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Messy but meaningful: exploring the transition to reform-based pedagogy with teachers of mathematics and coordinators in Ontario, Canada

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The RE4MUL8 Project involved the creation of an online/mobile resource for Intermediate Division (Grade 7 and 8) teachers of mathematics. This resource showcases video documentaries of seven key mathematics topic lessons (fractions, integers, proportional reasoning, composite shapes and solids, solving equations, and, patterning and algebraic thinking), as delivered by seven teachers in Ontario, Canada who were nominated by their respective District School Boards as being, or becoming, highly effective practitioners in the area of reform-based mathematics education. As part of a qualitative case study research design, these teachers, often along with their math coordinators, were then interviewed following the lesson, and shared reflections on the lesson itself and, more generally, on their ongoing journey towards reform-based mathematics teaching. This paper reports on three major themes that emerged from these discussions, namely, problem-based learning, the reality and necessity of 'messy time' transition to reform-based pedagogy, and, balancing instructional planning and practices.

Keywords: mathematics; education; problem-based learning; technology; professional learning; transition; change; reformulate

1. Problem-based and technology-enhanced mathematics learning

Reform-based mathematics – as researched, communicated, and championed by organizations such as the National Council of Teachers of Mathematics (NCTM) – is often characterized as featuring the following elements: the implementation of problem-based learning (PBL), cooperative groupwork strategies, the use of manipulatives, technology-rich classrooms, and varied assessment (Jarