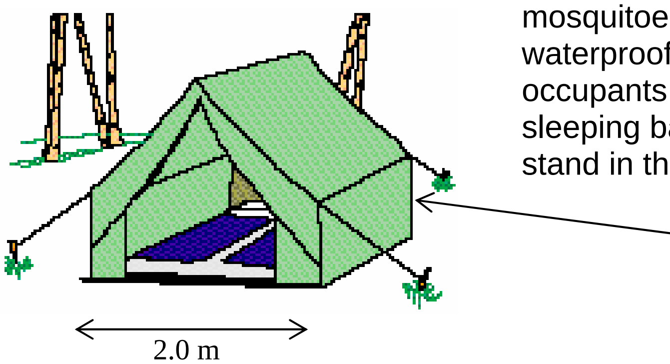
Home Activity or Further Classroom Consolidation

Complete the following questions.

Provide questions based on your assessment of students' work.

33.1: Tent Test

Camper's Choice!



This 2-person tent comes in a variety of colours. We recommend choosing a lighter colour that will not attract mosquitoes. Our tents are totally waterproof. This unique design allows occupants plenty of room for two sleeping bags and gear. You can even stand in this tent!

Footprint: 2.0 m x 3.0 m
Center Height: 2.0 m
Straight Side Height: 0.5 m
Price: $210.00
Item No. 39583749

Use the information on this advertisement to determine:

1. The amount of air space inside the tent.
2. The amount of material used to make the tent.
3. The amount of floor space per person.

Assessment CheckBric

Criteria	Level 1	Level 2	Level 3	Level 4
Computing and carrying out procedures				
Making convincing arguments, explanations and justifications				
Integrating narrative and mathematical forms				
Representing a situation mathematically				
Selecting and applying problem-solving strategies				

Developing Content and Reporting Targets for a Combined Grades 7 and 8 Mathematics Program

Crucial to planning an effective mathematics program for a combined Grade 7 and Grade 8 class is a study of the similarities and differences in the curriculum expectations for the two grades. To facilitate this comparison, the expectations are organized in parallel clusters in the Content and Reporting Targets chart.

The intent is that clusters positioned side-by-side in the Content and Reporting Targets chart are taught simultaneously to the two grades, in the sequence shown.

Five different types of comparisons between expectations for Grade 7 and Grade 8 are used to determine which clusters can be taught simultaneously:

1. Expectations that are virtually the same
These expectations are stated across the grade level columns and are identified by the expectation codes for the two grades.

Example:

Grade 7	Grade 8
7m87 and 8m94: evaluate data and make conclusions from the analysis of data; 7m104 and 8m113: evaluate arguments that are based on data analysis; 7m103 and 8m112: make inferences and convincing arguments that are based on data analysis.	

The same lesson or set of lessons can be taught to the entire class, with differentiated follow-up activities to accommodate the additional depth of skill and knowledge that is developed in Grade 8.

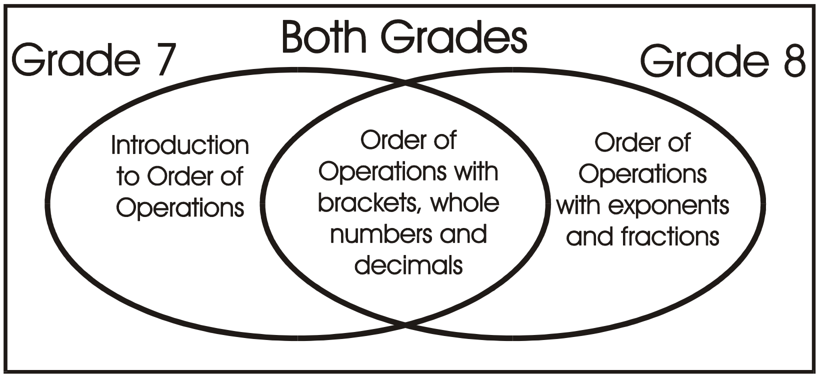
2. Expectations that are similar but have minor grade-specific differences
These expectations are stated separately across the grade level columns and are identified by the grade expectation code.

Example:

Grade 7	Grade 8
7m3: demonstrate an understanding of order of operations with brackets; 8m3: understand and apply order of operations with brackets and exponents in evaluating expressions that involve fractions; 7m20: demonstrate an understanding of the order of operations with brackets and apply the order of operations in evaluating expressions that involve whole numbers and decimals; 8m19: understand the order of operations with brackets and exponents and apply order of operations in evaluating expressions that involve fractions; 8m20: apply the order of operations (up to three operations) in evaluating expressions that involve fractions.	

In this example, expectations around the order of operations are similar, but expectations in Grade 8 require more depth of skill. Students in Grade 7 apply the order of operations using whole numbers and decimals. Students in Grade 8 would benefit from some review and practice of those concepts, but their skills must also be extended to include exponents and fractions, and they need more depth of understanding and skill in applying the order of operations.

A teacher's thinking, planning, and daily lessons might look like this:



Thinking of the differences between the Grade 7 and Grade 8 expectations in this way helps in planning the extensions and supports for this topic each time it is addressed throughout the mathematics program.

3. A few expectations in some strands have no expectations in the other grade that can be easily taught at the same time

These expectations are indicated by empty cells in the other grade column or by an obvious contrast in depth of treatment or level of abstraction between the two grades.

Example:

Patterning and Algebra	
Grade 7	Grade 8
	8m80: write an algebraic expression for the *n*th term of a numeric sequence; 8m82: use the concept of variable to write equations and algebraic expressions.

While students in both Grade 7 and Grade 8 are expected to describe in words how any term can be found in a sequence, students in Grade 8 must also write an algebraic statement for any term of a sequence. There is no extended learning for Grade 7. The empty cells in the column for Grade 7 point to the need for the students in Grade 8 to learn something not required in Grade 7. This would usually result in separate lessons for the two grades.

The need for separate lessons for the two grades is less obvious in the case illustrated below.

Fractions and Decimals	
Grade 7	Grade 8
7m18: add and subtract fractions with simple denominators using concrete materials, drawings and symbols; 7m19: relate repeated addition of fractions with simple denominators to the multiplication of a fraction by a whole number.	8m18: add, subtract, multiply, and divide simple fractions; 8m2: demonstrate a proficiency in operations with fractions.

The study of fractions and decimals are partially the same, but the depth of understanding and the additional expectations require extra learning for students in Grade 8, while in Grade 7, students spend more time on investigation, practice with manipulatives, and skill development. Both grades add and subtract fractions, however, the difference in the depth of understanding of multiplication and division requires separate lesson development for the two grades. As the students in Grade 8 develop understanding and proficiency in multiplication and division of fractions, students in Grade 7 spend extra time developing understanding of addition and subtraction.

Occasionally, the unmatched expectations can be included as an extension to a common lesson or as part of a discussion at a "teachable moment," instead of requiring a completely separate lesson.

4. Expectations that are different but can be addressed using the same type of class activity

These expectations are positioned in side-by-side in the Grade 7 and Grade 8 columns with separate cluster names for the two grades.

Example:

Grade 7	Grade 8
Volume of Rectangular Prism	Volume of Triangular Prism
7m42: develop the formula for finding the surface area of a rectangular prism using nets; 7m43: develop the formula for finding the volume of a **rectangular prism** (area of base × height) using concrete materials; 7m44: understand and articulate the relationship between the dimensions and the volume of a **rectangular prism**; 7m45: calculate the surface area and the volume of a rectangular prism in a problem-solving context; 7m46: sketch a **rectangular prism** given its volume; 7m31: apply volume formulas to problem-solving situations involving **rectangular prisms**.	8m50: develop the formula for finding the surface area of a triangular prism using nets; 8m51: develop the formula for finding the volume of a **triangular prism** (area of base × height); 8m52: understand and articulate the relationship between the dimensions and the volume of a **triangular prism**; 8m53: calculate the surface area and the volume of a triangular prism, using a formula in a problem-solving context; 8m54: sketch a **triangular prism** given its volume; 8m38: apply volume and area formulas to problem-solving situations involving **triangular prisms**.

The parallel structure of these expectations for the two grades can be addressed using the same type of rich task for both grades. Included are a series of lessons that illustrate how an investigation on volume and surface area, completed by the entire class, addresses the expectation regarding rectangular prisms (Grade 7) and triangular prisms (Grade 8).

5. Expectations address entirely different topics in the two grades

These expectations are positioned in side-by-side in the Grade 7 and Grade 8 columns with separate cluster names for the two grades.

Example:

Grade 7	Grade 8
Transformations	Pythagorean Relationship
7m50: explore transformations of geometric shapes; 7m51: understand, apply, and analyse key concepts in transformational geometry using concrete materials and drawings; 7m61: recognize the image of a two-dimensional shape under a translation, a reflections, and a rotations in a variety of contexts; 7m62: create and analyse designs that include translated, rotated, and reflected two-dimensional images using concrete materials and drawings, and computer applications. etc.	8m65: investigate the Pythagorean relationship using area models and diagrams; 8m70: apply the Pythagorean relationship to numerical problems involving area and right triangles; 8m73: explain the Pythagorean relationship.

To address the expectations of unrelated topics, separate lessons are taught to each grade for the entire unit. To minimize the disparity between the topics, a common theme might be used to link the two unrelated topics. For example, transformations have been taught throughout elementary school, with students in Grade 7 given a final opportunity to consolidate their skills. The Pythagorean relationship is introduced and developed in Grade 8. Although these topics are very different, they can be taught simultaneously, using *The Geometer's Sketchpad 4*® as a common tool and medium. A series of lessons are provided to assist teachers in presenting separate lessons.

There are a number of criteria to incorporate into a plan for implementing a math program for a combined Grade 7 and Grade 8 class. The clustering and sequencing shown in the Content and Reporting Targets chart is only one possibility. Other criteria include:
– ensuring necessary prior learning opportunities;
– providing opportunities to revisit key concepts and skills throughout the program;
– reporting requirements.

Combined Grades 7 and 8 Content and Reporting Targets

Term 1 – Content Targets	Term 2 – Content Targets	Term 3 – Content Targets
Number Sense and Numeration: • multiples and factors • prime numbers • exponents • square roots • integers • order of operations **Data Management and Probability:** • collecting data • organizing data • analyzing data • spreadsheets and databases **Patterning and Algebra:** • describing patterns • algebraic expressions • determining the *n*th term (Grade 8)	**Number Sense and Numeration:** • fractions • decimals • order of operations **Measurement:** • area and perimeter of 2-D shapes (Grade 7) • the circle (Grade 8) **Geometry and Spatial Sense:** • congruent figures (Grade 7) • angle relationships (Grade 8) **Patterning and Algebra:** • solving equations • inequalities (Grade 8)	**Number Sense and Numeration:** • ratio and unit rates • percent **Geometry and Spatial Sense:** • 2-D and 3-D geometry • transformations (Grade 7) • Pythagorean theorem (Grade 8) **Measurement:** • surface area and volume of rectangular prisms (Grade 7) • surface area and volume of triangular prisms (Grade 8) **Data Management and Probability:** • probability, tree diagrams, and frequency tables • experimental, theoretical and complex probability (Grade 8)

Across the strands and terms:
Problem Solving, Communication, Technology and Reasoning

Grade 7 and 8 Expectations to be applied to any/all content strands and terms:

7m8 and 8m6: use a calculator to solve number questions that are beyond the proficiency expectations for operations using pencil and paper;
7m52 and 8m60: use mathematical language effectively to describe geometric concepts, reasoning, and investigations;
7m12 and 8m14: explain numerical information in their own words and respond to numerical information in a variety of media;
7m16 and 8m7: justify the choice of method for calculations: estimation, mental computation, concrete materials, pencil and paper, algorithms (rules for calculations), or calculators;
7m24 and 8m31: explain the process used and any conclusions reached in problem solving and investigations.

7m7: explain, in writing, the process of problem solving using appropriate mathematical language;
7m23: ask "what if" questions, pose problems involving simple fractions, decimals and percents, and investigate solutions;
7m25: reflect on learning experiences and describe their understanding using appropriate mathematical language;
8m9: use mathematical language to explain the process used and the conclusions reached in problem solving;
8m30: ask "what if" questions, pose problems involving fractions, decimals, integers and percents, and rational numbers and investigate solutions.

7m28: demonstrate a verbal and written understanding of and ability to apply accurate measurement strategies that relate to their environment;
8m39: use listening, reading, and viewing skills to interpret and evaluate the use of measurement formulas.

7m34: research and report on uses of measurement instruments in projects at home, in the workplace, and in the community;
8m41: research, describe, and report on uses of measurement in projects at home, in the workplace, and in the community that require precise measurements.

Grade 7 Expectations to be applied to any/all content strands and terms:	**Grade 8 Expectations to be applied to any/all content strands and terms:**
7m52: use mathematical language effectively to describe geometric concepts, reasoning, and investigations.	8m28: use estimation to justify or assess the reasonableness of calculations; 8m32: reflect on learning experiences and interpret and evaluate mathematical issues using appropriate mathematical language; 8m43: ask questions to clarify and extend their knowledge of linear measurement, area, volume, capacity, and mass, using appropriate measurement vocabulary; 8m59: investigate and articulate geometric mathematical theories to solve problems and describe solutions.

Term One – Combined Grades 7 and 8 Sample Content and Reporting Targets

<table>
<tr><td rowspan="18">NUMBER SENSE and NUMERATION</td><th colspan="2">Multiples, Factors and Primes</th></tr>
<tr><td>7m11: generate multiples and factors of given numbers;
7m1: compare, order, and represent multiples, factors [and square roots].</td><td>8m13: represent composite numbers as products of prime numbers.</td></tr>
<tr><th colspan="2">Exponents and Square Roots</th></tr>
<tr><td colspan="2">7m1: compare, order, and represent [decimals, integers, multiples, factors and] square roots;
8m1: compare, order, and represent [fractions, decimals, integers, and] square roots.</td></tr>
<tr><td>7m 13: represent perfect squares and their square roots in a variety of ways.</td><td></td></tr>
<tr><td></td><td>8m10: represent whole numbers in expanded form using powers and scientific notation.</td></tr>
<tr><td colspan="2">7m4: understand and explain that exponents represent repeated multiplication;
7m15: understand that repeated multiplication can be represented as exponents;
8m17: express repeated multiplication as powers.</td></tr>
<tr><td>7m5: use estimation to justify or asses the reasonableness of calculations.</td><td>8m24: understand that the square roots of non-perfect squares are approximations;
8m25: estimate the square roots of whole numbers without a calculator.</td></tr>
<tr><td></td><td>8m26: find the approximate values of square roots of whole numbers using a calculator;
8m27: use trial and error to estimate the square root of a non-perfect square.</td></tr>
<tr><th colspan="2">Integers</th></tr>
<tr><td colspan="2">7m10: compare and order integers;
8m11: compare and order [fractions, decimals, and] integers;
7m1: compare, order, and represent [decimals,] integers, [multiples, factors and square roots];
8m1: compare, order, and represent [fractions, decimals], integers, [and square roots].</td></tr>
<tr><td>7m21: represent the addition and subtraction of integers using concrete materials, drawings, and symbols;
7m 22: add integers, with and without the use of manipulatives.</td><td>8m22: add and subtract integers, with and without the use of manipulatives.</td></tr>
<tr><td></td><td>8m21: discover the rules for the multiplication and division of integers through patterning;
8m5: demonstrate an understanding of the rules applied in the multiplication and division of integers;
8m23: multiply and divide integers;
8m5: demonstrate an understanding of the rules applied in the multiplication and division of integers.</td></tr>
<tr><td colspan="2">7m3: demonstrate an understanding of the order of operations with brackets;
8m4: understand and apply the order of operations with brackets for integers.</td></tr>
</table>

<table>
<tr><td rowspan="17">DATA MANAGEMENT and PROBABILITY</td><th colspan="2">Collecting, Organizing and Analyzing Data</th></tr>
<tr><td>7m82: recognize the different levels of data collection.</td><td>8m98: understand the relationship between a census and a sample;
8m97: collect primary data using both a whole population (census) and a sample of classmates, organize the data on tally charts and stem-and-leaf plots, and display the data on frequency tables.</td></tr>
<tr><td colspan="2">7m81: systematically collect, organize, and analyse data;
8m91: systematically collect, organize, and analyse primary data;
7m91: collect and organize data on tally charts and stem-and-leaf plots, and display data on frequency tables, using simple data collected by the students (primary data) and more complex data collected by someone else (secondary data).</td></tr>
<tr><td>7m89: demonstrate the pervasive use of data [and probability].</td><td></td></tr>
<tr><td>7m92: understand how tally charts and frequency tables can be used to record data.</td><td>8m105: discuss the quantitative information presented on tally charts, stem-and-leaf plots, frequency tables, and/or graphs.</td></tr>
<tr><td colspan="2">7m93: understand the difference between a spreadsheet and a database for recording and retrieving information;
7m94: search databases for information and interpret the numerical data;
8m101: search data bases for information and use the quantitative data to solve problems;
8m100: manipulate and present data using spreadsheets, and use the quantitative data to solve problems;
8m99: read a database or spreadsheet and identify its structure.</td></tr>
<tr><td colspan="2">7m83 and 8m92: use computer applications to examine and interpret data in a variety of ways;
7m105 and 8m115: explore with technology to find the best presentation of data;
7m85: construct graphic organizers using computer applications.</td></tr>
<tr><td colspan="2">7m95: understand that each measure of central tendency (mean, median, mode) gives different information about the data;
7m101: describe data using calculations of mean, median, and mode;
8m103: understand and apply the concept of best measure of central tendency.</td></tr>
<tr><td></td><td>8m109: determine the effect on a measure of central tendency of adding or removing a value.</td></tr>
<tr><td colspan="2">7m96: identify trends in graphs, using informal language to identify growth, clustering, and simple attributes;
8m93: interpret displays of data and present the information using mathematical terms;
8m102: know that a pattern on a graph may indicate a trend;
8m104: discuss trends in graphs to clarify understanding and draw conclusions about the data;
8m114: determine trends and patterns by making inferences from graphs.</td></tr>
<tr><td colspan="2">7m99: analyse bias in data-collection methods;
8m107: assess bias in data-collection methods.</td></tr>
<tr><td colspan="2">7m100: read and report information about data presented on bar graphs, pictographs, and circle graphs, and use the information to solve problems;
7m86: interpret displays of data and present the information using mathematical terms;
8m108: read and report information about data presented on line graphs, comparative bar graphs, pictographs, and circle graphs, and use the information to solve problems.</td></tr>
<tr><td>7m97: describe in their own words information presented on tally charts, stem-and-leaf plots, and frequency tables.</td><td>8m106: explain the choice of intervals used in constructing bar graphs or the choice of symbols in pictographs.</td></tr>
<tr><td>7m102: display data on bar graphs, pictographs, and circle graphs, with and without the help of technology;
7m98: use conventional symbols, titles, and labels when displaying data.</td><td>8m111: construct line graphs, comparative bar graphs, circle graphs, and histograms, with and without the help of technology, and use the information to solve problems;
8m110: understand the difference between a bar graph and histogram.</td></tr>
<tr><td colspan="2">7m104 and 8m113: evaluate arguments that are based on data analysis;
7m103 and 8m112: make inferences and convincing arguments that are based on data analysis.</td></tr>
</table>

	7m87: evaluate data and make conclusions from the analysis of data; 7m84: develop an appreciation for statistical methods as powerful means of decision making; 7m90: understand the impact that statistical methods have on decision making.	8m94: evaluate data and draw conclusions from the analysis of data.
PATTERNING and ALGEBRA	**Describing Patterns**	
	7m66 and 8m74: identify the relationships between whole numbers and variables; 7m73: interpret a variable as a symbol that may be replaced by a given set of numbers.	
	7m74: write statements to interpret simple formulas; 8m84: write statements to interpret simple equations.	
	7m67: identify, extend, create, and discuss patterns using whole numbers and variables; 7m69: apply and discuss patterning strategies in problem-solving situations; 7m70: describe patterns in a variety of sequences using the appropriate language and supporting materials; 7m71: extend a pattern, complete a table, and write words to explain the pattern.	8m75: identify, create and discuss patterns in algebraic terms; 8m81: find patterns and describe them using words and algebraic expressions; 8m79: describe and justify a rule in a pattern; 8m78: apply and defend patterning strategies in problem-solving situations.
	7m72: recognize patterns and use them to make predictions.	
		8m80: write an algebraic expression for the *n*th term of a numeric sequence; 8m82: use the concept of variable to write equations and algebraic expressions.
		8m85: present solutions to patterning problems and explain the thinking behind the solution process.

Term Two – Combined Grades 7 and 8 Sample Content and Reporting Targets

<table>
<tr><td rowspan="7">MEASUREMENT</td><th colspan="2">Measurement Concepts</th></tr>
<tr><td colspan="2">7m29 and 8m36: identify relationships between and among measurement concepts (linear, square, cubic, temporal, monetary).</td></tr>
<tr><td>7m32: create definitions of measurement concepts;
7m33: describe measurement concepts using appropriate measurement vocabulary.</td><td>8m40: explain the relationships between various units of measurement.</td></tr>
<tr><td colspan="2">7m35 and 8m42: make increasingly more informed and accurate measurement estimations based on an understanding of formulas and the results of investigations.</td></tr>
<tr><th>Area and Perimeter of 2-D Shapes</th><th>Circumference and Area of Circles</th></tr>
<tr><td>7m36: understand that that irregular two-dimensional shapes can be decomposed into simple two-dimensional shapes to find the area and perimeter;
7m30: solve problems related to the calculation and comparison of the perimeter and the area of irregular two-dimensional shapes;
7m37: estimate and calculate the perimeter and area of an irregular two-dimensional shape;
7m38: develop the formula for the finding the area of a trapezoid;
7m39: estimate and calculate the area of a trapezoid, using a formula;
7m40: draw a trapezoid given its perimeter and/or area;
7m41: develop the formulas for finding the area of a parallelogram and the area of a triangle.</td><td>8m44: measure the radius, diameter, and circumference of a circle using concrete materials;
8m45: recognize that there is a constant relationship between radius, diameter, and circumference of a circle, and approximate its value through investigation;
8m46: develop the formulas for finding the circumference and the formula for finding the area of a circle;
8m47: estimate and calculate the radius, diameter, circumference, and area of a circle, using a formula in a problem-solving context;
8m48: draw a circle given its area and/or circumference;
8m49: define radius, diameter, and circumference and explain the relationships between them;
8m69: construct a circle given its centre and radius, or centre and a point on the circle of three points on the circle;
8m37: solve problems related to the calculation of the radius, diameter, and circumference of a circle.</td></tr>
<tr><td rowspan="6">NUMBER SENSE and NUMERATION</td><th colspan="2">Fractions and Decimals</th></tr>
<tr><td>7m17: demonstrate an understanding of operations with fractions using manipulatives;
7m2: understand and explain operations with fractions using manipulatives.</td><td>8m15: demonstrate an understanding of operations with fractions.</td></tr>
<tr><td colspan="2">7m9: compare and order decimals;
8m11: compare and order fractions, decimals, [and integers].</td></tr>
<tr><td>7m18: add and subtract fractions with simple denominators using concrete materials, drawings and symbols;
7m19: relate repeated addition of fractions with simple denominators to the multiplication of a fraction by a whole number.</td><td>8m18: add, subtract, multiply, and divide simple fractions;
8m2: demonstrate a proficiency in operations with fractions.</td></tr>
<tr><td colspan="2">7m3: demonstrate an understanding of order of operations with brackets;
7m20: demonstrate an understanding of the order of operations with brackets and apply the order of operations in evaluating expressions that involve whole numbers and decimals;
8m3: understand and apply order of operations with brackets and exponents in evaluating expressions that involve fractions;
8m20: apply the order of operations (up to three operations) in evaluating expressions that involve fractions;
8m19: understand the order of operations with brackets and exponents and apply order of operations in evaluating expressions that involve fractions.</td></tr>
<tr><td colspan="2">7m27: solve problems that involve converting between fractions, decimals, and percents;
8m33: solve problems that involve converting between fractions, decimals, percents, unit rates, and ratios.</td></tr>
</table>

<table>
<tr><td></td><td colspan="2">7m26: solve problems involving fractions and decimals using the appropriate strategies and calculation methods;
8m8: solve and explain multi-step problems involving fractions, decimals, [integers, percents] and rational numbers.</td></tr>
<tr><td rowspan="2">GEOMETRY and SPATIAL SENSE</td><td>Congruent Figures</td><td>Angle Relationships</td></tr>
<tr><td>7m49: identify congruent and similar figures;
7m57: identify two-dimensional shapes that meet certain criteria;
7m58: explain why two shapes are congruent;
7m59: identify through investigation the conditions that make two shapes congruent;
7m60: create and solve problems involving the congruence of shapes.</td><td>8m63: identify the angle properties of intersecting parallel, and perpendicular lines by direct measurement: interior, corresponding, opposite, alternate, supplementary, and complementary;
8m64: explore the relationship to each other of the internal angles, in a triangle (they add up to 180°) using a variety of methods;
8m68: construct line segments and angles using a variety of methods;
8m71: describe the relationship between pairs of angles within parallel lines and transversals;
8m72: explain why the sum of the angles of a triangle is 180°;
8m57: identify and investigate the relationships of angles;
8m66: solve angle measurement problems involving properties of intersecting line segments, parallel lines, and transversals;
8m67: create and solve angle measurement problems for triangles;
8m58: construct and solve problems involving lines and angles.</td></tr>
<tr><td rowspan="10">PATTERNING and ALGEBRA</td><td colspan="2">Solving Equations</td></tr>
<tr><td colspan="2">7m76: evaluate simple algebraic expressions by substituting natural numbers for the variables;
8m76: evaluate algebraic expressions;
8m86: evaluate simple algebraic expressions, with up to three terms, by substituting fractions and decimals for the variables.</td></tr>
<tr><td>7m77: translate simple statements into algebraic expressions or equations.</td><td>8m87: translate complex statements into algebraic expressions or equations.</td></tr>
<tr><td colspan="2">7m68 and 8m77: identify, create, and solve simple algebraic equations.</td></tr>
<tr><td>7m78: solve equations of the form $ax = c$ and $ax + b = c$ by inspection and systematic trial, using whole numbers, with and without the use of a calculator;
7m79: solve problems giving rise to first-degree equations with one variable by inspection or by systematic trial.</td><td>8m88: solve and verify first-degree equations with one variable, using various techniques involving whole numbers and decimals.</td></tr>
<tr><td colspan="2">7m80: establish that a solution to an equation makes the equation true (limit to equations with one variable);
8m90: interpret the solution of a given equation as a specific number value that makes the equation true.</td></tr>
<tr><td></td><td>8m80: write an algebraic expression for the nth term of a numeric sequence;
8m89: create problems giving rise to first-degree equations with one variable and solve them by inspection or by systematic trial.</td></tr>
<tr><td></td><td>Inequalities</td></tr>
<tr><td></td><td>8m83: investigate inequalities and test whether they are true or false by substituting whole number values for the variables.</td></tr>
</table>

Term Three – Combined Grades 7 and 8 Sample Content and Reporting Targets

<table>
<tr><td rowspan="5">NUMBER SENSE and NUMERATION</td><th colspan="2">Ratio, Rate and Percent</th></tr>
<tr><td colspan="2">7m27: solve problems that involve converting between fractions, decimals, and percents;
8m33: solve problems that involve converting between fractions, decimals, percents, unit rates, and ratios.</td></tr>
<tr><td colspan="2">7m6: solve and explain multi step problems involving simple fractions, decimals, and percents;
8m8: solve and explain multi-step problems involving fractions, decimals, integers, percents, and rational numbers;
8m34: apply percents in solving problems involving discounts, sales tax, commission, and simple interest.</td></tr>
<tr><td></td><td>8m29: demonstrate an understanding of and apply unit rates in problem-solving situations.</td></tr>
<tr><td></td><td>8m12: mentally divide numbers by 0.1, 0.01, and 0.001.</td></tr>
<tr><td rowspan="7">GEOMETRY and SPATIAL SENSE</td><th colspan="2">Two- and Three-Dimensional Geometry</th></tr>
<tr><td colspan="2">7m47 and 8m55: identify, describe, compare, and classify geometric figures.</td></tr>
<tr><td colspan="2">7m53: recognize the front, side, and back views of three-dimensional figures;
8m61: recognize three-dimensional figures from their top, side, and front views.</td></tr>
<tr><td colspan="2">7m54: sketch front, top, and side views of three-dimensional figures with or without the use of a computer application;
7m55: sketch three-dimensional objects from models and drawings;
7m56: build three-dimensional figures and objects from nets;
8m62: sketch and build representations of three-dimensional figures from front, top, and side views.</td></tr>
<tr><td colspan="2">7m48: identify, draw, and construct three-dimensional geometric figures from nets;
8m56: identify, draw, and represent three-dimensional geometric figures.</td></tr>
<tr><th>Transformations</th><th>Pythagorean Relationship</th></tr>
<tr><td>7m50: explore transformations of geometric shapes;
7m51: understand, apply, and analyse key concepts in transformational geometry using concrete materials and drawings;
7m61: recognize the image of a two-dimensional shape under a translation, a reflection, and a rotation in a variety of contexts;
7m62: create and analyse designs that include translated, rotated, and reflected two-dimensional images using concrete materials and drawings, and computer applications;
7m63: identify whether a figure will tile a plane;
7m64: construct and analyse tiling patterns with congruent tiles;
7m65: describe designs in terms of images that are congruent, translated, rotated, and reflected.</td><td>8m65: investigate the Pythagorean relationship using area models and diagrams;
8m70: apply the Pythagorean relationship to numerical problems involving area and right triangles;
8m73: explain the Pythagorean relationship.</td></tr>
</table>

<table>
<tr><td rowspan="4">MEASUREMENT</td><th colspan="2">Surface Area</th></tr>
<tr><td>7m42: develop the formula for finding the surface area of a rectangular prism using nets.</td><td>8m50: develop the formula for finding the surface area of a triangular prism using nets.</td></tr>
<tr><th>Volume of Rectangular Prism</th><th>Volume of Triangular Prism</th></tr>
<tr><td>7m43: develop the formula for finding the volume of a rectangular prism (area of base × height) using concrete materials;
7m44: understand and articulate the relationship between the dimensions and the volume of a rectangular prism;
7m45: calculate the surface area and the volume of a rectangular prism in a problem-solving context;
7m46: sketch a rectangular prism given its volume;
7m31: apply volume formulas to problem-solving situations involving rectangular prisms.</td><td>8m51: develop the formula for finding the volume of a triangular prism (area of base × height);
8m52: understand and articulate the relationship between the dimensions and the volume of a triangular prism;
8m53: calculate the surface area and the volume of a triangular prism, using a formula in a problem-solving context;
8m54: sketch a triangular prism given its volume;
8m38: apply volume and area formulas to problem-solving situations involving triangular prisms.</td></tr>
<tr><td rowspan="7">DATA MANAGEMENT and PROBABILITY</td><th colspan="2">Probability</th></tr>
<tr><td colspan="2">7m109: apply knowledge of probability in sports and games of chance;
7m88: use and apply a knowledge of probability;
8m122: apply a knowledge of probability in sports and games, weather predictions, and political polling.</td></tr>
<tr><td colspan="2">7m107 and 8m118: list the possible outcomes of simple experiments by using tree diagrams, modelling, and lists.</td></tr>
<tr><td colspan="2">7m106: develop intuitive concepts of probability and understand how probability can relate to sports and games of chance;
8m95: identify probability situations and apply a knowledge of probability;
8m116: use probability to describe everyday events.</td></tr>
<tr><td colspan="2">7m108 and 8m119: identify the favourable outcomes among the total number of possible outcomes and state the associated probability.</td></tr>
<tr><td></td><td>8m117: identify 0 to 1 as a range from “never happens” (impossibility) to “always happens” (certainty) when investigating probability.</td></tr>
<tr><td></td><td>8m120: use definitions of probability to calculate complex probabilities from tree diagrams and lists;
8m121: compare predicted and experimental results;
8m96: appreciate the power of using a probability model by comparing experimental results with theoretical results.</td></tr>
</table>

Lesson Outline: Days 1 - 6

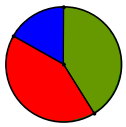
Combined Grades 7 and 8 class

BIG PICTURE

Students will:
- construct 3-D figures using linking cubes;
- develop the formulas for finding the surface area of a rectangular prism (Grade 7) and a triangular prism (Grade 8);
- investigate the relationship between the dimensions and the volume of a rectangular prism (Grade 7) and a triangular prism (Grade 8);
- develop the formula for finding the volume of a prism (both grades), a rectangular prism (Grade 7) and a triangular prism (Grade 8) using concrete materials;
- apply surface area and volume formulas to problem-solving situations involving rectangular prisms (Grade 7) and triangular prisms (Grade 8).

Day	Lesson Title	Description	Expectations
1	Surface Area of Prisms	• Draw the 3-D view of a cube and its net. • Develop the formula for finding the surface area of a cube. • Develop the formula for finding the surface area of a rectangular prism (Grade 7) or triangular prism (Grade 8).	7m42 8m36 CGE 2a, 3c
2	Surface Area of Rectangular Prisms (Grade 7) and Triangular Prisms (Grade 8)	• Develop and apply the formula for finding the surface area of rectangular prisms (Grade 7) and triangular prisms (Grade 8).	7m42, 7m74 8m38, 8m50 CGE 2a, 4b, 5a
3	Volume of a Prism	• Use linking cubes to build models of prisms. • Determine the volume of a prism by counting the number of cubes in its structure. • Develop and apply the formula for volume of a prism.	7m43 8m51, 8m52 CGE 3c, 4c, 5a
4	Volume of Rectangular Prisms (Grade 7) and Triangular Prisms (Grade 8)	• Investigate, develop, and apply the formula for volume of a rectangular prism (Grade 7) and triangular prism (Grade 8).	7m43, 7m44, 7m46 8m52, 8m54 CGE 3c, 4b
5	Performance Task	• Investigate the relationship between surface area and volume of rectangular prisms (Grade 7) or triangular prisms (Grade 8).	7m44, 7m45, 7m46, 7m55, 7m31 8m38, 8m52, 8m53, 8m54, 8m70 CGE 2c, 3c, 4f
6	Skills Test	• Grade 7: Write a test that assesses students' understanding of surface area and volume of rectangular prisms. • Grade 8: Write a test that assesses students' understanding of surface area and volume of triangular prisms.	7m42, 7m43, 7m44, 7m45, 7m46, 7m31 8m36, 8m38, 8m50, 8m51, 8m52, 8m53, 8m54

Day 1: Developing the Formula for Surface Area of a Cube
Grade 7: Surface Area of a Rectangular Prism
Grade 8: Surface Area of a Triangular Prism



Description
- Draw the 3-D view of a cube and its net.
- Develop the formula for finding the surface area of a cube.
- Develop the formula for finding the surface area of a rectangular (Grade 7) or triangular (Grade 8) prism.

Materials
- isometric dot paper (BLM 14.1)
- orthographic dot paper (BLM 14.2)
- BLM 1.1

Assessment Opportunities

Minds On ...

Whole Class → Guided Exploration

Display a collection of prisms.
Students discuss with a partner what they think "surface area" means and describe how it is different from area.
As a class, discuss and record various definitions. Discuss when it would be useful to determine the surface area of an object.

To further develop the concept of surface area, show the net of a cube using a cut-apart cubic box or a net drawn on chart paper.
Students identify and explain the connection between the area of the net and the surface area of the cube.

Give students a few minutes to individually determine a method for finding the surface area of a cube, then facilitate sharing of various methods. Discuss similarities between the methods highlighted.

Individual → Investigation

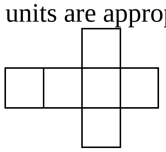
Pose the questions:
What is the area of each face, if the width, length, and height are 10 cm?
How could you calculate the total surface area of the cube?

Encourage multiple approaches for finding total surface area (including area of sides + top + base).

Curriculum Expectations/Observation/Rubric: Observe students and assess their inquiry and learning skills using BLM 1.1 Assessment Tool.
Note: The same processes can be observed over the next five classes allowing the teacher to focus on different groups of students each day.

Whole Class → Sharing

Discuss various solutions presented by students. As a class, identify which units are appropriate.



Students represent the relationship in a variety of ways (words, variables, and numbers).
All forms are equally acceptable.

Area of one face, $A = l \times w$
Total Surface Area = $A \times 6$

> Surface area is a new concept for students in Grade 7. Students in Grade 8 will benefit from a review of the concept.
>
> Use scissors or a knife to cut the edges of a large box so that it can be flattened into a net.
>
> For the whole class discussion, include boxes other than cubes – rectangular prisms, triangular-based prisms such as chocolate bar boxes, octagonal cleaning cloth boxes, cylindrical salt or oatmeal boxes, etc.
>
> Add vocabulary to the Word Wall: *orthographic, isometric, net, justify*

Action!

Pairs (Students of the same grade) → Visual Activity

Students in Grade 7 use rectangular prisms while students in Grade 8 use triangular prisms from the class collection. They measure the sides, use dot paper to draw a net, and calculate the surface area of the box. Encourage students to use a variety of different prisms, nets and methods.

Consolidate Debrief

Whole Class → Reflection

Students share their explanations of how they found the surface areas then write their methods in their journals.

Concept Practice

Home Activity or Further Classroom Consolidation

Write a description that you could use to find the surface area of any rectangular prism (Grade 7) or triangular prism (Grade 8).

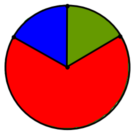
In your math journal explain your approach to finding the surface area.

1.1: Assessment Tool – Combined Grades 7 and 8 – Term 3: Measurement

Mathematical Process (Category)	Criteria	Below Level 1	Level 1	Level 2	Level 3	Level 4
Reasoning and Proving (Problem Solving)	Evidence of self-monitoring (makes own revisions)	- no evidence	- limited evidence	- some evidence	- evidence	- evidence of attending to subtleties
Communicating (Communication)	Clarity (explanations and presentations)	- unclearly	- with limited clarity	- with some clarity	- clearly	- precisely
	Use of conventions (accurately, effectively, fluently)	- demonstrates an undeveloped use of conventions	- demonstrates minimal skill in the use of conventions	- demonstrates moderate skill in the use of conventions	- demonstrates considerable skill in the use of conventions	- demonstrates a high degree of skill in the use of conventions

Learning Skills	Needs Improvement	Satisfactory	Good	Excellent
Independent Word				
• follows routines and instructions without supervision				
• persists with tasks				
Initiative				
• responds to challenges				
• demonstrates positive attitude towards learning				
• develops original ideas and innovative procedures				
• seeks assistance when necessary				
Use of Information				
• organizes information logically and creatively and manages it effectively				
• asks questions to clarify meaning and ensure understanding				

Day 2: Grade 7: Develop and Apply the Formula for Surface Area of a Rectangular Prism
Grade 8: Develop and Apply the Formula for Surface Area of a Triangular Prism



Description

- Develop and apply the formula for finding the surface area of rectangular prisms (Grade 7) or triangular prisms (Grade 8).

Materials

- isometric dot paper (BLM 14.1)
- orthographic dot paper (BLM 14.2)

Assessment Opportunities

Minds On ...

Whole Class → Guided Exploration & Student Presentation

Count the sides of a rectangular prism (Grade 7) and a triangular prism (Grade 8). Open and flatten the prisms into their nets. Re-count the rectangles (triangles) that form the net. Re-fold the prisms and discuss that the surface area is calculated by adding the areas of all the parts of the net, i.e., all the faces of the prism.

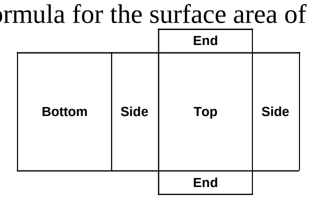
Students read their journal entry and explain their approach to calculating surface area of a prism.

Encourage students to use descriptive formulas until they are ready for symbolic forms.

Action!

Small Groups (Grade 7) → Conferencing

Students use their solutions for calculating surface area to develop an algebraic formula for the surface area of a rectangular prism.



Surface Area =
top + bottom + 2 sides + 2 ends
(descriptive formula)

Some students may need to scaffold their solutions, e.g., SA of top and bottom = 2 (b x h); SA of two ends = 2 (b x h); and SA of two sides = 2 (b x h)

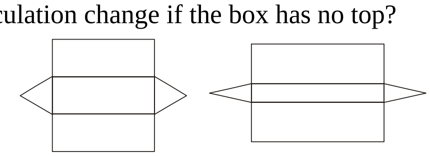
Total SA of rectangular prism
= __ + __ + __
= ____ units^2

Small Groups (Grade 8) → Conferencing

Students use their solutions for calculating surface area to develop an algebraic formula for the surface area of a triangular prism. Include triangular prisms where the triangles are not equilateral.

Discuss: How does the surface area calculation change if the box has no top?

How can congruent faces simplify the calculation of surface area?



Surface Area (SA) = 2 ends + 3 rectangles
OR
SA = 2 ends + 2 large rectangles + 3rd rectangle
(descriptive formula)

Curriculum Expectations/Exhibition/Rubric: Observe students and assess their inquiry skills and learning skills using BLM 1.1 Assessment Tool.
Note: This is the second of 5 days where the same processes can be observed.

Consolidate Debrief

Whole Class → Reflection

Students present their formulas. To assist students in Grade 7 as they move towards symbolic representation, discuss how the various representations convey the same information or result in the same answer. Highlight advantages to symbolic representation.

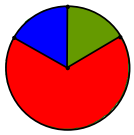
Students should use appropriate mathematical notation when presenting their solutions.

Home Activity or Further Classroom Consolidation

Concept Practice

Complete textbook questions for additional practice. (The teacher inserts textbook references.)

Day 3: Grades 7 and 8: Developing a Formula for Volume of a Prism



Description

- Use linking cubes to build models of prisms.
- Determine the volume of a prism by counting the number of cubes in its structure.
- Develop and apply the formula for volume of a prism.

Materials

- linking cubes
- BLM 3.1
- isometric dot paper (BLM 14.1)

Assessment Opportunities

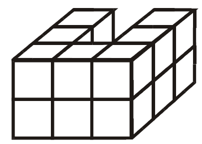
Minds On ...

Whole Class → Guided

Show a cube and ask: If the length of one side is 1 unit:

- What is the surface area of one face? (1 unit^2)
- What is the volume? (1 unit^3)

Using a "building tower" constructed from linking cubes, lead students through a discussion based on the model:



- Why is this a prism?
- What is the surface area of the base?
- What is the height of the building?

Count the cubes to determine the volume of the building.

Invite students to ask clarifying questions about the investigation to develop a formula for volume of a prism (BLM 3.1).

A prism has at least one pair of congruent, parallel faces.

Students gain better insight into the development of volume concepts and formulas by actually constructing the objects and counting the cubic units.

Action!

Pairs (Students in the same grade) → Investigation

Students create several more irregular prisms of various sizes using BLM 3.1 Building Towers. Students display their findings in the table. After investigating the problem with several samples, they generalize the formula for the volume of a prism:

Volume = Area of the Base x Height

Students test their formula for accuracy by constructing two other towers to test the formula.

Curriculum Expectations/Observation/Checklist: Observe students and assess their inquiry skills and learning skills using BLM 1.1 Assessment Tool.

Note: This is the third of 5 days where the same processes can be observed in small groups.

Provide some support for students who are having difficulty generalizing the formula.

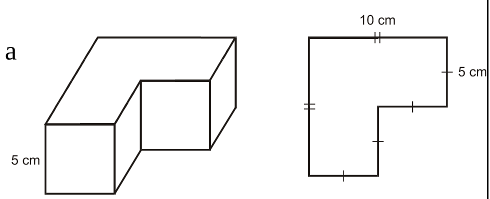
Consolidate Debrief

Whole Class → Student Presentation

As students present their findings, summarize the results of the investigation on a class chart.

Orally complete a few examples calculating volume of prisms given a diagram:

Reinforce the concept of cubic units.



Concept Practice
Application
Concept Practice
Exploration
Reflection
Skill Drill

Home Activity or Further Classroom Consolidation

A prism has a volume of 24 cm^3.

Draw prisms with this volume. How many possible prisms are there with a volume of 24 cm^3?

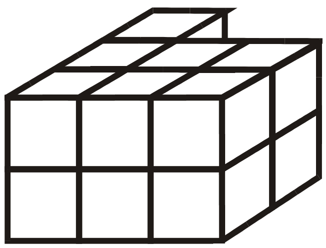
If students use decimal and fractional measures, an infinite number of prisms are possible.

3.1: Building Towers

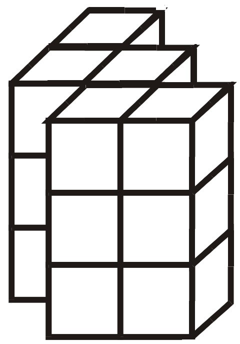
Name:
Date:

Each tower pictured here is a prism. Build each prism and determine the volume of each building by counting cubes.

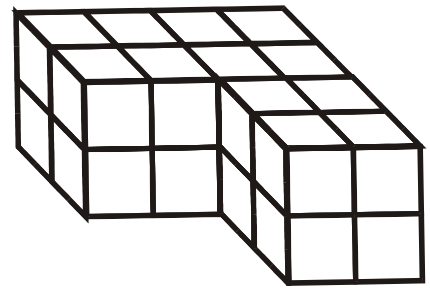
Tower A



Tower B



Tower C



Complete the table of measures for each Tower:

Tower	**Area of Base**	**Height of Tower**	**Volume (by counting cubes)**
A			
B			
C			

What relationship do you notice between volume, area of the base and height?

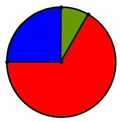
State a formula that might be true for calculating Volume of a prism when you know the Area of the Base and the Height of the prism.

Test your formula for accuracy by building two other prism towers and determining the volume. Sketch your towers and show calculations on this table.

Tower	**Area of Base**	**Height**	**Volume (by counting cubes)**	**Volume (using your formula)**
D				
E				

Is your formula accurate?

Day 4: Grade 7: Develop and Apply the Formula for Volume of a Rectangular Prism
Grade 8: Develop and Apply the Formula for Volume of a Triangular Prism



Description

- Develop, investigate, and apply the formula for volume of a rectangular prism (Grade 7) and triangular prism (Grade 8).

Materials

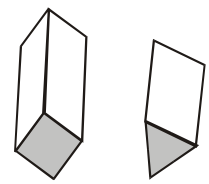
- models of triangular and rectangular prisms

Assessment Opportunities

Minds On ...

Whole Class → Sharing

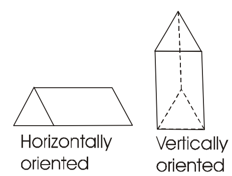
Have students share their diagrams and solutions for prisms with a volume of 24 cm^3.



Whole Class → Discussion

Using concrete samples of a rectangular prism and a triangular prism, ask students:

1. What can be altered in the volume of a prism formula to make the formula specific for a rectangular prism (Grade 7) or a triangular prism (Grade 8)
2. Will the volume be the same or different when the prisms are oriented vertically or horizontally?
3. What do we need to think about when applying the volume formula to a triangular prism?
4. What do we mean by "dimensions of a prism?"



For any prism:
V = Area of Base × height
For rectangular prisms:
$V = l \times w \times h$
For triangular prisms:
$V = ½bh \times H$

Action!

Pairs → Investigation

Students use a rectangular prism (Grade 7) or triangular prism (Grade 8) to develop a formula specific to their prism (based on the general formula of Volume = Area of the Base × Height.) They investigate how to use their specific formulas to calculate volumes of horizontally and vertically oriented prisms, including several examples and show their calculations to justify their conclusions.

Curriculum Expectations/Exhibition/Rubric: Observe students and assess their inquiry and learning skills using BLM 1.1 Assessment Tool.

When calculating volume of a rectangular prism, any of its faces can be thought of as the 'base.' However, when calculating the volume of a triangular prism, its 'base' is one of the triangles, not one of the rectangles.

Consolidate Debrief

One Grade at a Time (Grade 7 followed by Grade 8) → Reflection

Grade 7: Students share their investigation and justify their explanations using diagrams and calculations. They write their formula in their journals, then begin the question stated in the Home Activity.

Grade 8: Students share their investigation findings. Focus discussion on the need to identify the triangular face as the 'base' when using the formula V = Area of Base × Height for a triangular prism. Connect this discussion to the idea of stacking triangles either vertically or horizontally to generate the triangular prism. Discuss the need for *h* and *H* in the formula for volume, the fact *h* is perpendicular to *b*, and the fact that *H* is perpendicular to the triangular base. Discuss each of these ideas in relationship to rectangular prisms.

Some students may need to physically or mentally orient triangular prisms vertically in order to find the volume.

Exploration
Concept Practice

Home Activity or Further Classroom Consolidation

Grade 7: How many rectangular prisms with whole number dimensions can you sketch with volume: a) 27 cm^3? b) 48 cm^3?

Why are there many more solutions for 48 cm^3? Choose a volume for a rectangular prism that can be generated by several different sets of measurements.

Grade 8: Sketch a triangular prism whose whole number dimensions will produce a volume that is:

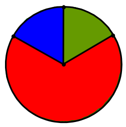
a) an even number b) an odd number c) a decimal value.

Explain your thinking in each case.

Complete the practice questions assigned. (The teacher inserts textbook references.)

Link measurement to the use of factors and multiples: The more factors a number has, the greater the number of rectangular prisms will have that volume.

Day 5: Grade 7: Investigating Surface Area and Volume of Rectangular Prisms
Grade 8: Investigating Surface Area and Volume of Triangular Prisms



Description

- Investigate the relationship between surface area and volume of rectangular prisms (Grade 7) or triangular prisms (Grade 8).

Materials

- BLMs 5.1, 5.2
- interlocking cubes
- calculators

Assessment Opportunities

Minds On ...

Whole Class → Sharing

Students share their findings about their home investigations based on the volume of rectangular and triangular prisms.

Whole Class → Discussion

Pose this question and have students orally make predictions:
If two rectangular prisms (Grade 7) or two triangular prisms (Grade 8) have the same volume, do they have the same surface area?

Grade 7:
Students would benefit from having interlocking cubes to help them visualize the various shapes and sizes of boxes.

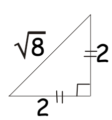
Action!

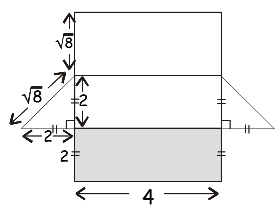
Pairs (same grade students) → Investigation

Introduce the problem using BLM 5.1 for Grade 7 investigation and BLM 5.2 for Grade 8 investigation.

Students investigate:

a) For prisms with the same volume, is the surface area also the same? *(No)*
b) What shape of rectangular/triangular prism has the largest surface area for a given volume?





Grade 8: Some students will require a review of the Pythagorean relationship in order to use the hypotenuse length to calculate the dimensions of the rectangles.

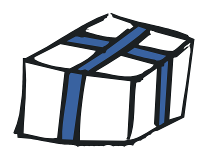
Curriculum Expectations/Exhibition/Rubric: Observe students and assess their inquiry skills and learning skills using a rubric adapted from BLM 1.1 Assessment Tool.

Individual → Written Report

Students individually prepare a written report of their findings.

Grade 8:
For students who benefit from visual and kinesthetic learning: prepare cut-out nets of the triangular prisms so they can better visualize the surface area and the different elongated shapes that are created.

As an extension, students in Grade 7 could also investigate the length of ribbon needed to tie around boxes with the same volume.



Consolidate Debrief

Whole class → Student Presentations

Students present their findings. Although the measurements will differ, the conclusions should be the same for both grades. (This debriefing may be conducted when their assessments are returned.)

Apply the mathematics learned in today's activity to answer this question:

- Why would a Husky dog curl up in the winter to protect himself from the cold winds when he is sleeping outdoors?

  (If the dog remains "long & skinny" he has greater surface area exposed to the cold. If he curls up, he has less surface area exposed to the cold, and thus he will lose much less body heat. Although his volume stays the same, his surface area decreases as he becomes more "cube-ish" or spherical.)

Solution:
The more elongated the prism, the greater the surface area. The closer the prism becomes to being cube-shaped or spherical, the less surface area it has.

Concept Practice

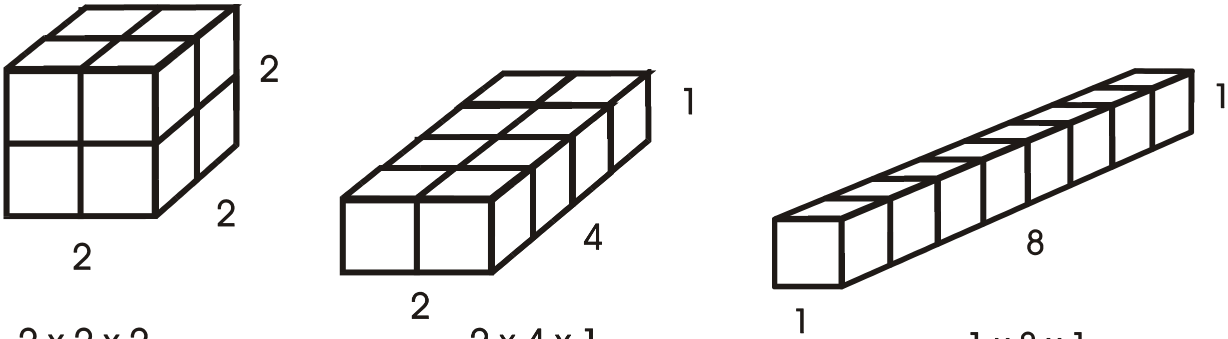
Home Activity or Further Classroom Consolidation

Complete the assigned surface area and volume problems in preparation for a skills test. (The teacher inserts textbook references.)

Use a marking scheme to assess the skills test.

5.1 Wrapping Gifts (Grade 7)

Three different rectangular prism-shaped boxes each have a volume of 8 cubic units.
Does each box require the same amount of paper to wrap? Let's investigate!



2 x 2 x 2   2 x 4 x 1   1 x 8 x 1

1. a) Verify that each rectangular prism illustrated above has a volume of 8 cubic units.
   b) Draw the net for each rectangular prism box.
   c) Determine the amount of paper required by calculating the surface area. (Ignore the overlapping pieces of paper you would need.)
   d) Describe your findings.

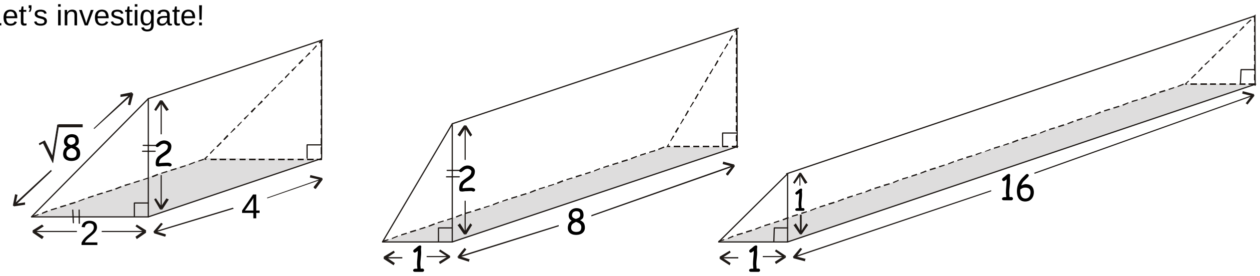
2. a) How many different rectangular prism boxes can be designed to have a volume of 24 cubic units?
   b) Draw several of the boxes, labelling the dimensions.
   c) How much paper is required to wrap each box?
   d) Describe your findings.

3. Investigate wrapping rectangular prism boxes with a volume of 36 cubic units.
   Determine the dimensions of the rectangular prism with the greatest surface area.

4. Write a report of your findings. Include the following information, justifying your statements.
   - Describe how surface area and volume are related, when the volume remains the same.
   - What shape of rectangular prism box uses the most paper?
   - What shape of rectangular prism box uses the least paper?

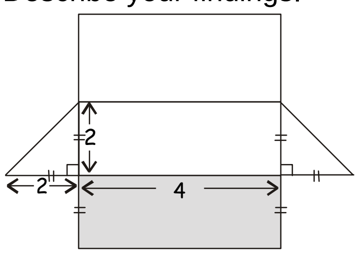
5.2 Wrapping Gifts (Grade 8)

Three different triangular prism gift boxes each have a volume of 8 cubic units.
Does each box require the same amount of paper to wrap?
Let's investigate!



2 x 2 x 4　　1 x 2 x 8　　1 x 1 x 16

1. a) Verify that each triangular prism illustrated above has a volume of 8 cubic units.
   b) Draw the net for each triangular prism box.
   c) Determine the amount of paper required by calculating the surface area. (Ignore the overlapping pieces of paper you would need.) The first one has been started for you.
   d) Describe your findings.



2. a) How many different triangular prism boxes can be designed to have a volume of 24 cubic units?
   b) Draw several of the boxes, labelling the dimensions.
   c) How much paper is required to wrap each box?
   d) Describe your findings.

3. Investigate wrapping triangular prism boxes with a volume of 36 cubic units. Determine the dimensions of the triangular prism with the greatest surface area.

4. Write a report of your findings. Include the following information, justifying your statements.
   - Describe how surface area and volume are related, when the volume remains the same.
   - What shape of triangular prism box uses the most paper?
   - What shape of triangular prism box uses the least paper?

Lesson Outline: Days 12 - 16 Grade 7

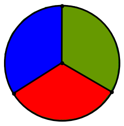
Grade 7 Lessons to Parallel Days 12 - 16 in TIPS Grade 8 for teachers with combined Grades 7 and 8

BIG PICTURE

Students will:

- explore transformations on a geoboard;
- investigate transformations using *The Geometer's Sketchpad 4*®;
- apply various types of transformation to investigate tessellations;
- explore the creation of Pentominoes and solve a problem involving transformations of Pentominoes;
- test a conjecture as to whether or not all triangles will tessellate.

Day	Lesson Title	Description	Expectations
12	Geoboards and Transformations	• Use various transformations to "move" a triangle from one position and orientation to another on a geoboard.	7m50, 7m51, 7m61, 7m62 CGE 5a, 4b
13	Investigating Transformations using *The Geometer's Sketchpad 4*®	• Use *The Geometer's Sketchpad 4*® to investigate transformations	7m50, 7m51 CGE5d
14	Applying Transformation: Tessellations	• Perform tessellations based on transformations: A: using *The Geometer's Sketchpad 4*®; OR B: using dot paper and coloured pencils.	7m50, 7m51, 7m62 CGE 2c, 2d
15	Pentominoes Puzzle	• Apply knowledge of transformations to solve a congruency problem based on Pentominoes.	7m49, 7m51, 7m52, 7m60, 7m61, 7m65 CGE 5e, 5a
16	Will It Tessellate?	• Apply knowledge of transformations to discover whether all types of triangles will tessellate.	7m52, 7m62, 7m63 CGE 3c



Description

- Use various transformations to "move" a shape from one position and orientation to another on a geoboard.

Materials
- geoboards
- geoboard dot paper (BLM 12.1)
- BLM 12.2

Assessment Opportunities

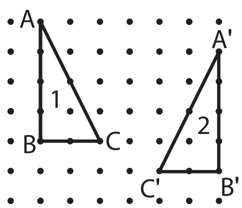
Minds On ...

Whole Class → Connecting to previous lesson and orientating students to an activity

Place two congruent triangles (Δ1 and Δ 2) on an overhead geoboard. Explain that students are to move Δ1 onto Δ2 several times using different combinations and/or sequences of transformations each time.

Shapes other then triangles could be used.

Review with the students in Grade 7 the precision needed for descriptions of transformations. For example,



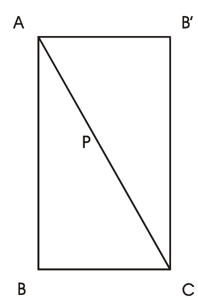
Flip Δ1 horizontally, then translate it 6 units right and 1 unit down.
OR
Translate Δ1 down 1 unit and right 6 units, then reflect it in side A'B'.

If a shape is rotated, the description should identify the centre of rotation.

If a shape is reflected, the description should identify the line of reflection.

Demonstrate that different types of transformations can result in the same image.

The diagram illustrates ΔABC being rotated 180° about point P to ΔCB'A, or ΔABC being reflected in AB, then reflected in the image of BC, then translated rightward and upward to ΔCB'A



Action!

Pairs → Exploration

Students use their geoboards to transform Δ1onto Δ 2 using translations, reflections, and rotations. Students record all transformations on BLM 12.2.

Curriculum Expectations/Question and Answer/Mental Note: Circulate among the groups, observing students' strategies for transforming Δ1 onto Δ2.

Use probing questions to prompt students thinking:
- What different types of transformations are there?
- Which combinations have you tried?

Challenge some students to perform the transformation in a specific number of moves.

Consolidate Debrief

Pairs → Making Connections and Summarizing

Different pairs of students check each other's descriptions by following the description and seeing whether the intended image results.

Students record more than one way to describe at least two of the examples on BLM 12.2.

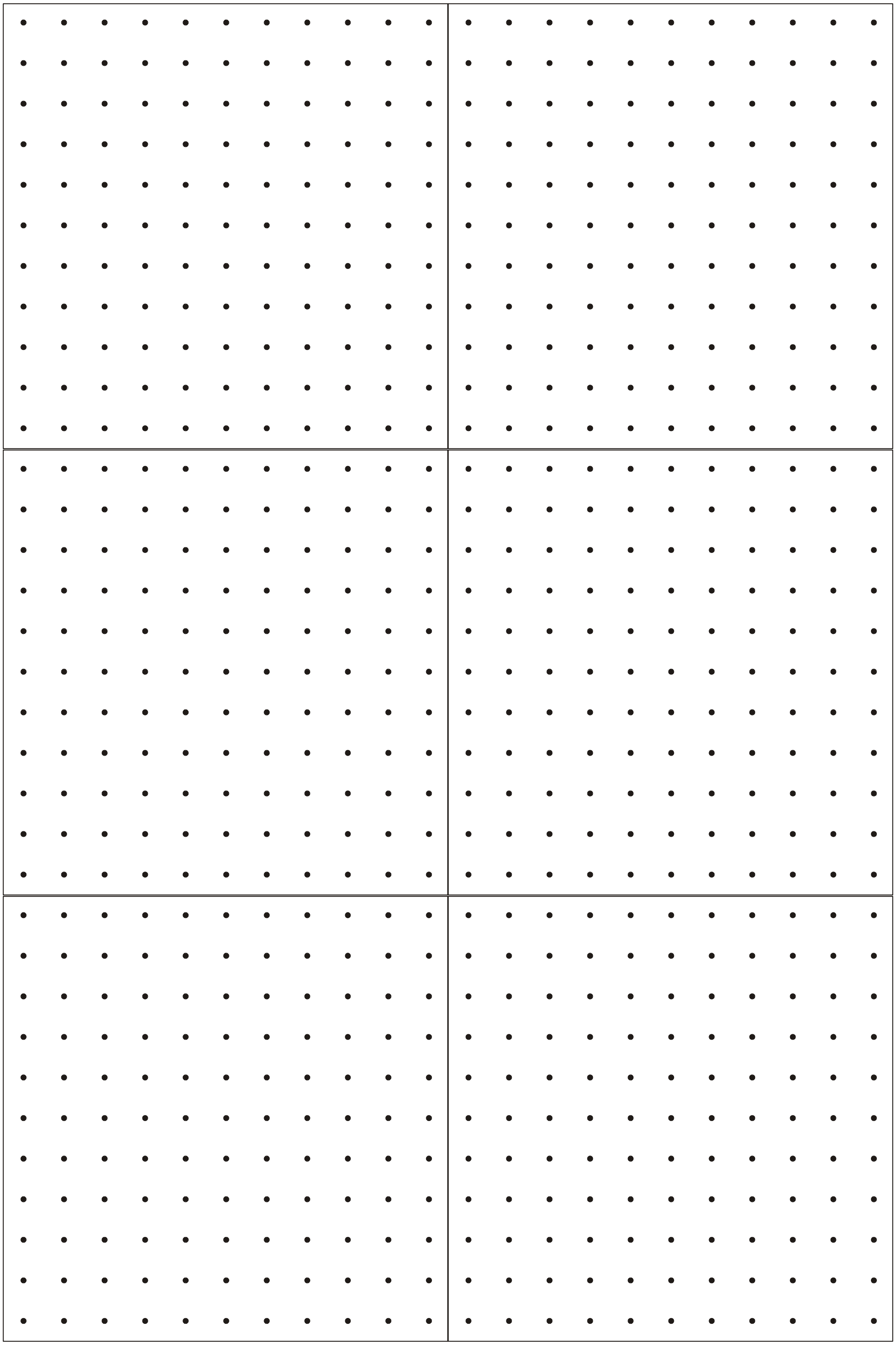
Application
Concept Practice
Reflection

Home Activity or Further Classroom Consolidation

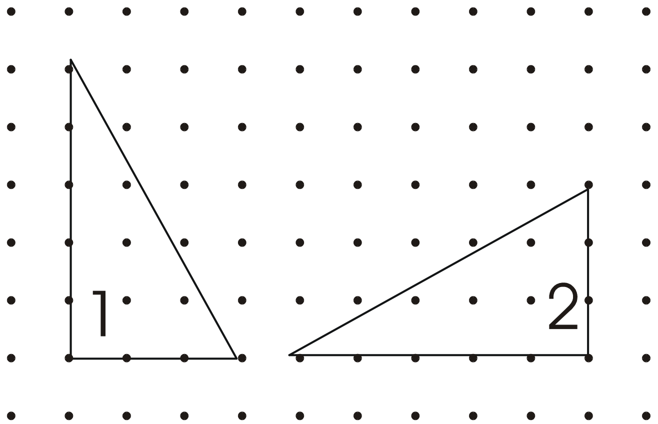
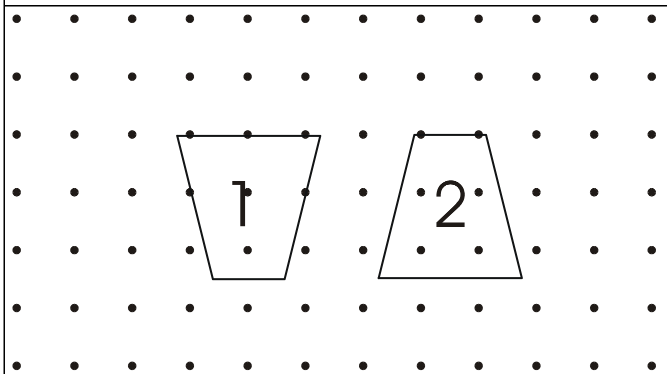
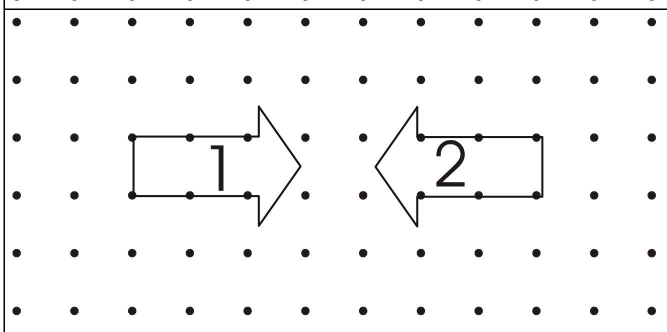
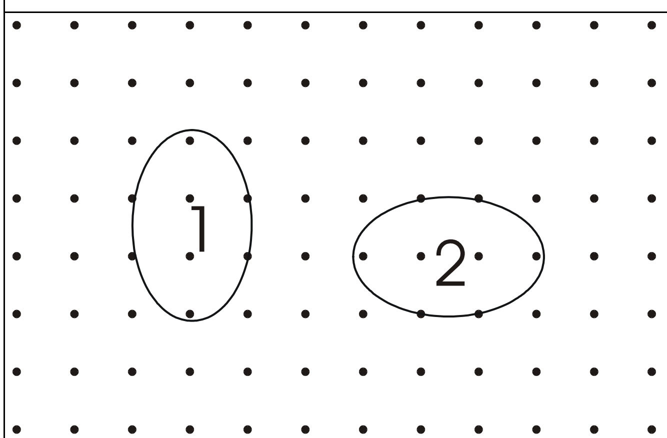
Create your own transformation challenge.

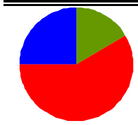
Complete your transformations in several ways. Trade challenges.

12.1 Geoboard Dot Paper



12.2 Transformations Recording Chart

Transformation diagram: Sketch each step in the transformation from figure 1 to figure 2.	**Description of Transformations:** Describe in words the transformation used for each sketch.
	
	
	
	



Description

- Use *The Geometer's Sketchpad 4®* to investigate transformations.

Materials

- The Geometer's Sketchpad 4®
- BLMs 13.1, 13.2

Assessment Opportunities

Minds on …

Pairs → Teacher Guided

Mixed-ability pairs work together at a computer with *The Geometer's Sketchpad 4®* (*GSP 4*) to construct a regular pentagon using BLM 13.1. Pairs share understanding.

In *GSP 4* there is a tool to construct a pentagon. Show the class this tool only after they understand the construction of a regular pentagon based on its geometric properties.

Action!

Pairs → Guided Exploration

Mixed-ability pairs use *The Geometer's Sketchpad 4®* and follow instructions on BLM 13.2 to work with various types of transformations.

Learning Skills (Co-operation with others)/Question & Answer/Checklist: Observe students as they work through the task.

Students who finish early can continue on with the Explore More section of BLM 13.2.

Consolidate Debrief

Whole Class → Demonstrating Understanding

Discuss the questions on BLM 13.2.

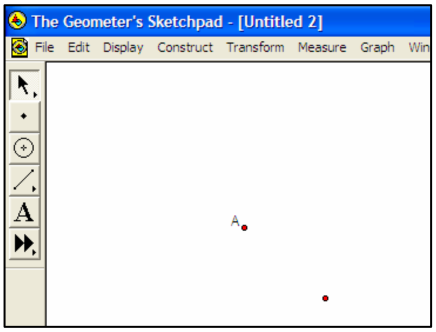
Application
Concept Practice

Home Activity or Further Classroom Consolidation

Create and describe several different designs based on transformations of a single shape.

Name your design to suggest the type of transformation(s) used in developing the design, e.g., Tilted Tiles, Spun Petals.

13.1 Constructing a Regular Pentagon

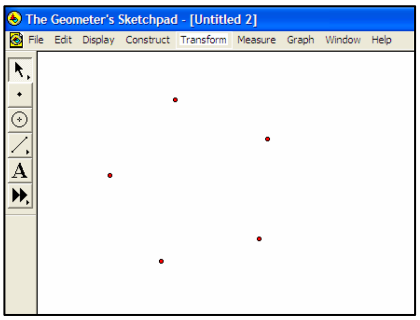


Using the Point Tool, construct 2 points on the screen as shown.

Select the Arrow Tool, click on any white space to deselect everything.

Click on point A; from the Transform Menu, select Mark Center.

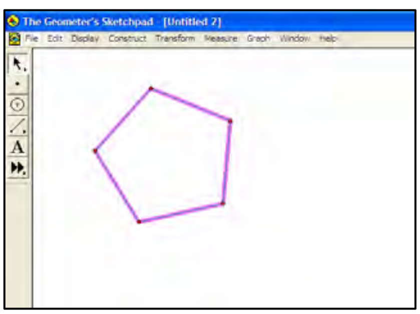
Click on the other point.



In Transform Menu, select Rotate; type 72 into the degree box, click on Rotate.

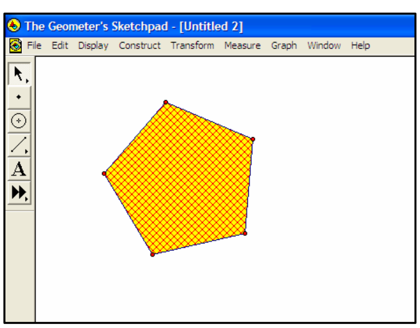
Select Rotate again; in the Rotate Box, click on Rotate, repeat 2 more times.

Click on point A; in the Display Menu, select Hide Point.



Select each of the 5 points in order clockwise.

In the Construct Menu, select Segments.



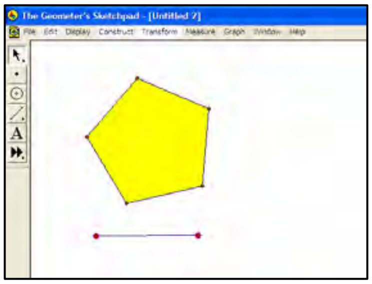
Deselect the segments and select each of the 5 points.

In the Construct Menu, select Interior.

Save your sketch.

13.2 Transformations in *The Geometer's Sketchpad 4*®

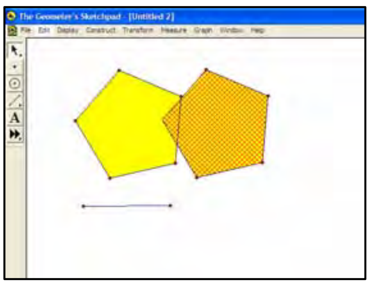
Part A: Translations



Open the Pentagon Sketch that you have saved, select the Segment Tool and draw a segment under your Pentagon. Select the Arrow Tool; deselect the segment, then select each of the two end points; from the Transform Menu, select Mark Vector.

Select the entire Pentagon; from the Transform Menu, select Translate. In the Translate Box click Translate.

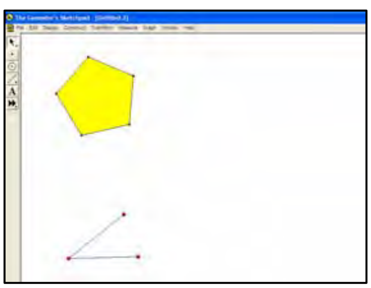
Describe in your notes what happened.

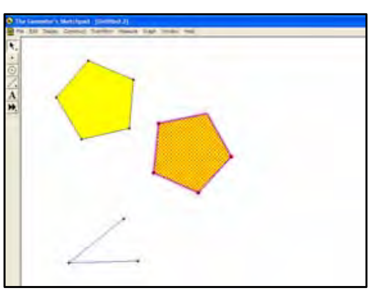


Select the right most point on your line and drag the point.

Describe in your notes what you observe. Compare the Translated image to the original. How are they the same, how are they different?

Part B: Rotations





Open the Pentagon Sketch that you have saved.

Using the Segment Tool, construct an angle below your Pentagon. Select the Arrow Tool, deselect the segment.

Select the 3 points of the angle in order counter clockwise; in the Transform Menu, select Mark Angle.

Select the vertex of the angle and in the Transform Menu select Mark Center.

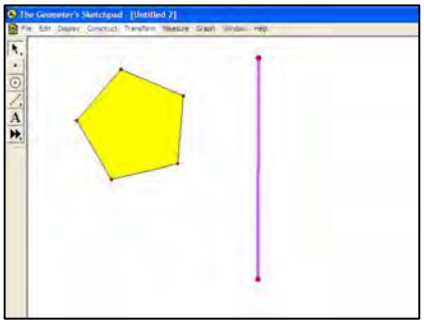
Select the entire Pentagon; in the Transform Menu, select Rotate. In the Rotate box, click Rotate.

Describe in your notes what happened.

Select one of the end points on your angle and drag it.

Describe in your notes what you observe. Compare the translated image to the original. How are they the same, how are they different?

Part C: Reflections

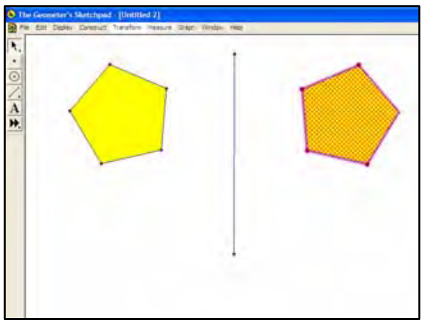


Open the Pentagon Sketch that you have saved.

Using the Segment Tool, construct a segment to the right of your Pentagon. In the Transform Menu, select Mark Mirror.

Select the Arrow Tool. Select your entire Pentagon
In the Transform Menu, select Reflect.

Describe in your notes what happened.



Select your mirror line and drag it.

Describe in your notes what you observe. Compare the translated image to the original. How are they the same, how are they different?

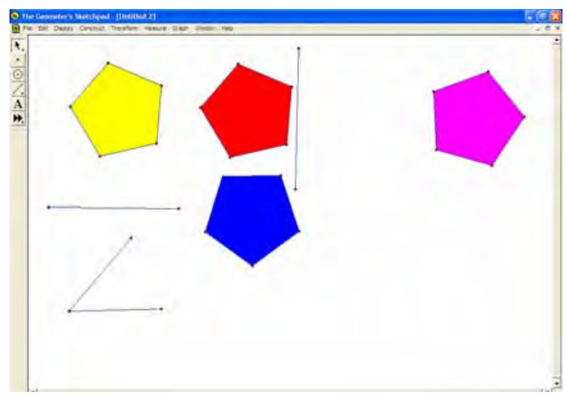
Part D: Put it all together

Open the Pentagon Sketch that you have saved.

Repeat each of the above transformations, this time not opening a new sketch each time. For each transformation select the image and change it to a different colour.

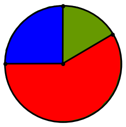
Your final sketch may have the images overlapping. You may need to drag your mirror line to achieve something similar to the screen shown here.

Considering the three images you have, explain whether it is possible for any of the three images to lie directly on top of one another. Experiment with your theory by dragging different parts of your sketch.



Part E: Explore More

1. Use various combinations of transformations to create a design.
2. Reflect an image over a line. Create a second reflection line parallel to the first line; reflect the image over the second line. Describe a single transformation that alone would have created the second reflected image. Repeat using two mirror lines that intersect.
3. Make up your own combination of transformations that could also be created by a single transformation.



Description

- Performing tessellations based on transformations:
  A: using *The Geometer's Sketchpad 4*®;
  OR
  B: using dot paper and coloured pencils.

Materials

- *The Geometer's Sketchpad 4*®
- BLM 14.3
  OR
  BLMs 14.1, 14.2
- coloured pencils

Assessment Opportunities

Minds On …

Whole Class → Introducing Problem

A: Explain the task: students use rotations to create an object and tessellate it using translations. Demonstrate using a design completed in *The Geometer's Sketchpad 4*®.

OR

B: Explain to the class that they are to create a shape on dot paper and then tessellate the shape on the dot paper.

Have one completed *GSP 4* design on a computer for students to see.

Action!

Individually → Explore

Students work collaboratively in pairs to complete the task on BLM 14.3. Alternately students use BLM 14.1 or 14.2 to create the tessellation illustrated on BLM 14.3.

Circulate and assist, as required.

Learning Skills (Co-operation with others)/Question & Answer/Checklist: Observe students as they work through the activity.

Students working collaboratively on one computer take turns manipulating the tessellation and giving advice or feedback.

Consolidate Debrief

Whole Class → Discussion

Debrief the class asking students to describe some of the "problems" and "fun" they had creating the tessellations. Discuss where they have seen tessellations used.

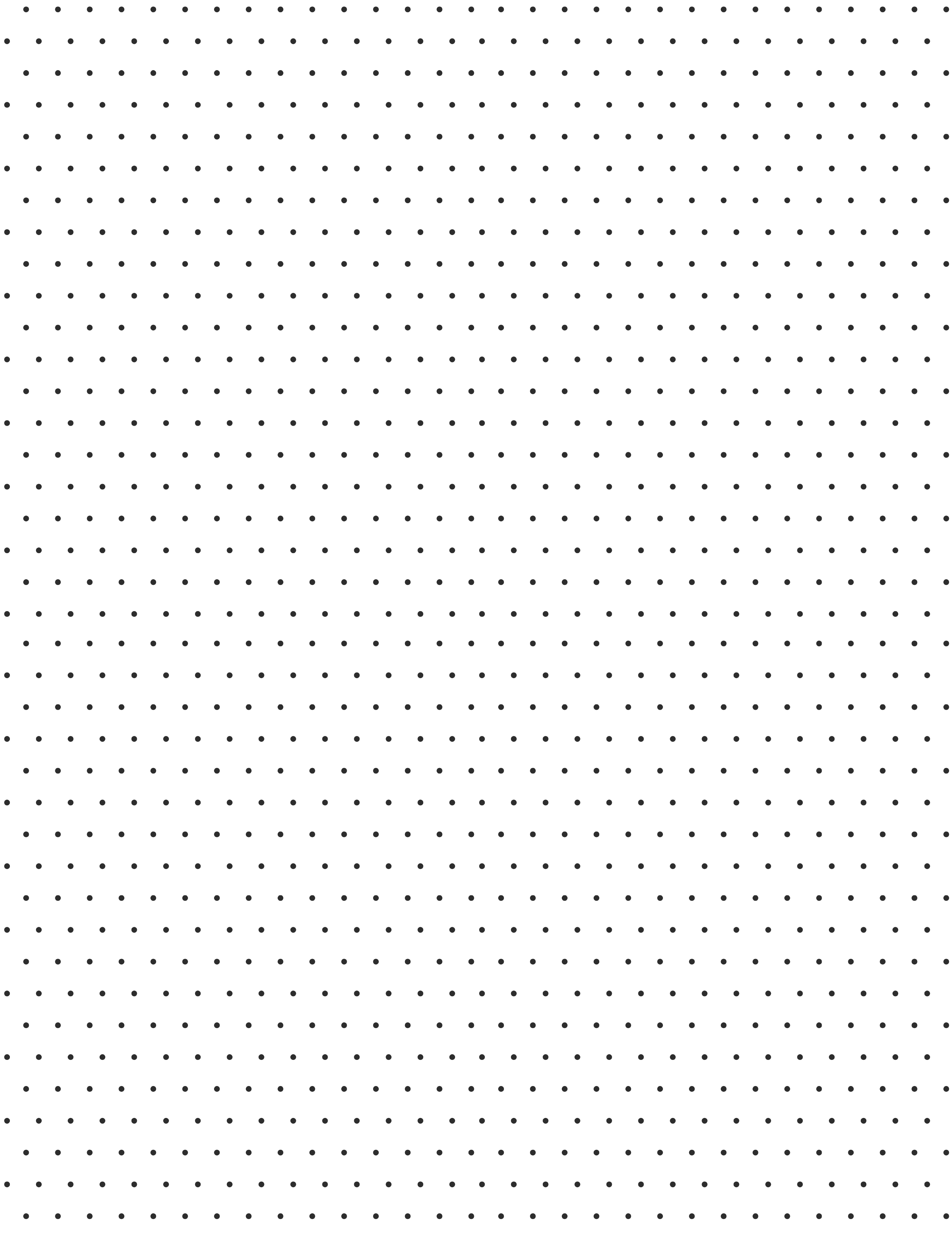
Challenge those students who finish early to create their own tessellation in *GSP 4*.

Application
Concept
Practice

Home Activity or Further Classroom Consolidation

Create a tessellation using more then one shape. Colour the results creating an interesting piece of art. Find examples where tessellations have been used around your home, neighbourhood, and school.

14.1 Isometric Dot Paper



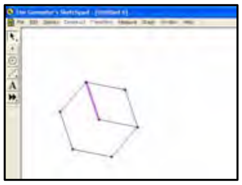
14.2 Orthographic Dot Paper

14.3 Tessellations with *The Geometer's Sketchpad 4*®

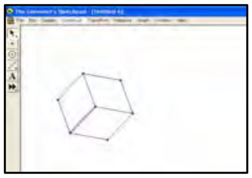
Create a regular hexagon: Using the Point Tool construct two points on the screen. Select the Arrow Tool, click on any white space to deselect everything. Click on the left most point; using the Transform Menu, Mark Center. Click on the other point. In the Transform Menu, select Rotate and type 60 into the degree box; click on Rotate. Select Rotate again and click on Rotate; repeat three more times.

Select each of the 6 points in order clockwise. In the Construct Menu, select Segments.

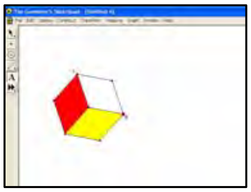
Create a cube.
Select the centre point and one other point. Construct the segment.



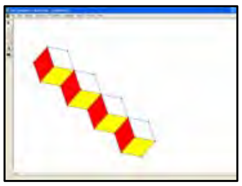
Select the centre point and not the next point but the one after it, construct the segment.



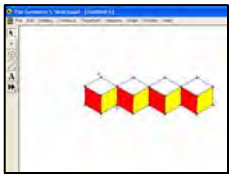
Repeat to complete the cube.



Select the four points that form the vertices of one of the square faces you have represented. In the Construct Menu, select Quadrilateral Interior. Repeat for another square. Change the colour of the second square using the Display Menu.
Select the two diagonal points of the top square shown on the screen as A and B, in the Transform Menu select Mark Vector.

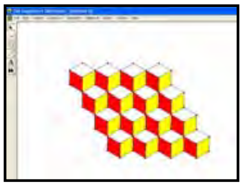


Select the entire cube and from the Transform Menu; select Translate. Repeat a few more times.



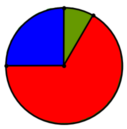
Drag the left most corner of the original cube and "straighten" out the line of cubes.

Mark a vector by selecting the two points indicated on screen as A and B.



Select all the cubes and translate them. Repeat a few times.

Describe the different shapes and effects you see in your diagram.



Description

- Apply knowledge of transformations to solve a congruency problem based on Pentominoes.

Materials

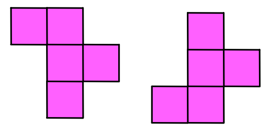
- Square tiles
- BLMs 15.1, 15.2
- dot paper

Assessment Opportunities

Minds On …

Whole Class→ Teacher Guided

Explain the task: Create different shapes using 5 square tiles. Demonstrate one shape that can be made using the five squares. Construct a congruent shape as it would appear under a transformation, explaining that these two shapes are considered to be the same shape.



If square tiles are not available, copy BLM15.1 onto card stock and have students cut the squares to be used.

Action!

Pairs →Explore

Ask students to find different Pentominoe shapes and draw them on the dot paper.

Curriculum Expectations/Question & Answer/Mental Note: Circulate, observing students' strategies for finding as many different shapes as possible.

There are 12 different Pentomino shapes.

Whole Class→ Teacher Guided

Use an enlarged overhead copy on the overhead of BLM 15.2 to guide students in identifying pair A and 1 as congruent under a rotation of 90° clockwise, followed by a translation. Students fill in the chart. The translation will vary depending on the centre of the rotation. Students may complete the chart in the following sequence:

- identify all of the pairs of congruent Pentominoes;
- identify whether a rotation or reflection is needed;
- describe the amount and direction of rotation or the type of reflection;
- mark the centre of rotation or the reflection line and identify the amount and direction of translation.

Prompt students who are having difficulty with the task.

Think, Pair→ Problem Solving

Students work on the next 11 pairs individually for a set amount of time. Ask pairs to share their results and discuss any differences

Students are likely to find the last part of the description the most challenging. Using cut-outs of the Pentomino shapes may help.

Consolidate Debrief

Whole Group → Discussion

Invite a pair of students to explain a pair that they found, using an overhead of BLM 15.2. One student could write the description while the other student demonstrates the transformation with cut-outs. Encourage students to communication clearly, using mathematical terminology. For each pair of Pentomino shapes, ask students to describe the transformation differently, and explain it.

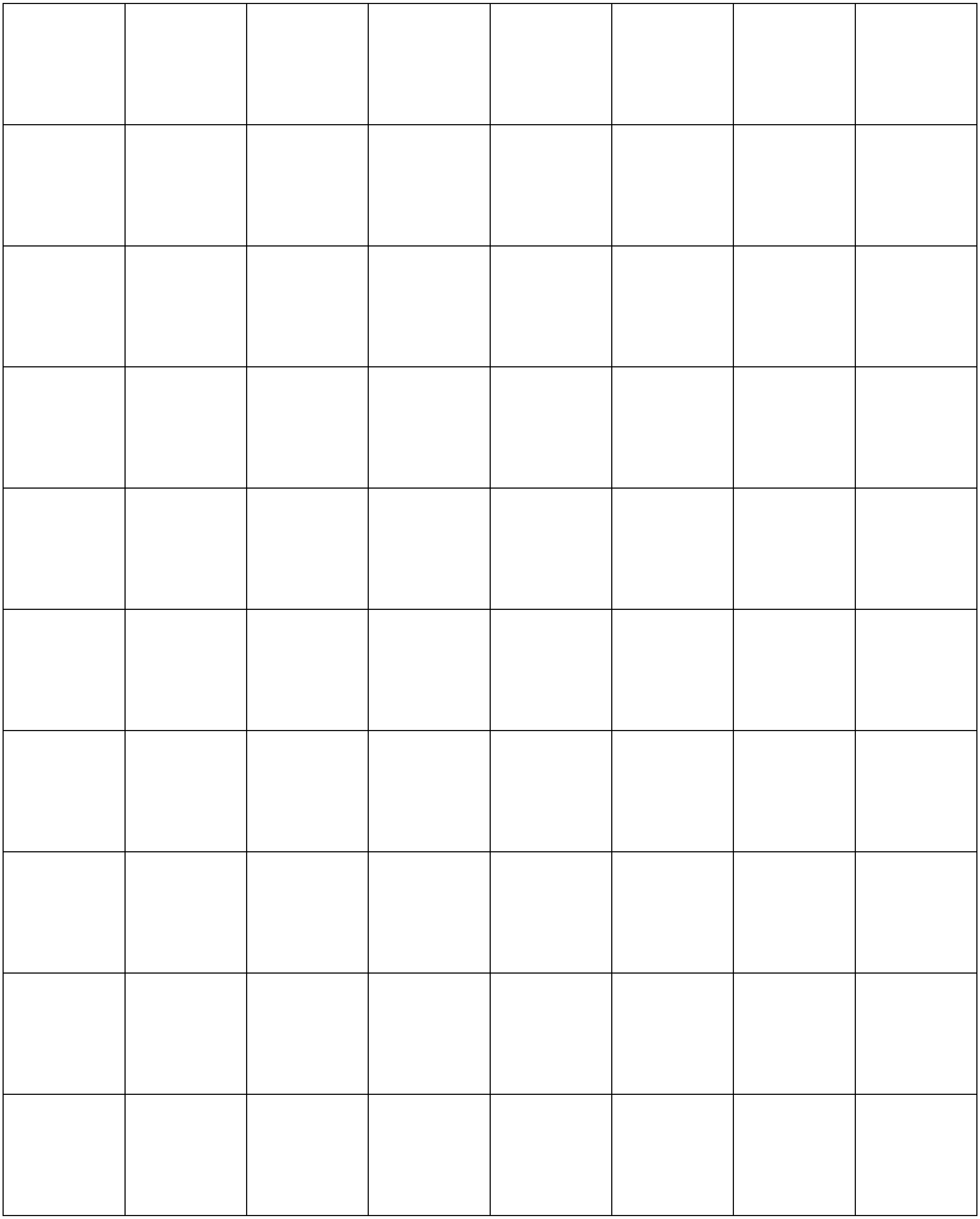
Curriculum Expectations/Learning Skills/Presentation/Rubric/Checklist: Assess student presentations for understanding of concepts, communication, application of procedures, and problem-solving skills.

Concept Practice

Home Activity or Further Classroom Consolidation

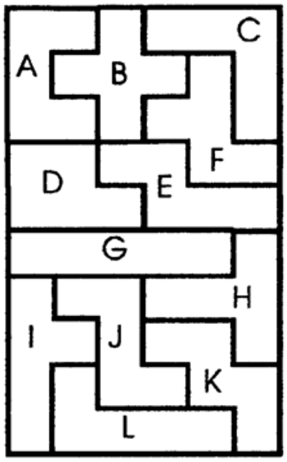
Find another arrangement of Pentominoes that form a rectangle.

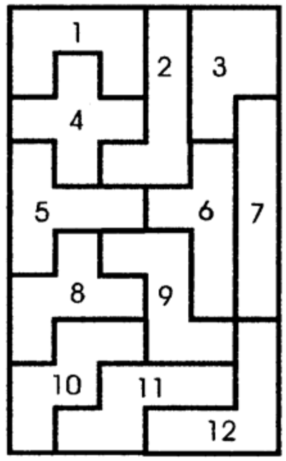
15.1 Square Tiles



15.2 Pentominoes

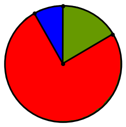
Two different rectangular arrangements of the 12 Pentomino pieces:





Find all pairs of congruent Pentomino pieces and describe the transformation that has been applied.

Pairs	Description of Transformation
A and ___	
B and ___	
C and ___	
D and ___	
E and ___	
F and ___	
G and ___	
H and ___	
I and ___	
J and ___	
K and ___	
L and ___	



Description

- Apply knowledge of the transformations to discover whether all types of triangles will tessellate.

Materials

- BLM 16.1 Assessment Tool
- geoboards, dot paper
- *The Geometer's Sketchpad 4*®

Assessment Opportunities

Minds On …

Whole Class → Introduction

Present the problem: Is this statement true? – All Types of Triangles Will Tile a Plane.

Groups of 4 → Brainstorm

Students brainstorm various ways in which they can solve the problem. They discuss different approaches, using materials and tools available, but do not take notes.

Learning Skills/Observation/Mental Note: Circulate, noting students contributions to the discussion.

The group time should be brief to allow time for students to individually complete the performance task.

Action!

Individual → Problem Solving

Students work independently to solve the problem in as many ways as they can, using the ideas generated during their brainstorming group session. Remind students that they can use any materials that they suggested in their groups.

See the *Grade 7 Exemplars 2002* for scored samples of student work.

See TIPS Section 2 – Developing Perimeter and Area for samples of different ways students could approach the problem.

Consolidate Debrief

Curriculum Expectations/Performance Task/Rubric/Marking Scheme: Collect and assess student work using BLM 16.1 Assessment Tool.

Comment on the students' strengths and next steps that they can take to improve performance.

When returning graded work to students, consider photocopying samples of Level 3 and 4 responses with student names removed. Select and discuss, samples that show a variety of strategies.

Home Activity or Further Classroom Consolidation

Reflection

Reflect on your solution to the problem presented in the assessment task and answer the following questions in your math journal:

- Did you find it easy or difficult to test the theory that all types of triangles will tile a plane? Explain why.
- What tools helped you?
- Select one of your tiling methods and describe how you thought of using it to show that this type of triangle will tile a plane.

16.1 Assessment Tool: Will All Types of Triangles Tile a Plane?

Name:
Date:

Criteria	Below Level 1	Level 1	Level 2	Level 3	Level 4
Making Connections (Understanding of Concepts)					
Tests types of triangles	- little or no evidence that various types of triangles should be tested	- uses very few types of triangles	- uses some types of triangles	- uses most types of triangles	- uses all types of triangles
Makes appropriate use of tools to test for tiling	- inappropriate for the problem	- with limited appropriateness for the problem	- moderate appropriateness	- appropriate	- highly appropriate
Reasoning and Proving (Problem Solving)					
Makes convincing argument, explanations and justifications	- no evidence of logic	- limited logic evident	- somewhat logical	- logical	- highly logical
	- no conclusion reached	- major omissions in arriving at conclusion	- some omissions in arriving at conclusions	- thorough	- complete and extended
Communicating (Communications)					
Uses mathematical language	- undeveloped use of conventions	- minimal skill in the use of conventions	- moderate skill in the use of conventions	- considerable skill in the use of conventions	- high degree of skill in the use of conventions
Explains the processes and conclusions	- unclear/ confusing	- limited clarity	- some clarity	- clear	- precise
Knowing Facts and Procedures (Application)					
Uses tools	Assess for correctness, using a marking scheme.				
Manipulates triangles to test for tiling a plane					

Comments:

Grade 9 Applied Year Outline

Term	Cluster of Curriculum Expectation	Reference	Number of Lessons Planned	Lesson Time Available - "Instructional Jazz"	Total Lesson Time
Intro	Graphing in the Cartesian Plane	TIPS 1-3	3	2	5
1	Measurement Relationships	TIPS 1-4, 10, 20 LMS 5-9, 12, 14, 16	21	2	23
2	Describing Relationships	TIPS 1-4, 10-11, 14	19	2	21
3	Tell me a Story Using Words and Equations	TIPS 1, 3	16	2	18
4	Plane Geometry	TIPS 1-3	9	2	11
5	Consolidation Emphasizing Algebraic Models	TIPS 1, 3-7	9		9
	EQAO		3		3
			80 days	**10 days**	**90 days**

Targeted Implementation and Planning Supports (TIPS)
Leading Math Success (LMS)

The number of prepared lessons represents the lessons that could be planned ahead based on the range of student readiness, interests, and learning profiles that can be expected in a class. The extra time available for "instructional jazz" can be taken a few minutes at a time within a pre-planned lesson or taken a whole class at a time, as informed by teachers' observations of student needs.

The reference numbers are intended to indicate which lessons are planned to precede and follow each other. Actual day numbers for particular lessons and separations between terms will need to be adjusted by teachers.

Grade 9 Applied Activating Prior Knowledge on Composite Figures

75 min.

Description

- Activate students' prior knowledge of terminology related to identifying geometry shapes.
- Determine students' readiness to identify geometric figures in composition and use appropriate calculations for perimeter and area.

Materials

- BLM 1.D.1, 1.D.2, one per pair,
- BLM 1.D.3, one per student

Assessment Opportunities

Minds On…

Whole Class → Orientation

Outline the procedure for the day, including the purposes of each component of the lesson (**Minds On** – activate prior knowledge of shapes, **Action** – review their measurement skills and **Consolidate** - demonstrate these skills in an activity). Explain that assessment will allow you to plan upcoming lessons according to their current levels of understanding and that the results will not influence their grade in the course.

Small Groups → Pass the Paper

Each group lists as many geometric figures as they can find in the diagram BLM 1.D.1. Circulate to provide direction and guidance, as necessary.

Students pass one piece of paper around the group, writing a response each time they receive the paper. Students may pass. Responses should include rectangles, squares, triangles, trapezoids, circles (semi-circles) and may include adjectives as in 'equilateral triangle.' Encourage effective communication by referring to group work skills.

Pass the Paper is typically a timed activity, usually taking no more than 2 minutes.

Learning Skill (Teamwork)/Observation/Mental Note: Observe students as they work.

Action!

Pairs → Composite Figures

Students answer the question on BLM 1.D.1. Circulate to encourage discussion and to clarify information regarding the diagrams.

Whole Class → Discussion

Using an overhead of BLM 1.D.2, lead a discussion in which students share their answers.

Question 2 requires students to determine the hypotenuse of a right triangle. The intent is to determine if they can apply the Pythagorean theorem to find the hypotenuse only at this time.

Consolidate Debrief

Small Group → Placemat

Describe the procedure for completing a placemat activity. The centre oval is used to document the consensus of the group.

Students individually respond to the question on BLM 1.D.3 for 5 minutes. The members of each group share their procedures then complete the centre portion of the placemat providing a model solution to the question. Collect the placemats for assessment.

Ask students to label their "place" on the placemat for reference later.

Curriculum Expectation/Placemat/Checkbric: Circulate and observe students as they complete a solution, recording individual strengths and needs. The results of individual work will allow you to differentiate the next two lessons appropriately.

Whole Class → Discussion

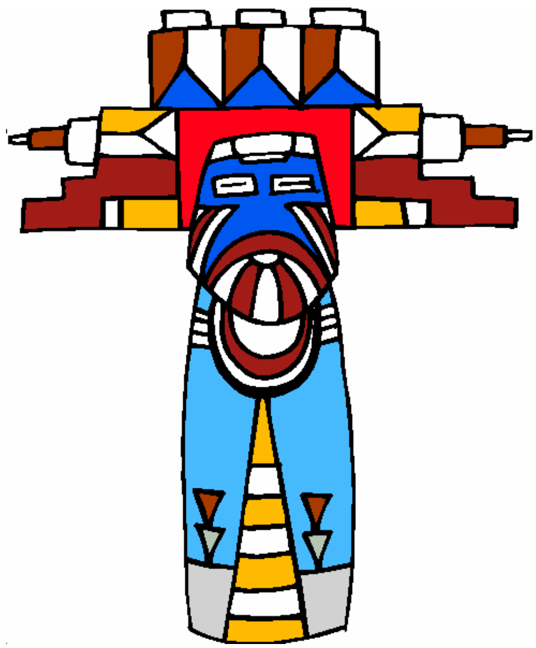
Facilitate group sharing of approaches to the posed question.

Home Activity or Further Classroom Consolidation

Choose an activity appropriate to the context of the pre-planned lesson.

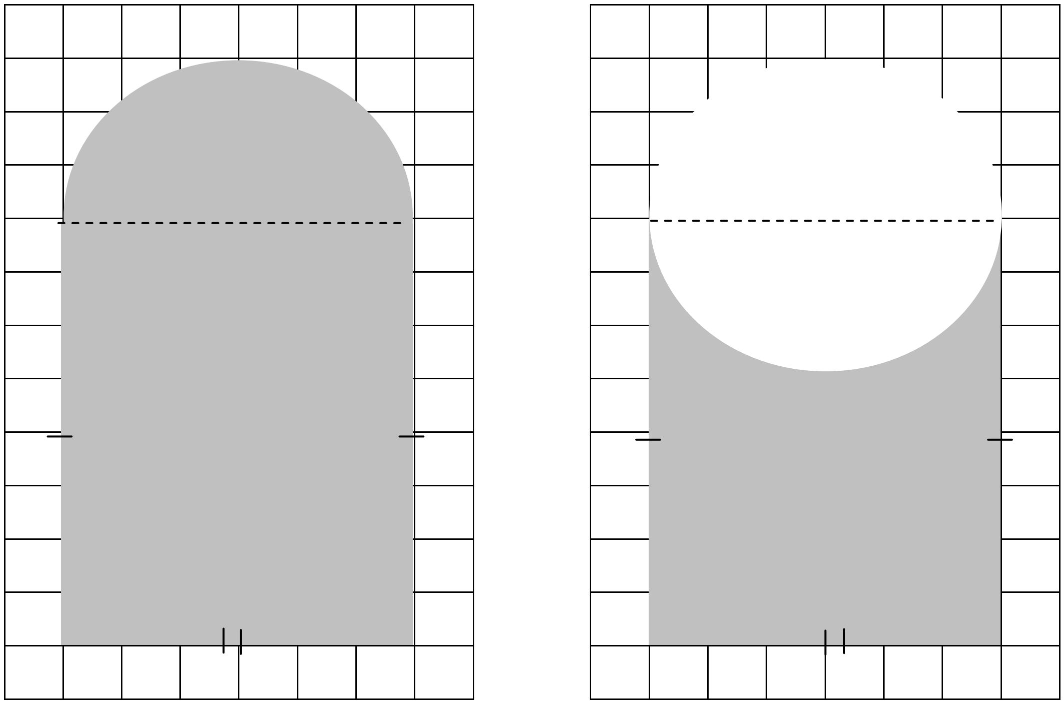
1.D.1

Name the geometric shapes, using arrows to connect names of shapes to the diagram.



1.D.2

1. Consider the two composite figures below.
   (a) Identify the geometric shapes in each. Write the names on the diagrams.



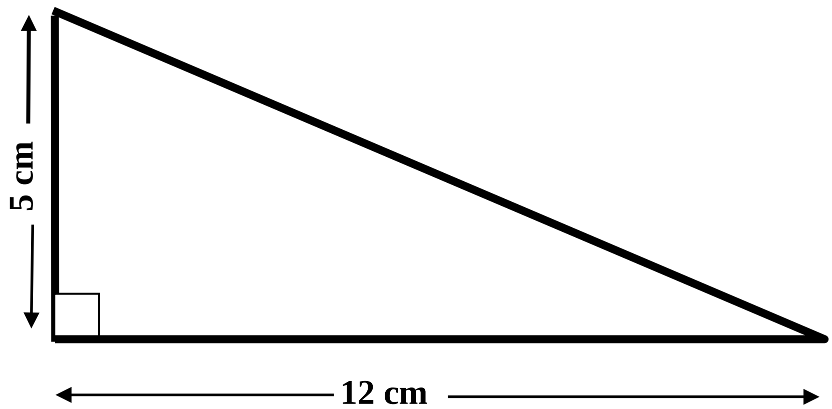
   (b) Calculate the area of the shaded regions and describe the features and calculations that are:
       i) the same   ii) different

   (c) What is the difference in the area of the two figures?

   (d) Use a coloured pencil to outline the perimeters of the two figures. How do these two perimeters compare?

1.D.2 (continued)

2. Use the diagram below to calculate the Area and Perimeter of the triangle.



a) What dimensions are needed to determine the perimeter?

b) What dimensions are needed to determine the area?

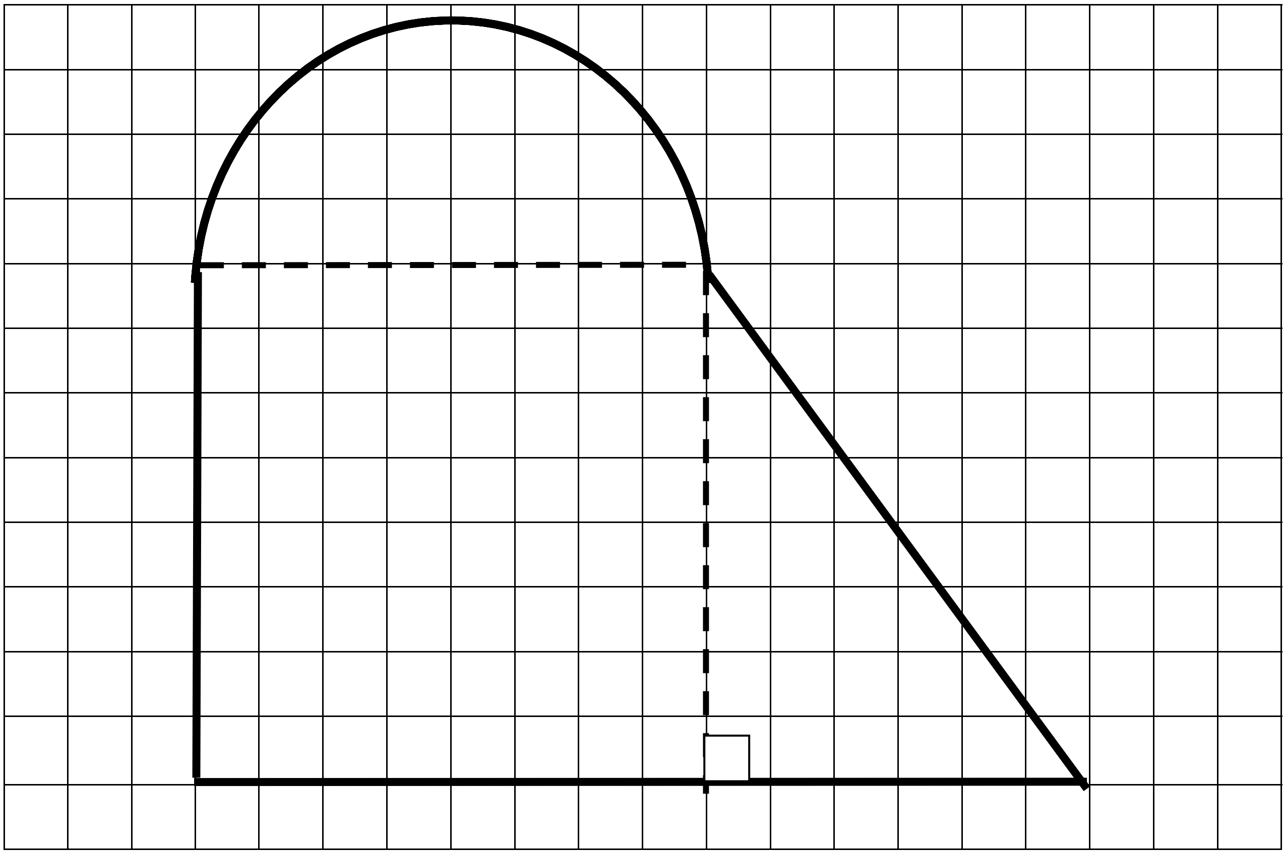
c) Describe a method to determine the missing dimension(s).

d) Calculate:

Area Perimeter

3. Provide an example in daily life of a figure that involves more than one geometric shape.

1.D.3



Individually on the placemat, list the steps required to determine the area of the figure.

Discuss the steps from #1 (on worksheet 1.D.2) then, as a group, provide one complete solution to the problem in the centre of the placemat.

Unit 1: Measurement Relationships

Grade 9 Applied

Lesson Outline

BIG PICTURE

Students will:

- describe relationships between measured quantities;
- connect measurement problems with finding the optimal solution;
- apply knowledge and understanding of 3-D formulas to simple problems in context;
- develop numeric facility in a measurement context;
- work as effective members of learning teams.

Day	Lesson Title	Description	Expectations
1	What is the Largest Rectangle? *Lesson included*	• Use an inquiry process to determine the largest rectangle with integral sides that can be constructed for a given perimeter. **Note:** Focus on the Explore stage of the Inquiry Process.	NA3.05, NA4.03, MG1.01, MG1.04, MG2.03, AG3.05 CGE 2a, 5a
2	What is the Largest Rectangle Revisited? *Lesson included*	• Connect the sum of the side lengths and the perimeter formula. **Note:** Focus on the Model and Hypothesize stages of the Inquiry Process.	RE1.01, RE1.03 - 1.07, RE2.04, NA4.03 CGE 4b
3	On Frozen Pond *Lesson included*	• Extend fixed perimeter problems to ones that do not have integral solutions. **Note:** Focus on the Manipulate/Transform and Infer/Conclude stages of the Inquiry Process.	MG1.01, MG1.04, MG2.03, RE1.01, RE1.03 - 1.07, RE2.04, AG3.05, NA3.05, NA4.03 CGE 5a
4	Down by the Bay *Lesson included*	• Apply the Inquiry Process to a problem that requires enclosing a rectangular space on only three sides. • Explore the results for a variety of What if? conditions.	MG1.01, MG1.04, MG2.03, RE1.01, RE1.03 - 1.07, RE2.04, AG3.05, NA3.05, NA4.03 CGE 4c

Day	Lesson Title	Description	Expectations
5	Formative Assessment Task *New: Lesson included*	• Manipulate given data and a scatterplot for a fixed perimeter context, e.g., a garden plot, fenced area, sandpit. • Discover the need to collect more data in the region of the graph where an optimal value occurs. • Make inferences and conclusions.	RE1.05 - 1.07, MG1.01, MG1.04, NA4.03, AG3.05 CGE 2b, 3c
6	Greenhouse Commission *New: Lesson included*	• Investigate the optimal solution when a rectangular area is fixed and the perimeter must be minimized through problems involving, for example, the cost of fencing or the number of tiles around a pool.	MG2.03, MG2.05, RE1.04, RE1.05 - 1.07, AG3.05, NA1.02, NA2.02 CGE 3b, 5a
7	All Cooped Up *New: Lesson included*	• Investigate the optimal solution when a rectangular area is fixed and the perimeter must be minimized, including enclosures of only two or three sides of a rectangular prism	MG2.03, MG2.05, RE1.04, RE1.05 - 1.07, AG3.05, NA1.02, NA2.02 CGE 3c, 4e
8	Composite Figures *New: Lesson included*	• Solve composite area and perimeter problems	MG2.01, MG2.02 - 2.05, NA1.04, NA2.01, NA3.03, NA3.04, NA3.05 CGE 3c, 4b, 5a
9	Pythagorean Theorem and Composite Figures *New: Lesson included*	• Use the Pythagorean theorem to solve composite rectangle and triangle problems.	MG2.01, MG2.02 - 2.05, NA1.04, NA2.01, NA3.03, NA3.04, NA3.05 CGE 2b, 4e, 5g
10	Camp Olympics *Lesson included*	• Determine optimal dimensions based on factors that include function, volume, and surface area. • Use straightforward application of spreadsheets.	MG2.03, RE1.01, RE1.03 - 1.07, RE2.04, AG3.05, NA2.01, NA4.03 CGE 5a

75 min.

Description

- Manipulate given data and scatterplot for a fixed perimeter context.
- Discover the need to collect more data in the region of the graph where an optimal value occurs.
- Make inferences and conclusions.

Materials

- BL.M 1.5.1
- graphing calculators

Assessment Opportunities

Minds On…

Whole Class → Read a Story to Introduce Investigations

Students read the poem "The Kittens with Mittens Come to Math Class" that introduces the problem. Each student reads one line.

Students identify the two different geometry problems presented in the poem and any important information.

Writing the names on each line on the teacher copy ahead of time will speed up the process and allow for flexibility; alternatively assign a line to each student and give them time to practise.

Action!

Whole Class → Demonstration

Provide each student with a graphing calculator. Using the presentation software file "Scatterplots on the Graphing Calculator," demonstrate some functions of the calculator.

See TIPS Downloads at http://www.curriculum.org/occ/tips/downloads/gr9u1scatter.ppt

Pairs → Investigation

Students complete the questions on BLM 1.5.1. Circulate to assist them as they work.

Curriculum Expectations/Observation/Mental Note: Observe if students recognize that the area of a four-sided rectangle is maximized when the figure is a square. For a three-sided figure, the area is maximized when the length is twice the width.

Students who complete this task early may be challenged to present their solutions in a style similar to the poem.

Consolidate Debrief

Whole Class → Discussion

Choose students who used different strategies and formulas to share their solutions and conclusions.

Discuss dependent and independent variables, and discrete vs. continuous data.

Bring out the ideas that:

- Scatter plots are appropriate when values of the variables between plotted points do not belong to the relationship being graphed.
- The data shown in a scatter plot is called discrete data. Points are not connected.
- Broken-line graphs are appropriate when values of the variables between plotted points do belong to the relationship being graphed.
- The data shown in a broken-line graph is called continuous.
- For the Kittens with Mittens investigations, there is no reason that the length, width, and area measures have to be whole numbers. Fractional measures would make sense. Therefore, broken-line graphs are most appropriate for modelling the investigations.

Reflection

Home Activity or Further Classroom Consolidation

Write a response in your journal: One thing I did well is… or I need…

1.5.1: The Kittens with Mittens Come to Math Class

The clock must have stopped as we sat in math class that day.
We'd never get out, I was certain there's no way.
Hally and I, we sat there, it's true.
Perimeter and area, we weren't sure what to do.
It was an investigation the teacher wanted us to complete.
We were to be very careful and especially neat.
"Look at me!" said the teacher, "Look at me now!
Regular shapes with the same perimeter, you have to know how!"

But as hard as I tried I could not stay awake,
Until a big BUMP caused me to shake.
I opened my eyes and our teacher was gone
And the Kittens with Mittens were out on the lawn.
They strolled into our room with a box over their heads.
"Get ready to have fun!" is what they both said.
"In this box you will find something fickle!
Two little variables to get you out of this pickle."

They jumped on the box and opened the lock,
And both of us were too excited to talk.
But slowly out of the box came Variable Two and Variable One.
They looked rather sad. They didn't want to have fun.
They explained to us that they were in a real bind.
They hadn't done their homework and they were really behind.
Their problem was in math as you could probably guess.
It was area and perimeter. What a coincidence? Yes?

They had 50 m of rope with which to enclose a rectangular ground for play.
Whoever enclosed the biggest area was champion for the day.
Two designs were required to be written down with our pen, this was not cool,
A 4-sided enclosure and also a 3-sided enclosure attached to the school.
Hally and I knew they needed our help, but what could we do?
We didn't listen to the area and perimeter lesson, too.
But then Hally jumped up and started to shout.
"We'll do the investigation so we can figure it out!"
That's what we did for Variable One and Variable Two.
We found the answer, can you find it too?

1.5.1: The Kittens with Mittens Come to Math Class (continued)

Solving the Kittens with Mittens problem by creating graphical models on the graphing calculator.

1. Clear all lists by pressing **2nd, + [Mem]**, then choose **RESET**. Press **ENTER, ENTER .**

2. To begin entering data, press **STAT**, then choose **1:Edit**. Press **ENTER.**

3. Enter the width data into the calculator. **L1** will represent the width, **L2** will represent the length and **L3** will represent the area. Remember to press **ENTER** after each entry.

4. Move the curser to the top of L2 (on top of the letters) and press **ENTER**. Enter the formula for length. Remember that **2nd, 1** gives you L1.

5. Move the curser to the top of L3 (on top of the letters) and press **ENTER.** Enter the formula for area. Remember that **2nd, 1** gives you L1 and **2nd, 2** gives you L2.

6. To plot the data using a broken line graph, press **2nd, Y=** for **[STATPLOT]**. Select **1:Plot 1…Off** and press **ENTER**.

   Using the arrow keys **<** and **>** and the **ENTER** key:

   - Turn the graph on by setting **On-Off** to **On.**
   - Set the **Type** to a **Line Graph** (second picture on top row)
   - Check that the **Xlist** is **L1**.
   - Change the **Ylist** to L3 using **2nd, 3.**
   - Set the **Mark** to □.

7. To set the viewing window for your graph, press **ZOOM** and use the arrow keys to select **9:ZoomStat**.

8. To view the graph press **ENTER**.

9. Use the Trace feature to view the co-ordinate values of each point. Press **TRACE**. When you press the arrow keys, you will be able to see the *x* and *y* values for each point.

1.5.1: The Kittens with Mittens Come to Math Class (continued)

Investigation 1: The Four-Sided Enclosure

1. Copy your data from the graphing calculator for the 4-sided enclosure in the table below.

Perimeter (m)	L1 Width (m)	L2 Length (m)	L3 Area (m^2) $l \times w$
50	0		
50	2		
50	4		
50	6		
50	8		
50	10		
50	12		
50	14		
50	16		
50	18		
50	20		
50	22		
50	24		

2. Draw a picture of the broken line graph on the screen of the calculator.

3. What variable is represented on the horizontal axis? _______________________

4. What variable is represented on the vertical axis? _________________________

5. Which variable is:
   independent?

   dependent?

6. Describe what happened to the area as the width increased.

Investigation 2: The Three-sided Enclosure

Enter data for the 3-sided enclosure in the table below.

Perimeter (m)	L1 Width (m)	L2 Length (m)	L3 Area (m^2) $l \times w$
50	0		
50	2		
50	4		
50	6		
50	8		
50	10		
50	12		
50	14		
50	16		
50	18		
50	20		
50	22		
50	24		

1. Make a broken line graph of the data.

2. What appears to be the relationship between the width and the area?

3. Make a scatter plot of the same data using the graphing calculator. To do this, follow steps 1 to 5 from the calculator instructions. This time set the **Type** to a **Scatter Plot** (first picture on top row). Continue with the rest of steps 6 to 9.

4. How does this scatterplot compare to the one you completed above?

1.5.1: The Kittens with Mittens Come to Math Class (continued)

5. Which type of graph better models the problem – a broken-line graph or a scatter plot? Explain.

6. What recommendation would you make for the 4-sided enclosure?

7. What recommendation would you make for the 3-sided enclosure?

8. Refer back to the Kittens with Mittens Come to Math Class story to decide if you will be the "champion of the day?" Explain.

Description

- Investigate the optimal solution when a rectangular area is fixed and the perimeter must be minimized through problems involving, for example, the cost of fencing or the number of tiles around a pool.

Materials

- BLM 1.6.1
- rulers
- grid paper

75 min.

Assessment Opportunities

Minds On ...

Whole Class → Discussion

Introduce the task: Greenhouse Commission (BLM 1.6.1). Read the instructions and clarify the problem, so that students understand the difference between this problem and the previous problems that had fixed perimeter. This time the area is fixed.

Ask prompting questions:

How is this problem different from the Kittens with Mittens tasks?
What is the measure you need to minimize in this problem?
How can you be sure that you have found the minimum perimeter?

Focus on the Manipulate/ Transform and Infer/Conclude stages of the Inquiry Process

Action!

Pair/Share → Guided Investigation

Students explore possible greenhouse proportions and share strategies for selecting designs with smaller perimeters.

Students investigate the dimensions of a sufficient number of these rectangles to make and justify a conclusion.

Curriculum Expectations/Observation/Mental Note: Observe and question students as they complete the activity.

Students should be encouraged to hypothesize the optimal dimensions and work from that hypothesis.

Students could begin their calculations by considering widths that produce a square first, then testing width values immediately above and below. They could also use the symmetrical properties of the table to minimize calculations.

Consolidate Debrief

Whole Class → Discussion

Discuss the investigation addressing how this problem is different from their previous experiences and how it is the same. Students share strategies about how they completed the table. Students listen and question their peers to improve their own understanding.

Discuss the use of the square root to determine minimum perimeters for areas that are not perfect squares. Be sure to ascertain that all students are able to use the square root key on their calculator.

Application
Concept Practice

Home Activity or Further Classroom Consolidation

Complete textbook questions.

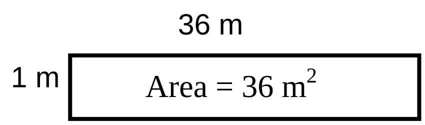
Select 2 or 3 appropriate textbook questions to reinforce using square root in solving perimeter and area problems.

1.6.1: Greenhouse Commission

Elaine and Daniel are building a rectangular greenhouse. They want the area of the floor to be 36 m^2. Since the glass walls are expensive they want to minimize the amount of glass wall they use. They have commissioned you to design a greenhouse which minimizes the cost of the glass walls.

Explore

It is possible to build a long narrow greenhouse.



$$\begin{aligned}\text{Perimeter} &= 2l + 2w \\ &= 2(36) + 2(1) \\ &= 74 \text{ m}\end{aligned}$$

Sketch *three* more greenhouses that have a perimeter smaller than this greenhouse. Label the dimensions on the sketch and calculate the perimeter.

Hypothesize

Based on your exploration, predict the length and the width of the greenhouse with the least perimeter.

Model

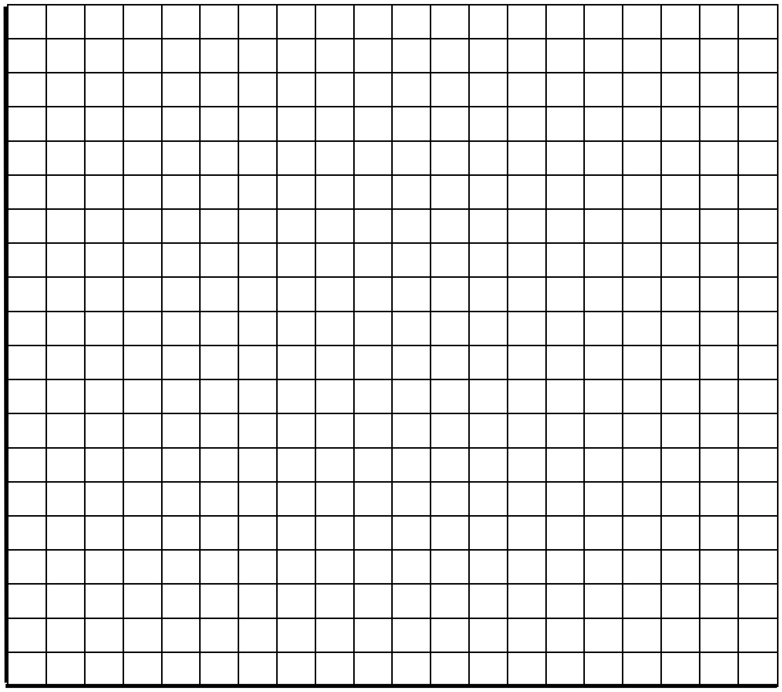
Complete as much of the table as required to determine the dimensions that result in the least perimeter. You may not need to fill in the whole table.

Area (m^2)	**Width (m)**	**Length (m)**	**Perimeter (m) (P = 2*l* + 2*w*)**
36	1	36	2(36) + 2(1) = 74
36	2	18	2(18) + 2(2) =
36	3		
36			
36			
36			
36			
36			
36			

What happens to the perimeter of the greenhouse as the width increases?

1.6.1: Greenhouse Commission (continued)

Construct a graph of Perimeter vs. Width



Conclude

Write a report for Elaine and Daniel advising them of the dimensions that would be the best for their greenhouse. Justify your recommendation using both the table and the graph. Include a sketch and the perimeter of the greenhouse that you are recommending.

Apply

1. If the greenhouse is to have a height of 2 m and the price of the glass from Clear View Glass is $46.75/m^2, what will it cost to purchase the glass for the walls of the greenhouse? Show all of your work.

2. Translucent Inc. charges $50/m^2 for the first 30 m^2 and then they give a 20% reduction on the rest of the glass. From which company should Elaine and Daniel purchase the glass? Explain fully and show all of your calculations.

Unit 1: Day 7: All Cooped Up

Grade 9 Applied

75 min.

Description

- Investigate the optimal solution when a rectangular area is fixed and the perimeter must be minimized, including enclosures of only two or three sides of a rectangular prism.

Materials

- BLM 1.7.1
- rulers

Assessment Opportunities

Minds On ...

Whole Class → Discussion

Introduce the task: All Cooped Up (BLM 1.7.1). Read the instructions and clarify the problem.

Ask prompting questions:

- How is this problem different from the Greenhouse Commission?
- Do you think the answer will be the same or different if you need to fence only 3 sides instead of 4 sides?
- How will you be sure that you have found the minimum perimeter?

Focus on the Manipulate/ Transform and Infer/Conclude stages of the Inquiry Process.

Action!

Pair/Share → Guided Investigation

Students explore possible chicken coops and share strategies for selecting designs with smaller perimeters.

Students investigate the dimensions of a sufficient number of these rectangles to make and justify a conclusion.

Students who complete BLM 1.7.1 quickly can continue to explore these relationships with BLM 1.7.2, which requires access to a spreadsheet. This investigation requires students to consider non-integral solutions. Since only selected students will complete this activity, the conclusions should not be used as part of an assessment.

Curriculum Expectations/Observation/Mental Note: Observe and question students as they complete the activity.

Students could begin their calculations by considering widths that produce a square first, based on their conclusions from Greenhouse Commission, then test width values immediately above and below. They will notice that a square does not minimize perimeter in this case.

Consolidate Debrief

Whole Class → Discussion

Discuss the investigation addressing how this problem is different from their previous experiences and how it is the same. Students share strategies about how they completed the table. Students listen and question in order to improve their own understanding that the length = 2 × width for the minimum perimeter when 3 sides of the rectangle are fenced.

Students who completed the second investigation could share their strategies and results with the class.

Students are not expected to memorize the result that length = 2 × width when 3 sides are fenced. Rather, they need to appreciate that the shape is not a square, and know how to discover the appropriate shape.

Application
Concept Practice

Home Activity or Further Classroom Consolidation

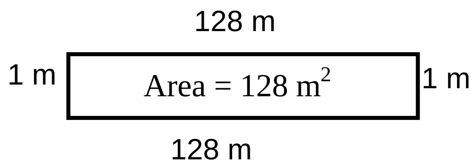
Complete the worksheet, A Pool Walkway.

1.7.1: All Cooped Up

You want to construct a new chicken coop at the side of a barn. Since the barn will make one of the sides, you only need to fence off 3 sides of the coop. The chicken coop must have an area of 128 m^2. A clever fox has been trying to get into the old coop and has caused a lot of damage. Since it is likely that you will be constantly repairing this coop you want to minimize its perimeter so you can save money.

Explore

It is possible to build a long narrow chicken coop.



$$\begin{aligned} \text{Perimeter} &= l + 2w \\ &= 128 + 2(1) \\ &= 130 \text{ m} \end{aligned}$$

Sketch *three* more chicken coops that have a perimeter smaller than this chicken coop. Label the dimensions on the sketch and calculate the perimeter.

Hypothesize

Based on your exploration, predict the length and the width of the chicken coop with the least perimeter.

Model

Complete the table with possible combinations of width and length for the chicken coop. Think about patterns you noticed in previous tables.

Area (m^2)	Width (m)	Length (m)	Perimeter (m) ($P = l + 2w$)
128	1	128	128 + 2(1) = 130
128	2	64	64 + 2(2) = 68
128	4		
128			
128			
128			
128			
128			
128			
128			
128			

What happens to the perimeter of the chicken coop as the width increases?

1.7.1: All Cooped Up (continued)

Construct a graph of Perimeter vs. Width

Conclude

Write a report outlining the dimensions that would be the best for your chicken coop. Justify your recommendation, using both the table and the graph. Include a sketch and the perimeter of your chicken coop.

Apply

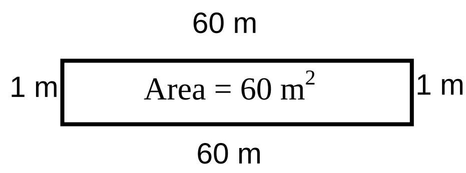
If fencing costs $2.49/m, what is the total cost of the fencing needed to build the chicken coop, including taxes of 15%?

1.7.2: A Pool Walkway

You want to construct a rectangular pool in your backyard with a water surface area of 60 m^2. The pool will be built in the back corner of your lot so that it will be bordered on two sides by a fence. You will make a walkway on the other two sides of the pool. Since the walkway will be constructed from blue slate tiles you want to minimize the number of tiles that will be used.

Explore

It is possible to build a long narrow pool.



Perimeter = $l + w$
= 60 + 1
= 61 m

Sketch *three* more pools that have a perimeter smaller than this pool. Label the dimensions on the sketch and calculate the perimeter.

Hypothesize

Based on your exploration, predict the length and the width of the pool with the least perimeter.

Model

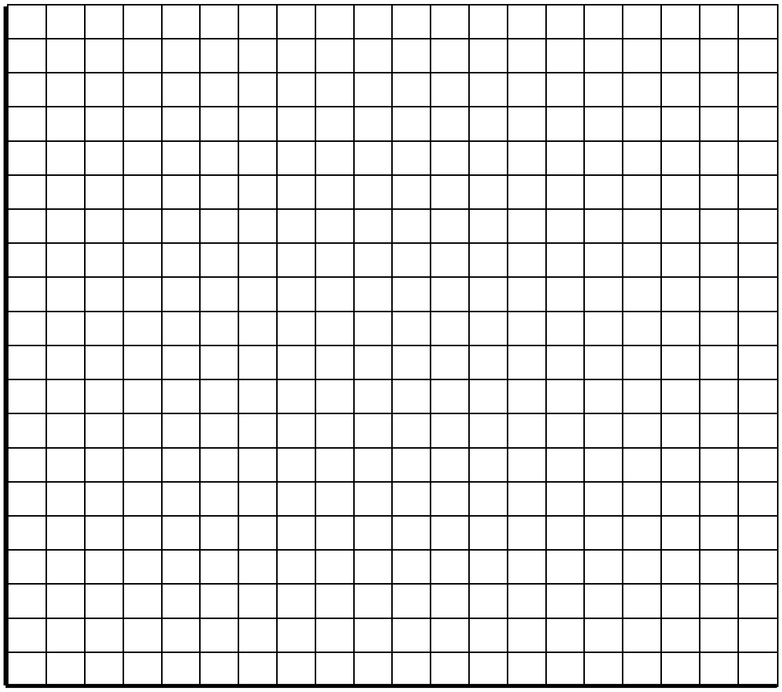
Complete the table with possible combinations of width and length for the pool.

Area (m^2)	**Width (m)**	**Length (m)**	**Perimeter (m) (P = $l + w$)**
60	1	60	60 + 1 = 61
60	2	30	30 + 2 = 32
60	3		
60	4		
60	5		
60	6		
60	8		
60			
60			
60			
60			
60			
60			

What happens to the perimeter of the pool as the width increases?

1.7.2: A Pool Walkway (continued)

Construct a graph of Perimeter vs. Width



Circle the area on the graph where the minimum occurs.

Use a spreadsheet to determine the dimensions of the pool with the least perimeter.
Choose values for the width in the area you have circled on your graph.
Add four entries from the spreadsheet to the chart below. One of the entries must correspond to the least perimeter.

Area (m^2)	**Width (m)**	**Length (m)**	**Perimeter (m) (P = *l* + *w*)**
60			
60			
60			
60			

Conclude

Write a report outlining the dimensions that would be the best for your pool. Justify your recommendation, using both the table and the graph. Include a sketch and the perimeter your pool.

75 min.

Description

- Solve composite area and perimeter problems.

Materials

- BLM 1.8.1

Assessment Opportunities

Minds On…

Pairs → Problem Solving

Pairs complete BLM 1.8.1. Provide scaffolding, as necessary.

Whole Class → Discussion

Facilitate student sharing of methods for determining area and perimeter of the shapes, in particular, the variety of solutions for the first shape. A key message is that one method for determining a measurement is counting squares and another is using formulas. Composite figures may be divided in many different ways.

Differentiate for students who need a review of composite figures by having them access the PowerPoint: Composite Figures. (TIPS Download)

Action!

Pairs → Problem Solving with Composite Shapes

Select or create three or four area questions involving a variety of composite shapes, including rectangles, circles, triangles, parallelograms, and trapezoids. Consider shapes such as a rectangle with a circle cut out of the centre.

Select or create two or three additional questions involving perimeter of composite figures constructed only with rectangles and circles. **Note:** These may include some of the diagrams used for area questions.

Students work in pairs to solve the questions, sharing strategies with their partner.

Post answers to the questions a few minutes before it is time to consolidate understanding, so students know which questions they need to correct.

Curriculum Expectations/Observation/Anecdotal: Circulate and observe students as they complete solutions, providing feedback on strengths and concepts or skills that need further work.

Consolidate Debrief

Whole Class → Discussion

Students vote on three questions that they would like to see the solutions for. Select students to share their solutions to the three questions on the board.

Circulate as students compare their answers to solutions being presented, helping students identify and correct their errors, particularly where pairs used a different approach than that shown on the board.

Application
Concept Practice

Home Activity or Further Classroom Consolidation

Find a 2-D shape around the school or in the community that is considered a composite figure. This may be a face of a 3-D object.

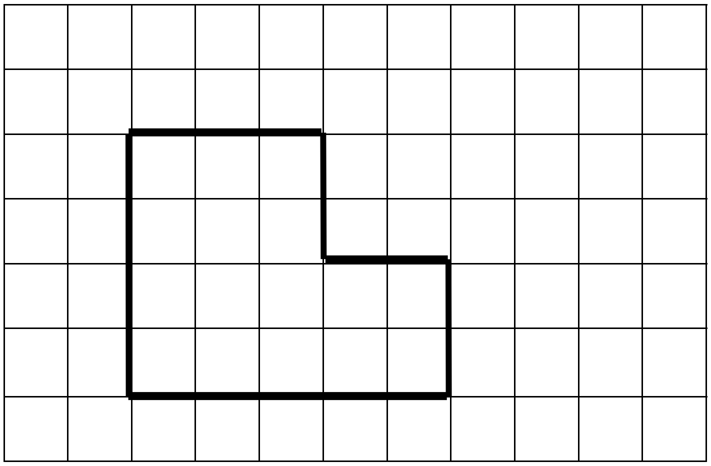
Draw a sketch of the 2-D face or shape and include estimated measurements.

Calculate or estimate the area of the figure.

Explain how you made your estimations of measurements and explain why you could or could not calculate the area.

1.8.1: Area and Perimeter of Composite Figures

1.



a) Determine the area of the above shape using two different methods.

Method 1:

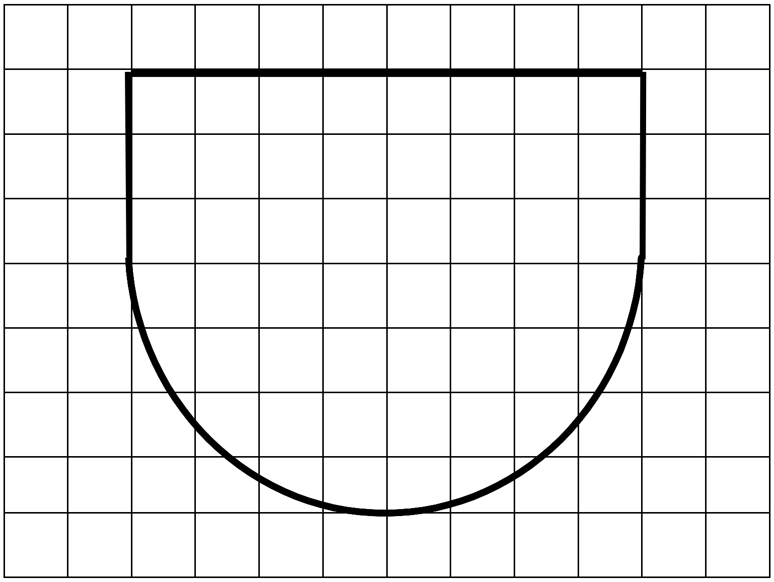
Method 2:

b) Determine the perimeter of the above shape. Explain your method.

Perimeter:

Explanation:

2.



Estimate the area of the above shape by counting squares.

Calculate the area of the above shape, accurate to one decimal place. Explain your method.
(HINT: Use the formula for area of a circle to determine the area of the semi-circle.)

Area: Explanation:

Estimate the perimeter of the above shape by counting squares.

Calculate the perimeter of the above shape, accurate to one decimal place. Explain your method.
(HINT: Use the formula for circumference of a circle to determine the length of the curved part.)

Perimeter: Explanation: Composite Figures

Unit 1: Day 9: Pythagorean Theorem and Composite Figures

Grade 9 Applied

75 min.

Description

- Use the Pythagorean theorem to solve composite rectangle and triangle problems.

Materials

- Sets of right-angled triangles
- BLM D.1.5
- rulers

Assessment Opportunities

Minds On…

Small groups → Pencil and Paper

Students review how to calculate the area of the figure on BLM 1.D.5 using formulas. (Diagnostic Assessment)

As a Class

Quickly review the area formulas of rectangles, circles and triangles as students share their solutions. Discuss the appropriate use of units.
Using the same diagram, students calculate the perimeter of the figure using formulas. Some students will have difficulty applying the Pythagorean theorem to calculate the length of the hypotenuse, which leads into the lesson. Do not correct or discuss these solutions at this time.

Prepare sufficient different right-angled triangles so not all groups are working on the same triangles.

Constructing triangles out of different coloured cardstock allows the teacher to see quickly which measurements and calculations students should find.

Action!

Whole Class → Brainstorm/Discussion

Ask students to volunteer any information that they have about the Pythagorean theorem and record it on a mind map on the board or overhead.
Discuss key points:

- Pythagorean theorem works with right-angled triangles only.
- The hypotenuse is always opposite the right angle.
- The hypotenuse is the longest side.
- The square root key is handy for solving for side lengths whose square is not a perfect square.
- Pythagorean triples include: 3-4-5, 5-12-13, 8-15-17 and multiples of these triples.
- The Pythagorean theorem can sometimes be used to find lengths of sides in composite figures.

Help students identify the challenge in using formulas to calculate the perimeter of the figure. Guide students to use the Pythagorean Theorem to determine the length of the hypotenuse and eventually the perimeter of the figure. Stress appropriate units.

Pairs or Small Groups → Problem Solving

Have five right triangles of different colours and sizes constructed out of cardstock, with enough sets to ensure that there are 3 triangles for each group. Side lengths should be multiples of Pythagorean triples. Groups select 3 different triangles and measure two sides in order to calculate the third side. For the first triangle, students calculate the hypotenuse. For the second, students calculate the shortest side. For the third, students calculate the side of medium length. Each student completes the calculations in their notebook and groups conduct peer checks of solutions. Circulate and facilitate peer checks.

Consolidate Debrief

Individual → Problem Solving

Students calculate the perimeter of three or four composite figures that require the use of Pythagorean theorem. Circulate and assist students as necessary.

Curriculum Expectations/Paper and Pencil/Analytic Marking: Collect one solution from each student to mark.

Choose some questions that require only the Pythagorean theorem and some that involve composite figures.

Concept Practice

Home Activity or Further Classroom Consolidation

Complete assigned questions from the textbook.

Unit 1: Day 12: Boxed In

75 min.

Description

- Pose an authentic packaging problem that requires students to investigate the optimal surface area for a fixed volume.
- Use the inquiry process to rationalize optimal container size.
- Enter formula into spreadsheets or graphing calculators to facilitate calculations and data manipulations.

Materials

- linking cubes
- BLM 1.12.1
- placemats
- computers or graphing calculators

Assessment Opportunities

Minds On…

Small Groups → Placemat (Teacher TIP 9)

Each student in the group identifies within their placement section applications of three-dimensional geometry in the world. They can consider home, work, school and play. Provide examples, as needed. Students dialogue with their group to identify the applications that can involve *rectangular* prisms, and record these in the centre of the placemat.

Designate a scribe for each group.

Whole Class → Discussion

Groups share their applications of rectangular prisms.

Read through the instructions on BLM 1.12.1 with the student to clarify the task.

Discuss the use of an organized list to find possible combinations.

Action!

Triads → Guided Investigation

Students work in groups of three to determine the optimal packaging for the paint set and the friendship bracelet. Groups should reach a reasonable hypothesis before moving on to the second investigation.

Curriculum Expectations/Observation/Mental Note: Circulate and observe students as they complete the models, assessing how they apply their reasoning skills to optimize the surface area.

Students could use a spreadsheet or graphing calculator.

Note that repetition of dimensions will occur, e.g., 24 × 1 × 1 versus 1 × 24 × 1, producing identical surface areas.

Consolidate Debrief

Whole Class → Discussion

Students compare their final conclusions with their original hypotheses and indicate any revisions that they made giving a rationale. They compare their findings with others, identify reasons for the different conclusions, and justify their reasoning.

Pose additional problems.

Application
Exploration
Reflection

Home Activity or Further Classroom Consolidation

Identify two rectangular prisms in your home with similar volumes. Hypothesize which prism has the smaller surface area. Calculate the volume and surface area of both your prisms to confirm or refute your hypothesis

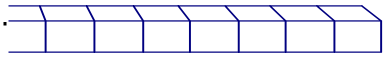
Discuss the students' responses during the next class. Clarify variations. Discuss the rationale for many prisms not being optimized for surface area.

1.12.1: Boxed In

You have been hired by Bottles, Boxes and Bows to design a case for an exclusive paint set. The box will be coated with 14 K gold so they wish to minimize the cost. The box is to have a volume of 24 cm^3.

Explore

Use linking cubes to build a long narrow box.
Estimate the surface area.
Build two more different boxes, using the cubes.
Compare the surface areas of the three boxes.



Hypothesize

Based on your exploration, predict the length, the width and the height of the paint case with the least surface area.

Model

Complete the table by adding four more possible combinations of length, width and height for the case.

Volume (cm^3)	Length (cm)	Width (cm)	Height (cm)	Surface Area (cm2) SA = $2lw + 2lh + 2wh$
24	1	1	24	2(1)(1)+2(1)(24)+2(1)(24) = 98
24	1	2	12	2(1)(2)+2(1)(12)+2(2)(12) =

Note the dimensions, length, width and height, as the surface area decreases. How do they appear to be related?

Revise your hypothesis, if necessary.

1.12.1: Boxed In (continued)

Bottles, Boxes and Bows wants you to construct a silver case to hold a friendship bracelet. The case is to have a volume of 8 cm^3. The case must have minimal surface area to reduce the production costs.

Explore

Use the linking cubes to create *three* different models for your case.

Hypothesize

Based on your exploration predict the length, the width, and the height of the case with the least surface area.

Model

Complete the table for the 3 possible combinations of length, width and height for the treasure case.

Volume (cm^3)	Length (cm)	Width (cm)	Height (cm)	Surface Area (cm^2) $SA = 2lw + 2lh + 2wh$
8				
8				
8				

Note the dimensions, length, width, and height as the surface area decreases. How do they appear to be related?

What model provides the least surface area? What shape is this?

Refer to the paint case. Using your findings for the bracelet case, try to find a lower surface area for the paint case by using dimensions which have decimal values for the volume of 24 cm^3. Add these values to your paint case chart.

What are the approximate dimensions of the paint case with the least surface area?

Conclude

What shape produces a minimum surface area for rectangular prisms of a fixed volume?

If Bottles, Boxes and Bows wants to create a larger box which has a volume of 64 cm^3 at a minimum cost, what recommendation about the dimensions of the boxes could you make?

75 min.

Description

- Pose a cylinder optimization problem involving a fixed volume and investigate the least expensive packaging.
- Use spreadsheet technology or graphing technology to allow students to focus on the investigative process.

Materials

- cans with similar volumes
- projection unit, computer, and spreadsheet software

Assessment Opportunities

Minds On…

Whole Class → Discussion

Show the students different sizes of cans, preferably with similar volumes. Ask why they think manufacturers use different sizes and shapes of cans. Answers may include references to packaging cans into larger shipping containers, least expensive for a given volume, design of label or stacking limitations. Students identify the can that they think represents minimal surface area for that volume.

Using a net or the GSP file, discuss how the formula for the surface area of a cylinder is derived. Students rearrange the volume of a cylinder formula to isolate *h*.

Point out that the surface area includes the top and bottom.

Action!

Pairs → Guided Investigation

Students work in pairs to complete BLM 1.14.1. Circulate and encourage pairs to share challenges and successes with other pairs.

Students may need guidance entering the correct formulae into the appropriate columns and rows of the spreadsheet.

Consolidate Debrief

Whole Class → Discussion

Students share strategies that they used to complete different portions of the investigations. They should think about how their findings relate to any other findings of this unit. Ask: Are there similarities in results when optimizing rectangles, rectangular prisms and cylinders?

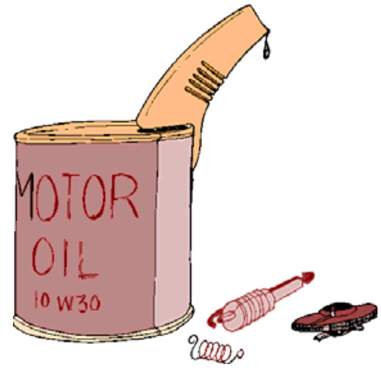
Learning Skills/Observation/Mental Note: Observe students as they engage in group discussion: Do they listen actively? Do they reflect on their work? Do they question their peers?

Application
Differentiated
Exploration
Reflection

Home Activity or Further Classroom Consolidation

Make a list of as many cylindrical containers as you can find at home. The list should contain at least five items. For each cylinder record how closely the dimensions approximate the optimal shape and state reasons why the dimensions may vary from one that would minimize the surface area and, therefore, the cost of materials.

1.14.1: Bottle Neck



You have been hired by Bottles, Boxes and Bows to design a small cylindrical container for motor oil. The container will be made out of aluminium and they wish to minimize the cost. The container is to have a volume of 314 cm^3.

Explore

Look at the various styles of cans. Which style appears to have the lowest surface area to volume ratio?

Hypothesize

Based on your exploration, predict the radius and the height of the container with the least surface area.

Model

Complete the missing height and surface area values.

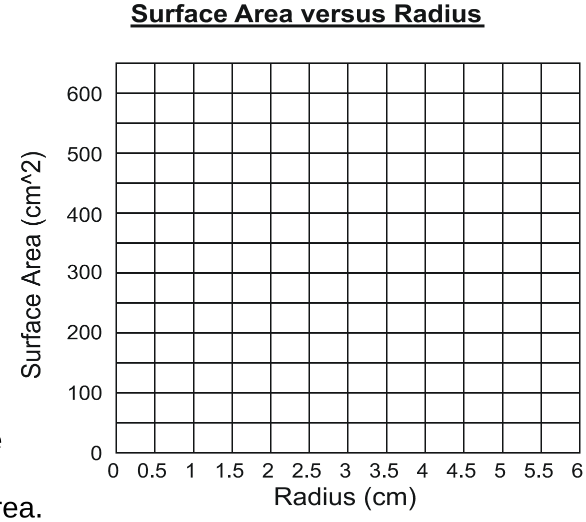
Volume (cm^3) $V = \pi r^2 h$	**Radius (cm)**	**Height (cm)** $h = \frac{314}{3.14r^2} = \frac{100}{r^2}$	**Surface Area (cm^2)** $SA = 2\pi r^2 + 2\pi rh$
314	1	100	$2(3.14)(1)^2+3(3.14)(1)(100) = 634.28$
314	1.5	44.44	$2(3.14)(1.5)^2+3(3.14)(1.5)(44.44) = 432.8$
314	2.0		
314	2.5		
314	3.0	11.11	
314	3.5	8.16	
314	4.0	6.25	
314	4.5	4.94	
314	5.0		
314	5.5	3.3	
314	6.0	2.78	

What happens to the surface area of the container as the radius increases?

1.14.1: Bottle Neck (continued)

Construct a scatter plot of *Surface Area* vs. *Radius*.

Circle the area on the graph where the minimum occurs.



Use a spreadsheet to determine the dimensions of the cylinder with the least surface area.
Choose values for the radius in the area you have circled on your graph.

Add four entries from the spreadsheet to your table for the motor oil can.
Highlight the entry that corresponds to the least surface area.

Conclude

If Bottles, Boxes and Bows wants to create cylindrical containers that cost the least to produce what dimensions would you suggest?

Extension

Boxes, Bottles and Bows pays $1.25/cm^2 for aluminium purchased from Mel's Metal. From your investigation, what is the minimum cost to produce one container? Show all of your calculations.

If Boxes, Bottles and Bows finds another supplier, Popeye's Scrap, which charges 85% of the cost of Mel's Metal then what is the minimum cost to produce one container?

75 min.

Description

- Compare the volume of a cylinder to a cone.
- Extend the comparison to prisms and pyramids with congruent bases and equal heights.
- Introduce the formula for finding the volume of a sphere.

Materials

- three-dimensional figures with equal base and height
- BLM 1.16.1 (overhead transparency)
- BLM 1.16.2

Assessment Opportunities

Minds On…

Think/Pair/Share → Problem Solving

Introduce the problem on BLM 1.16.1 on the overhead without showing the details. Students think about how they would determine if the cone will hold the ice cream as it melts and share their responses with a partner. They identify information that they require to solve the problem and any assumptions that they made.

Pairs share the process they would use; the information that they identified as needed; and any assumptions that they would make.

Share the details on the overhead. Students should recognize that they require the formula for volume of a sphere and that the relative size of the ice cream sphere and cone will affect their responses. Ask students to identify the meaning of the variables in the formula.

Students complete the calculations to determine if the ice cream will fit in the cone and share their reasoning with classmates. Ask for clarification, as appropriate.

Note: A cone and a hemisphere with equal radii and cone height equal to diameter have the same volume. Substituting the above equalities into the volume cone formula yields the corresponding volume of a hemisphere formula.

Action!

Whole Group → Exploration

Using a cylinder and a cone with equal radius and height, students identify which object has the larger volume. They predict how many times larger the volume of the cylinder is and compare the volumes of the two figures using sand, water or other pourable material. Students perform the task documenting the comparison each time. Discuss the potential for error in the activity.

Students identify other pairs of three dimensional figures that could have the same relationship. Confirm each hypothesis by repeating the pouring activity.

It is important to level the material in the cone (pyramid) before pouring.

Consolidate Debrief

Whole Class → Discussion

Establish that the volume of any prism can be calculated as V_{prism} = (Area of the Base)(Height).

Students establish a comparable formula for the volume of pyramids.

i.e., $V_{pyramid} = \frac{1}{3}$ (Area of the Base)(Height). Students share their reasoning.

Although cylinders are not classified as prisms, the relationship established is still maintained.

Individual → Journal

Students complete BLM 1.16.2 as a journal entry.

Curriculum Expectations/Pencil and Paper/Anecdotal: Collect the journal and provide feedback to students about their current understanding and areas of focus for them prior to the Summative assessment.

This is a good opportunity to highlight correct use of mathematical language.

Application
Concept Practice
Differentiated
Exploration
Reflection

Home Activity or Further Classroom Consolidation

Compile a list of composite three-dimensional figures in your environment, identifying the individual 3-D solids in each figure.

Examples could include most buildings, silos, rural mail boxes, door knobs, vases, light bulbs.

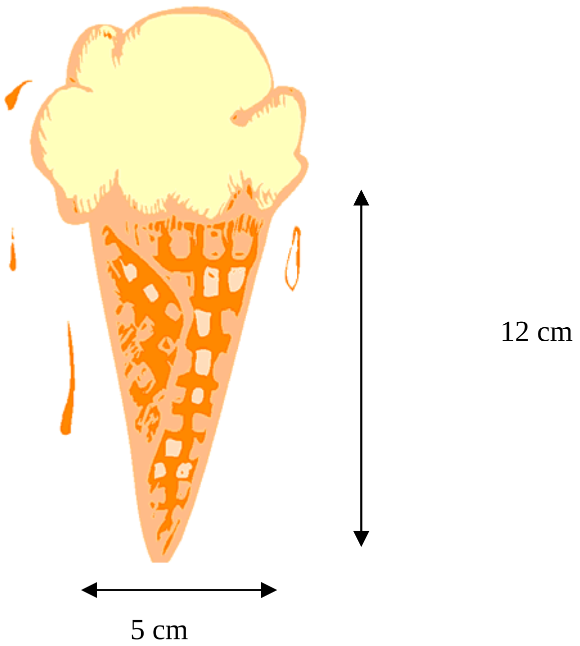
1.16.1 Jack and Jill

On a hot summer day, Jack bought an ice-cream cone for his friend Jill as a surprise!

As he headed to the park to meet her, the sun started melting the ice cream. Jack worried that the cone would not hold all the melted ice cream.

Help him do the math!

Some details:



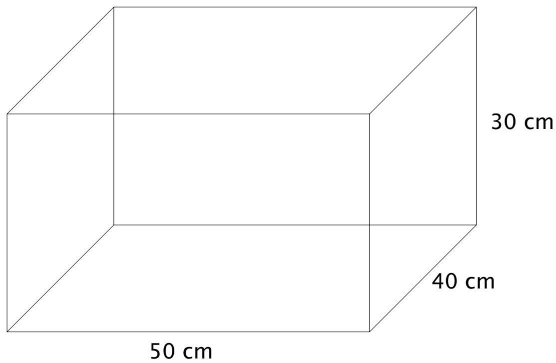
1.16.2: Journal Entry

Name:

Relationships in Measurement

A family pet is a dog named Emily Carr.

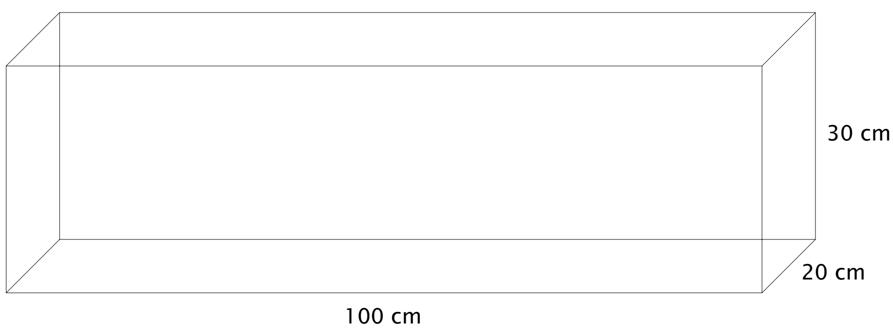
a) When Emily is cold she curls up into a ball to keep warm.
The shape that her body forms when she is curled can be modelled by the prism below.
Calculate the volume and the surface area of this prism.



V =

S.A. =

b) When Emily is hot she sprawls out to cool down.
The shape that her body forms can be modelled by the prism below. Calculate the volume and the surface area of this prism.



V =

S.A. =

c) Using what you have learned from this unit, explain why Emily curls up to keep warm and sprawls out to cool down.

d) Find one more occurrence in everyday life that can be explained using the relationship between surface area and volume. Explain your reasoning.

What Differentiating Instruction Is, and What It Is Not

Within an era of standards-based, international mathematics education reform *and* standardized provincial assessment, differentiated instruction represents a promising teaching method that may facilitate both high levels of student *engagement* and curricular *achievement*. Carol Ann Tomlinson, a leading proponent of differentiated instruction, maintains that these goals are indeed consistent:

> There is no contradiction between effective standards-based instruction and differentiation. Curriculum tells us *what* to teach: Differentiation tells us *how*. Thus, if we elect to teach a standards-based curriculum, differentiation simply suggests ways in which we can make that curriculum work best for varied learners. In other words, differentiation can show us how to teach the same standards to a range of learners by employing a variety of teaching and learning modes. (2000, pp. 8-9)

Differentiated instruction is based on the idea that because students differ significantly in their interests, learning styles, and readiness, teaching strategies and decisions involving issues of content, process, and product should vary accordingly (see Figure 1. Tomlinson's Differentiation of Instruction Model, 1999, p. 3). These ideas are in keeping with the National Council of Teachers of Mathematics' *Principles and Standards for School Mathematics* (2000), as explained by Pierce (2004):

> The tenets of differentiated instruction support both the Equity Principle and the Teaching Principle of the Principles and Standards for School Mathematics (NCTM, 2000). These principles direct us to select and adapt content and curricula to meet the interests, abilities, and learning styles of our students; to recognize our students' diversity; and to encourage them to reach their full potential in mathematics. (p. 60)

This concept echoes our own provincial curriculum mathematics documents:

> Teachers are responsible for developing a range of instructional strategies based on sound learning theory. They need to address different student needs and bring enthusiasm and a variety of teaching approaches to the classroom. Good teachers know that they must persevere and make every reasonable attempt to help all students develop their interests and abilities to the fullest extent. (Ontario Ministry of Education and Training, 1997, p. 4)

> Students in a mathematics class typically demonstrate diversity in the ways they best learn. It is important, therefore, that students have opportunities to learn in a variety of ways—individually, cooperatively, independently, with teacher direction, through hands-on experience, through examples followed by practice. …Because there is no single, correct way to teach or to learn mathematics, the nature of this curriculum demands that a variety of strategies be used in the classroom. (Ontario Ministry of Education and Training, 1999b, p. 8)

A multi-strategy approach to mathematics education from Grades 7-12 has been highlighted and elaborated upon within the messages and examples of both the resource document *Targeted Implementation and Planning Strategies: Grades 7, 8, 9 Applied Mathematics (TIPS)* (Consortium of Ontario School Boards, 2003) and the recently-released expert panel report on student success, *Leading Math Success: Mathematical Literacy, Grades 7-12* (Ontario Ministry of Education, 2004). TIPS says that, "Teachers must be observers of their students and make adjustments in their instruction and assessment on the basis of what they observe." (pp. 51-52)
To avoid possible misconceptions, it is helpful to first consider what differentiated instruction is, and what it is not.

Differentiating instruction does include:

- Using a variety of groupings to meet student needs;
- Providing alternative instruction/assessment activities;
- Challenging students at an appropriate level, in light of their readiness, interests, and learning profiles.

Differentiating instruction does not include:

- Doing something different for every student in the class;
- Disorderly and undisciplined student activity;
- Using groups that never change, or isolating struggling students within the class;
- Never engaging in whole-class activities with all students participating in the same endeavour.

Sometimes, Not Always

Much of the planning and preparation can be done before a teacher begins to teach a new class or course. Diagnostic and formative assessment tools, tiered assignment structures, flexible grouping strategies, etc., can all be carefully designed well in advance. This organization offers opportunities for teachers to focus on student learning and understanding during the early part of the course/year, and to effectively differentiate curriculum and activities. Differentiated instruction does not negate the need for activities in which all students are working on the same learning task at the same time, whether individually, in small groups, or as a whole class. However, within a differentiated framework, the teacher will frequently choose to assign different tasks to different students (individually or small groups) based on observations of the students in the class. For example, some students will require remediation activities while others may need extensions. With practice, the teacher can respond to such diverse needs on the spot.

Low-Prep Examples

There are aspects of differentiating instruction that require minimal preparation by the teacher. The teacher has different students work on different tasks using materials that are already available or using strategies that are reasonably easy to implement (e.g., increasing 'wait time' when posing questions to the whole class). In many cases, it may be for just a few minutes that different students work on these different tasks.

High-Prep Examples

Some aspects of differentiating instruction require significant preparation by the teacher—developing new instructional and assessment materials, and using strategies that depend on students' having acquired specific social skills, e.g., Jigsaw. When teachers build up their repertoires and share resource materials, high-prep examples become lower-prep.

Research and Literature Related to Differentiated Instruction

A review of the relevant research and literature regarding differentiated instruction reveals that while few empirical studies have yet been undertaken in this particular area, much is available for analysis by way of qualitative research and testimonial accounts of differentiated instruction in progress (Hall, 2003).

References

Consortium of Ontario School Boards (2003). *Targeted Implementation and Planning Supports: Grades 7, 8, and 9 Applied Mathematics*. Toronto, ON: Queen's Printer for Ontario.

Hall, T., N. Strangman, and A. Meyer (2003). *Differentiated Instruction and Implications for UDL Implementation*. National Center on Accessing the General Curriculum. Retrieved October 21, 2004, from http://www.cast.org/ncac/downloads/DI_UDL.pdf

Ontario Ministry of Education and Training (1997). *The Ontario Curriculum, Grades 1-8: Mathematics*. Toronto, ON: Queen's Printer for Ontario.

Ontario Ministry of Education and Training (1999b). *The Ontario Curriculum, Grades 9 & 10: Mathematics*. Toronto, ON: Queen's Printer for Ontario.

Ontario Ministry of Education (2004). *Leading Math Success - Mathematical Literacy, Grades 7-12: The Report of the Expert Panel on Student Success in Ontario.* Toronto, Ontario: Queen's Printer for Ontario.

Pierce, R.L. and C.M. Adams (2004). "Tiered Lessons: One Way To Differentiate Mathematics Instruction." *Gifted Child Today*, 27(2), pp. 58-66.

Tomlinson, C.A. (1999b). *The Differentiated Classroom: Responding to the Needs of All Learners*. Alexandria, VA: Association for Supervision and Curriculum Development.

Tomlinson, C.A. (2000). "Reconcilable Differences? Standards-Based Teaching and Differentiation." *Educational Leadership*, 58(1), pp. 6-11.

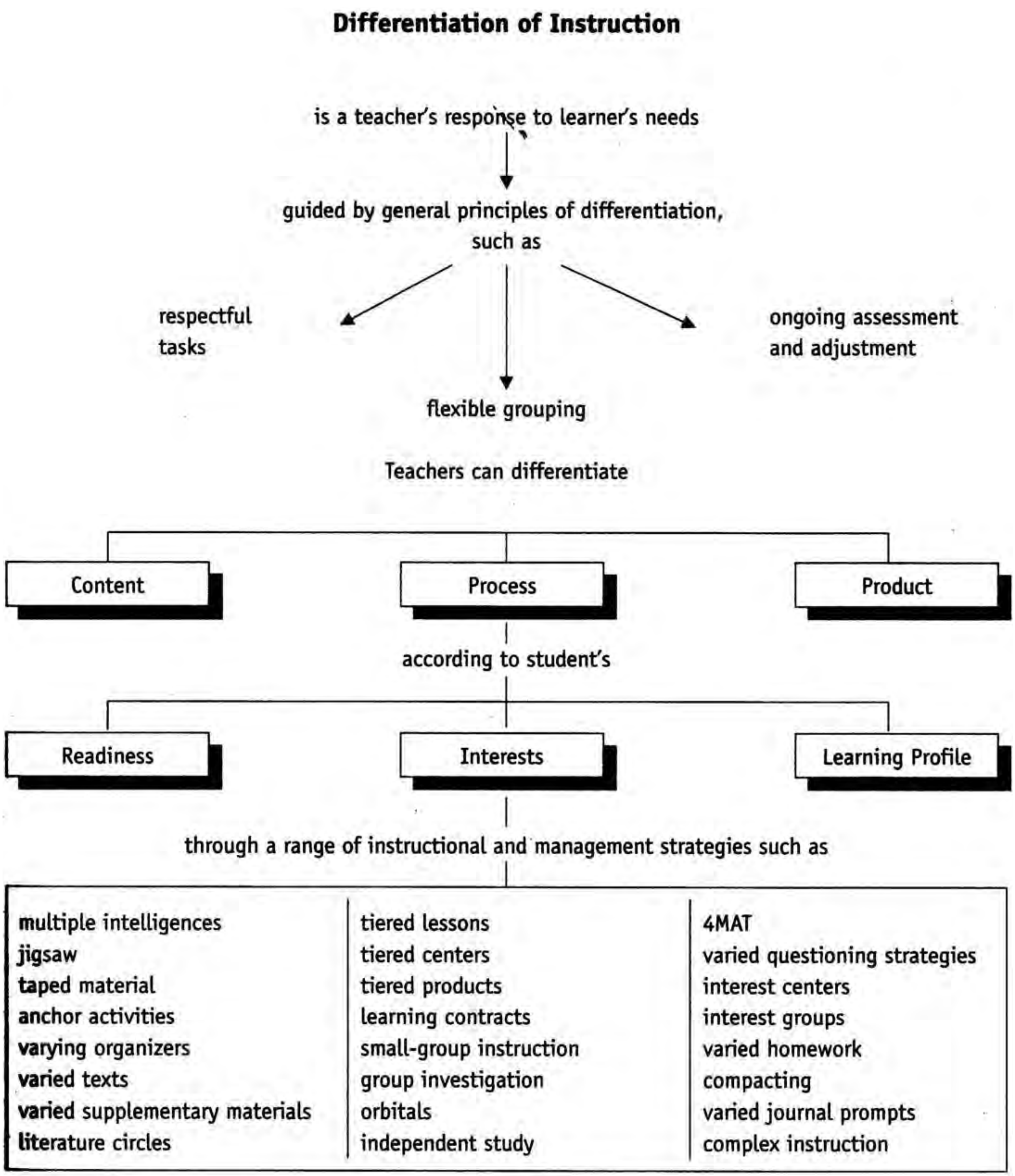


Figure 1. Tomlinson's Differentiation of Instruction Model (Tomlinson, 1999b, p. 15)

Differentiation Based on Student Readiness

> Equality in education does not require that all students have exactly the same experiences. Rather, education in a democracy promises that everyone will have an equal opportunity to actualize their potential, to learn as much as they can. (Fiedler, 2002, p. 111)

What is it?

Readiness is the current knowledge, understanding, and skill level a student has related to a particular sequence of learning. It is important to note that readiness is not merely a synonym for general ability level, but rather, "it reflects what a student knows, understands, and can do *today* in light of what the teacher is planning to teach *today*" (Tomlinson & Eidson, 2003, p. 3; emphasis added). Differentiating instruction based on student readiness involves knowing where particular students are on the learning continuum of important mathematics, then planning program features to move them along this continuum.

Why is it important?

Struggling students have to overcome many conceptual obstacles and are often not at the conceptual developmental level that their age might suggest. To help these students move forward, the teacher needs to determine the particular conceptual stumbling block(s); search out strategies to help the students overcome the learning obstacles; and provide the needed materials, time, and support that will help the students develop conceptual understanding with practice.

Differentiated instruction draws upon the work of Russian psychologist and educational theorist Lev Vygotsky. One of the theories that Vygotsky (1930/1978) developed was the Zone of Proximal Development (ZPD)—the difference between the learner's capacity to solve problems on his/her own, and his/her capacity to solve them with assistance. According to this theory, the teacher's role is to provide appropriate instructional scaffolding and non-intrusive, relational support in order to maximize student achievement within his/her ZPD. In differentiated instruction, teachers scaffold and tailor learning episodes to individual students' needs and understanding, while providing emotional support.

What are low-prep examples?

Select two sets of home or follow-up consolidation questions from the students' textbook where some questions are common to both sets: Target one set of questions for students who think they need more time to develop the concept; target the other set for students who think they are ready to apply the concept in new contexts.

What are high-prep examples?

Provide all students, including the struggling and the gifted, with challenging and different assignments; 'compact the curriculum' for advanced learners; and develop and complete learning contracts.

What are some considerations/cautions?

The recent resurgence of interest in differentiated instruction began with a focus on the needs of the gifted learner within a mixed-ability classroom (Fiedler, 2002; Winebrenner, 2000). However, it is significant to note that differentiation, when applied globally, reaps benefits for all students, including the gifted, low-achieving, special needs, and average, grade-level learners (Bender, 2002). Karp and Voltz note that special educators can be viewed as particularly valuable resources, in terms of their skills and experiences in weaving instructional strategies in inclusive settings (2000, p. 207).

Kapusnick and Hauslein (2001) explain how the brain requires appropriate levels of stimulation:

> When a student experiences a learning situation, the brain responds with the release of the chemical noradrenaline. Students who feel intimidated and rejected because their level of readiness is over-challenged experience an overproduction of noradrenaline, causing the brain to be over-stimulated. Attention is diverted from learning and focused on self-protection, resulting in misbehaviour or withdrawal, with more time being spent on learning to cope rather than learning concepts. Conversely, if student readiness is beyond what is needed for a particular task, the brain is, quite literally, not engaged, releasing fewer neurochemicals. The advanced student often feels apathetic because his or her brain is under-stimulated (Tomlinson, 1999). Diverse learning styles, interests, and abilities act as filters for student experiences, while emotional safety, challenges, and self-constructed meaning determine how students make sense of information. (2001, p. 156)

References

Bender, W.N. (2002). Differentiating Instruction For Students With Learning Disabilities: Best Teaching Practices For General And Special Educators. Thousand Oaks, CA: Corwin Press.

Fiedler, E.D., R.E. Lange, and S. Winebrenner (2002). "In Search Of Reality: Unraveling The Myths About Tracking, Ability Grouping, and the Gifted." *Roeper Review*, 24(3), pp. 108-111.

Kapusnick, R.A. and C.M. Hauslein (2001). "The 'Silver Cup' of Differentiated Instruction." *Kappa Delta Pi Record*, 37(4), p. 156.

Karp, K.S. and D.L. Voltz (2000). "Weaving Mathematical Instructional Strategies into Inclusive Settings." *Intervention in School and Clinic*, 35(4), pp. 206-215.

Tomlinson, C.A. and C.C. Eidson (2003). *Differentiation In Practice: A Resource Guide for Differentiating Curriculum, Grades 5-9*. Alexandria, VA: Association for Supervision and Curriculum Development.

Vygotsky, L. (1978). *Mind in Society: The Development Of Higher Psychological Processes*. Cambridge, MA: Harvard University Press. (Original work published 1930)

Winebrenner, S. (2000). "Gifted Students Need an Education, Too." *Educational Leadership*, 58(1), pp. 52-56.

Differentiation Based on Student Interest

> To differentiate instruction is to recognize students' varying background knowledge, readiness, language, preferences in learning and interests, and to react responsively. (Hall, 2003, pp. 2-3)

What is it?

Interest is that which a student enjoys learning about, thinking about, and doing. Differentiating instruction based on student interest involves the variation of themes, examples, and projects accordingly.

Why is it important?

Adolescents are undergoing tremendous personal changes during the middle years of schooling. Physical, emotional, and social changes make concentrating on classroom activities and taking responsibility for their own learning all the more difficult for the adolescent learner (*TIPS, Consortium of Ontario School Boards*, 2003, p. 26). In differentiating instruction according to students' interests, a teacher attempts to increase the likelihood that any given lesson or project is at once highly engaging and personally meaningful for each student in the class.

What is a low-prep example?

Use the names of students in the class when creating problems in contexts that appeal to these particular students; situate mathematics problems in contexts that will appeal to individual students in the class.

What is a high-prep example?

Prepare multiple versions of assignments that allow students to have choice and pursue their own interests.

What are some considerations/cautions?

Brain research is one of the most exciting areas of educational research being developed now at the beginning of the 21st century. As we discover more about how the human brain functions, we will be better able to design and tailor curriculum to take full advantage of these insights. Tomlinson and Kalbfleisch note:

> Brain research suggests three broad and interrelated principles that point clearly to the need for differentiated classrooms, that is, classrooms responsive to students' varying readiness levels, varying interests, and varying learning profiles: (i) Learning environments must feel emotionally safe for learning to take place; (ii) To learn, students must experience appropriate levels of challenge; and (iii) Each brain needs to make its own meaning of ideas and skills. (2001, pp. 53-54)

Safe learning environments, appropriate challenges, and meaning-making are indeed paramount if teachers are to be successful in using differentiated instruction to engage all learners.

References

Consortium of Ontario School Boards (2003). *Targeted Implementation and Planning Supports: Grades 7, 8, and 9 Applied Mathematics*. Toronto, ON: Queen's Printer for Ontario.

Hall, T., N. Strangman, and A. Meyer (2003). *Differentiated Instruction and Implications for UDL Implementation*. National Center on Accessing the General Curriculum. Retrieved October 21, 2004, from http://www.cast.org/ncac/downloads/DI_UDL.pdf

Tomlinson, C.A. and M.L. Kalbfleisch (1998). "Teach Me, Teach My Brain: A Call For Differentiated Classrooms." *Educational Leadership*, 56(3), pp. 52-55.

Differentiation Based on Student Learning Profile

> Individuals learn in different ways and at different rates; it is the major responsibility of schools to accommodate these differences to maximize each student's education. Rather than assuming that all students can and do learn in the same way and at the same rate, it is imperative for educators to acknowledge those differences.
> (Wang & Walberg, cited in Tieso, 2003, p. 34)

What is it?

A learning profile is a student's preferred mode of learning. Individual learning profiles are influenced by learning style, intelligence preference, gender, and culture (Tomlinson & Eidson, 2003). Differentiating instruction based on a student's profile requires that teachers: be aware of the demographics of their school population and classroom; think of understanding in terms of the multiple ways in which individual students learn best; and, ensure that classroom teaching and learning reflect this diversity in cognitive mechanics.

Why is it important?

Gardner's *Theory of Multiple Intelligences* (1993) emphasizes the unique learning capabilities and aptitudes of individual students. At its core is the validation of students' divergent talents and areas of expertise. Gardner has listed eight of these areas that he claims represent the complex workings of the human brain and how these complexities translate into thinking, learning, and doing in the classroom: verbal/linguistic; logical/mathematical; visual/spatial; musical/rhythmic; bodily/kinesthetic; interpersonal; intrapersonal; and naturalist. Learning styles are also important to consider when differentiating instruction—Do students prefer more auditory, visual, tactile, or kinesthetic modes of internalizing, processing, and communicating information?

Although this standard list of learning styles is perhaps most familiar, others have been considered and researched, including one recent project in mathematics education. Strong, Thomas, Perini, and Silver (2004, pp. 74-75), interested in helping mathematics teachers find the proper balance between unity of instructional purpose and models of differentiation, created a series of research workshops. Based on their findings, they presented the following four types of learning styles: **Mastery** (people in this category tend to work step-by-step); **Understanding** (tend to search for patterns, categories, and reasons); **Interpersonal** (tend to learn through conversation and personal relationship and association); and the **Self-Expressive** (tend to visualize/create images and pursue multiple strategies). These researchers draw the following conclusions:

> These different mathematical learning styles provide a map of cognitive diversity among mathematics students. Understanding these styles helps teachers address student strengths and weaknesses as learners. If teachers incorporate all four styles into a math unit, they will build in computation skills (Mastery), explanations and proofs (Understanding), collaboration and real-world application (Interpersonal), and non-routine problem solving (Self-Expressive). (pp. 74-75)

What are some low-prep examples?

Use pre-made TIPS lessons, developed to appeal to a wide range of learning profiles, and provide a balanced variety of activities across them.

What are some high-prep examples?

Prepare multiple versions of certain lessons/units to allow students with differing styles to explore multiple paths.

What are some considerations/cautions?

The Strong et al. (2004) learning styles mentioned above correspond well with the *Mathematical Processes* as discussed and modelled repeatedly within the *TIPS* (Consortium of Ontario School Boards, 2003) document: Knowledge; Reasoning and Proving; Making Connections; and Communicating, respectively. Students require support and challenge in areas of difficulty and strength.

References

Consortium of Ontario School Boards (2003). *Targeted Implementation and Planning Supports: Grades 7, 8, and 9 Applied Mathematics.* Toronto, ON: Queen's Printer for Ontario.

Gardner, H. (1993). *Frames of Mind: The Theory of Multiple Intelligences.* (10th ed.) New York: Basic Books.

Strong, R., E. Thomas, M. Perini, and H. Silver (2004). "Creating A Differentiated Mathematics Classroom." *Educational Leadership*, 61(5), pp. 73-78.

Tieso, C.L. (2003). "Ability Grouping Is Not Just Tracking Anymore." *Roeper Review*, 26(1), pp. 29-36.

Tomlinson, C.A., and C.C. Eidson (2003). *Differentiation In Practice: A Resource Guide for Differentiating Curriculum, Grades 5-9.* Alexandria, VA: Association for Supervision and Curriculum Development.

Differentiation Based on Mathematics Content

> It is sometimes necessary for a teacher to have different students in the class work on different content, e.g., Students in combined grade classes work on different curriculum expectations for the different grades; students in a regular classroom focus on different curriculum expectations based on readiness to move from concrete to abstract.

What is it?

Content is what students should know, understand, and be able to do as a result of a section of study. It is typically derived from a combination of sources: international standards; provincial curriculum documents; local curriculum guides and textbooks; and the teacher's knowledge of the subject and of pedagogy. Differentiating instruction based on mathematics content requires an understanding of multiple developmental continua and each student's placement along these various continua. Further, it calls for instructional repertoires and materials that help move each student along each separate continuum of mathematical learning and understanding.

Why is it important?

In Ontario elementary schools and secondary schools where teachers are assigned to combined grades or have repeating students, classrooms are often comprised of multi-age learners. Not entirely unlike the one-room school dynamics of the past, differentiated instruction provides contemporary educators with a powerful approach to meet the needs of multiage students and capitalize upon the unique benefits of combined configurations. One key strategy used to achieve these ends is that of flexible grouping.

> The flexible grouping strategies utilized by multiage classroom teachers allow opportunities for students to form small groups based on common interests and shared tasks. Peer learning takes place in structured, purposely planned instruction, as well as in less-structured situations that occur in the classroom every day as students are flexibly grouped for instruction. …A multiage classroom is an ideal environment for capitalizing on peer learning opportunities; in fact, a hallmark of multiage classrooms is their collaborative environments. (Hoffman, 2002, pp. 48-52)

> In differentiated classrooms, the teacher coordinates what students learn (curricular content) by engaging in such practices as providing the broad content framework; delineating between essential learning in which *all* students will be involved and areas of learning in which students can *choose* to be involved; considering the impact of varying student needs, abilities, strengths, and interests on the identified content areas; locating and securing a variety of resources; and, when appropriate, integrating the curricular content across subject areas. (Kronberg et al., 1997, p. 37)

What are some low-prep examples?

Use commercial materials that support a different content focus for combined grades. Use commercial diagnostic and formative assessment tasks to match students to materials developed to move students forward from various points along learning continua.

What are some high-prep examples?

Create custom-made materials to match different readiness levels, interests, and learning profiles. Use course compacting for high-level and gifted students while engaging them in whole-class learning activities on a regular basis. Modify curriculum according to students' IEPs and collaborate regularly with other teachers supporting struggling students in order to optimize student success on content.

What are some considerations/cautions?

Assessment and evaluation must be based on the provincial curriculum expectations and the achievement levels outlined in the curriculum documents. Some exceptional students and students who have not been identified as exceptional but who are receiving special education programs and services may need to have the curriculum expectations modified in keeping with their special needs. Such students may be provided with modified curriculum expectations (Ontario Ministry of Education Tradition, 1999a, pp. 24-25).

References

Hoffman, J. (2002). "Flexible Grouping Strategies In The Multiage Classroom." *Theory into Practice*, 41(4), p. 47.

Kronberg, R., J. York-Barr, K. Arnold, S. Gombos, S. Truex, B. Vallejo, and J. Stevenson. (1997). *Differentiated Teaching and Learning in Heterogeneous Classrooms: Strategies for Meeting the Needs of All Students*. Minneapolis, MN: Institute on Community Integration. (ERIC Document Reproduction Service No. ED418538)

Ontario Ministry of Education and Training (1999a). Ontario Secondary Schools, Grades 9-12: Program and Diploma Requirements. Toronto, ON: Queen's Printer for Ontario.

Differentiation Based on Process or Sense-Making Activity

> I soon realized that if I were willing to let go of ineffective practices refuted by research, I could find time for activities that allowed me to reach a wide range of students, representing a wide range of needs and abilities. (Cronin, 2003, p. 49)

What is it?

Process refers to the activities, both mental and physical, in which a student strives to make sense out of the new learning, whether it be procedural skill attainment or deeper conceptual understanding. Differentiating instruction based on process or sense-making activity involves the use of a combination of teaching strategies.

Processes can be differentiated by allowing students to learn in their preferred learning style. Mathematical *models* can be differentiated by teaching students how to work with alternatives such as numerical models, verbal descriptions, physical models, graphical organizers, scale drawings, and not-to-scale diagrams, as well as algebraic models (*Leading Math Success*, Ontario Ministry of Education, 2004). Working with these different types of models allows students to make sense of the mathematics in different ways.

Why is it important?

To teach mathematics using a wide variety of strategies, a generous mix of mathematical models and manipulatives, and with due deference to learning styles and multiple intelligences is to significantly increase the likelihood of high student comprehension—in terms of both the number of students affected and the level of individual student understanding.

One specific strategy for differentiating mathematics curriculum is the *Math Workshop* (Heuser, 2000). The format of the math workshop is similar to that of the writing workshop, consisting of a mini-lesson, an activity period, and reflection, with workshops in two varieties: teacher-directed and student-directed.

> During the activity period, children can follow their abilities and interests. Each period is a self-differentiated inquiry session…Like the teacher-directed variety, student-directed workshops end with a reflection period. Before they clean up their objects, students share their creations with others. (Heuser, 2000, p. 36)

What are some low-prep examples?

Use *TIPS* lessons developed with the MATCH (Minds On, Action, Time, Consolidation, Home Extension) framework (Consortium of Ontario School Boards, 2003); create a positive classroom environment in which students feel safe and encouraged to participate and take risks.

What are some high-prep examples?

When some students have not learned something important, reteach it with another strategy while other students work on extensions.

What are some considerations/cautions?

Using a textile metaphor, authors Karp and Voltz (2000) describe a creation and a caveat:

> As teachers learn and practice various teaching strategies, they expand the possibilities for weaving rich, authentic lessons that are responsive to all students' needs. ...The combination of subject matter knowledge, pedagogical knowledge, and knowledge of learner characteristics gives strength to the weave. Then as each individual student is considered, the pattern and texture of the cloth emerge. In many paradigms, the adherence to a single approach will create an instructional situation that will leave some students unravelled and on the fringe. (p. 212)

References

Consortium of Ontario School Boards (2003). *Targeted Implementation and Planning Supports: Grades 7, 8, and 9 Applied Mathematics*. Toronto, ON: Queen’s Printer for Ontario.

Cronin, M. K. (2003). “Rejecting Senseless Things: Promoting Differentiation.” *English Journal*, 92(4), pp. 47-53.

Heuser, D. (2000). “Reworking the Workshop for Math And Science.” *Educational Leadership*, 58(1), pp. 34-37.

Karp, K.S. and D.L. Voltz (2000). “Weaving Mathematical Instructional Strategies into Inclusive Settings.” *Intervention in School and Clinic,* 35(4), pp. 206-215.

Differentiation Based on Product

> In differentiated classrooms, assessment and instruction are interwoven and assessment is viewed as an ongoing process of feedback that occurs throughout a unit or course. (Kronberg et al., 1997, p. 53)

What is it?

A product is a means by which students demonstrate what they have come to know, understand, and be able to do. Products are often considered as major or culminating demonstrations of student learning. Differentiating instruction based on product involves the generation (by teacher and/or students) of various assessment tools and strategies that will be used to demonstrate new learning that has taken place in the lesson, unit, or course.

Why is it important?

The purpose of assessment is to improve student learning by providing feedback and generating data to inform and guide instruction. These ideas are described in detail in *Leading Math Success* (Ontario Ministry of Education, 2004):

> Multiple strategies—such as observations, portfolios, journals, rubrics, tests, projects, self-assessments, and peer assessments—tell students that the teacher appreciates their daily contributions and does not base evaluations solely on test results (Consortium of Ontario School Boards, 2003). Information gathered through assessment helps teachers determine students' strengths and weaknesses in achieving the curriculum expectations. As part of assessment, teachers provide students with descriptive feedback that guides their efforts to improve. …Assessment *for* learning puts the focus on using diagnostic assessment immediately before learning and formative assessment in the middle of learning to plan and adjust instruction. The emphasis moves from making judgments to coaching students and planning the next steps in teaching and learning. (pp. 33, 50)

When individuals are viewed in light of differentiated instruction, multiple forms of assessment are simply extended to pairs or groups of students, simultaneously. Not only can this make for a more meaningful and appropriate assessment experience for each student, but the classroom itself can become the repository of multiple exciting and stimulating projects, made only more so through presentations of student work to peers. Hall, Strangman, & Meyer (2003) describe some of the salient features of differentiated product assessment:

> Initial and on-going assessment of student readiness and growth are essential. Meaningful pre-assessment naturally leads to functional and successful differentiation. Incorporating pre- and ongoing assessment informs teachers so that they can better provide a menu of approaches, choices, and scaffolds for the varying needs, interests and abilities that exist in classrooms of diverse students. Students are active and responsible explorers. Teachers respect that each task put before the learner will be interesting, engaging, and accessible to essential understanding and skills. Each child should feel challenged most of the time. Vary expectations and requirements for student responses. Items to which students respond may be differentiated so that different students can demonstrate or express their knowledge and understanding in different ways. A well-designed student product allows varied means of expression and alternative procedures and offers varying degrees of difficulty, types of evaluation, and scoring. (pp. 3-4)

What is a low-prep example?

Spend one-on-one time with students who have not done well on a particular assessment task, giving them different ways to demonstrate their knowledge and skill.

What is a high-prep example?

Create a variety of project types that allow students to choose how they will demonstrate required knowledge and skills, e.g., photo essay, musical rap, poster display, performance task.

What are some considerations/cautions?

Clear expectations must be identified and communicated to students for *all* variations of assessment methods. Students and parents need to feel confident that all students in the class are being held to the same high standard.

References

Hall, T., N. Strangman, and A. Meyer (2003). Differentiated Instruction and Implications for UDL Implementation. *National Center on Accessing the General Curriculum.* Retrieved October 21, 2004, from http://www.cast.org/ncac/downloads/DI_UDL.pdf

Kronberg, R., J. York-Barr, K. Arnold, S. Gombos, S. Truex, B. Vallejo, and J. Stevenson (1997). *Differentiated Teaching and Learning in Heterogeneous Classrooms: Strategies for Meeting the Needs of All Students*. Minneapolis, MN: Institute on Community Integration. (ERIC Document Reproduction Service No. ED418538)

Ontario Ministry of Education (2004). Leading Math Success - Mathematical Literacy, Grades 7-12: The Report of the Expert Panel On Student Success in Ontario. Toronto, Ontario: Queen's Printer for Ontario.

Bibliography

Articles and Books Related to Differentiating Instruction

Bender, W.N. (2002). *Differentiating Instruction for Students with Learning Disabilities: Best Teaching Practices for General and Special Educators*. Thousand Oaks, CA: Corwin Press.

Consortium of Ontario School Boards (2003). *Targeted Implementation and Planning Supports: Grades 7, 8, and 9 Applied Mathematics*. Toronto, ON: Queen's Printer for Ontario.

Cronin, M. K. (2003). "Rejecting Senseless Things: Promoting Differentiation." *English Journal*, 92(4), pp. 47-53.

Davies, P. (2000). "Differentiation: Processing and Understanding in Teachers' Thinking and Practice." *Educational Studies*, 26(2), pp. 191-203.

Fiedler, E.D., R.E. Lange, and S. Winebrenner (2002). "In Search Of Reality: Unraveling the Myths about Tracking, Ability Grouping, and the Gifted." *Roeper Review*, 24(3), pp. 108-111.

Gardner, H. (1993). *Frames of Mind: The Theory of Multiple Intelligences.* (10th ed.) New York: Basic Books.

Goldberg, M.F. (2002). *15 School Questions and Discussion: From Class Size, Standards, and School Safety to Leadership and More*. Lanham, MD: Scarecrow Press.

Gregory, G. (2003). *Differentiated Instructional Strategies in Practice: Training, Implementation, and Supervision*. Thousand Oaks, CA: Corwin Press.

Gregory, G.H. and C. Chapman (2002). *Differentiated Instructional Strategies: One Size Doesn't Fit All*. Thousand Oaks, CA: Corwin Press.

Hall, T., N. Strangman, and A. Meyer. (2003). "Differentiated Instruction and Implications For UDL Implementation." *National Center on Accessing the General Curriculum*. Retrieved October 21, 2004, from http://www.cast.org/ncac/downloads/DI_UDL.pdf

Heuser, D. (2000). "Reworking the Workshop for Math and Science." *Educational Leadership*, 58(1), pp. 34-37.

Hoffman, J. (2002). "Flexible Grouping Strategies in the Multiage Classroom." *Theory into Practice*, 41(1), p. 47.

Holloway, J.H. (2000). "Preparing Teachers for Differentiated Instruction." *Educational Leadership*, 58(1), pp. 82-83.

Kapusnick, R.A and C.M. Hauslein (2001). "The 'Silver Cup' Of Differentiated Instruction." *Kappa Delta Pi Record*, 37(4), p. 156.

Karp, K.S. and D.L. Voltz (2000). "Weaving Mathematical Instructional Strategies into Inclusive Settings." *Intervention in School and Clinic*, 35(4), pp. 206-215.

Kronberg, R., J. York-Barr, K. Arnold, S. Gombos, S. Truex, B. Vallejo, and J. Stevenson (1997). *Differentiated Teaching and Learning in Heterogeneous Classrooms: Strategies for Meeting the Needs of All Students*. Minneapolis, MN: Institute on Community Integration. (ERIC Document Reproduction Service No. ED418538)

National Council of Teachers of Mathematics (2000). *Principles and Standards for School Mathematics.* Reston, VA.

National Research Center on the Gifted and Talented (2004). Hott linx: A link to differentiated curriculum. University of Virginia. Retrieved October 16, 2004, from http://www.hottlinx.org/

Nordlund, M. (2003). *Differentiated Instruction: Meeting The Educational Needs of All Students in Your Classroom*. Lanham, MD: Scarecrow Press.

Ontario Ministry of Education and Training (1997). *The Ontario Curriculum, Grades 1-8: Mathematics.* Toronto, ON: Queen's Printer for Ontario.

Ontario Ministry of Education and Training (1999a). *Ontario Secondary Schools, Grades 9-12: Program and Diploma Requirements*. Toronto, ON: Queen's Printer for Ontario.

Ontario Ministry of Education and Training (1999b). *The Ontario Curriculum, Grades 9 & 10: Mathematics*. Toronto, ON: Queen's Printer for Ontario.

Ontario Ministry of Education (2004). *Leading Math Success - Mathematical Literacy, Grades 7-12: The Report of The Expert Panel on Student Success in Ontario*. Toronto, Ontario: Queen's Printer for Ontario.

Page, S.W. (2000). "When Changes For The Gifted Spur Differentiation for All." *Educational Leadership*, 58(1), pp. 62-64.

Pettig, K.L. (2000). "On the Road to Differentiated Practice." *Educational Leadership,* 58(1), pp. 14-18.

Pierce, R.L. and C.M. Adams (2004). "Tiered Lessons: One Way to Differentiate Mathematics Instruction." *Gifted Child Today*, 27(2), pp. 58-66.

Reis, S.M., S.N. Kaplan, C.A. Tomlinson, K.L. Westbert, C.M. Callahan, and C.R. Cooper (1998). "A Response: Equal Does Not Mean Identical." *Educational Leadership,* 56(3), pp. 74-77.

Silver, H.E. and R.W. Strong (2003). *Learning Style Inventory*. Ho-Ho-Kus, NJ: Thoughtful Education Press.

Smutny, J.F. (2003). *Differentiated Instruction*. Bloomington, IN: Phi Delta Kappa Educational Foundation.

Smutny, J.F. and S.E. von Fremd (2004). *Differentiating for the Young Child: Teaching Strategies Across The Content Areas (K-3)*. Thousand Oaks, CA: Corwin Press.

Strong, R., E. Thomas, M. Perini, and H. Silver (2004). "Creating A Differentiated Mathematics Classroom." *Educational Leadership*, 61(5), pp. 73-78.

Tieso, C.L. (2003). "Ability Grouping Is Not Just Tracking Anymore." *Roeper Review*, 26(1), pp. 29-36.

Tomlinson, C.A. (1999a). "Leadership for a Differentiated Classroom." *School Administrator,* 56(9), p. 11.

Tomlinson, C.A. (1999b). *The Differentiated Classroom: Responding to the Needs of All Learners*. Alexandria, VA: Association for Supervision and Curriculum Development.

Tomlinson, C.A. (2000a). *Differentiation of Instruction in the Elementary Grades*. Champaign, IL: ERIC Clearinghouse on Elementary and Early Childhood Education. (ERIC Document Reproduction Service No. 443572)

Tomlinson, C.A. (2000b). "Reconcilable Differences? Standards-Based Teaching and Differentiation." *Educational Leadership*, 58(1), 6-11.

Tomlinson, C.A. (2001). *How to Differentiate Instruction in Mixed-Ability Classrooms*. (2nd ed.) Alexandria, VA: Association for Supervision and Curriculum Development.

Tomlinson, C.A. (2003). *Fulfilling the Promise of the Differentiated Classroom: Strategies and Tools for Responsive Teaching*. Alexandria, VA: Association for Supervision and Curriculum Development.

Tomlinson, C.A. (2004a). "Differentiation in Diverse Settings." *School Administrator,* 61(7), pp. 28-30.

Tomlinson, C.A. (2004b). "Research Evidence for Differentiation." *School Administrator*, 61(7), p. 30.

Tomlinson, C.A. (2004c). "Sharing Responsibility for Differentiating Instruction." *Roeper Review*, 26(4), pp. 188-189.

Tomlinson, C.A. and S.D. Allan (2000). *Leadership for Differentiating Schools and Classrooms*. Alexandria, VA: Association for Supervision and Curriculum Development.

Tomlinson, C.A. and C.C. Eidson (2003). *Differentiation In Practice: A Resource Guide For Differentiating Curriculum, Grades 5-9*. Alexandria, VA: Association for Supervision and Curriculum Development.

Tomlinson, C.A. and M.L. Kalbfleisch (1998). "Teach Me, Teach My Brain: A Call for Differentiated Classrooms." *Educational Leadership*, 56(3), pp. 52-55.

Vygotsky, L. (1978). *Mind In Society: The Development of Higher Psychological Processes*. Cambridge, MA: Harvard University Press. (Original work published 1930)

Winebrenner, S. (2000). "Gifted Students Need an Education, Too." *Educational Leadership,* 58(1), pp. 52-56.

Mathematics Education Resource Books

Bennett, B. and C. Rolheiser (2001). *Beyond Monet: The Artful Science of Instructional Integration*. Toronto, ON: Bookation.

Colgan, L. (2003). *Elementary Mathematics in Canada: Research Summary and Classroom Implications*. Toronto, ON: Pearson Education Canada.

Doctorow, R. (2002). *Mathematics Education: A Summary of Research, Theories, and Practice*. Scarborough, ON: Thomson Nelson.

Erlauer, L. (2003). *The Brain-Compatible Classroom: Using What We Know about Learning to Improve Teaching*. Alexandria, VA: Association for Supervision and Curriculum Development.

Flewelling, G. and W. Higginson (2000). *A Handbook on Rich Learning Tasks*. Kingston, ON: Queen's University.

Haylock D. and M. D'Eon (1999). *Helping Low Achievers Succeed at Mathematics: Grades 2-8*. Toronto, ON: Trifolium Books.

Kilpatrick, J., W.G. Martin, and D. Schifter, eds. (2003). *A Research Companion to Principles and Standards for School Mathematics.* Reston, VA: National Council of Teachers of Mathematics.

Kilpatrick, J., J. Swafford, and B. Findell (Eds.) (2001). *Adding it Up: Helping Children Learn Mathematics*. Washington, DC: National Academy Press.

Mirra, A.J. (2003). *Administrator's Guide: How to Support and Improve Mathematics Education in Your School*. Reston, VA: National Council of Teachers of Mathematics.

Reys, R., M. Lindquist, D. Lambdin, N. Smith, and M. Suydam (2003). *Helping Children Learn Mathematics.* (7th ed.) Hoboken, NJ: Wiley.

Shellard, E., and P.S. Moyer (2002). *What Principals Need to Know about Teaching Mathematics*. Alexandria, VA: National Association of Elementary School Principals, Educational Research Services.

Sutton, J., and A. Krueger, eds. (2002). *EDThoughts: What We Know About Mathematics Teaching and Learning*. Aurora, CO: Mid-continent Research for Education and Learning.

Van de Walle, J.A. (2004). *Elementary and Middle School Mathematics: Teaching Developmentally.* (5th ed.) New York: Pearson.

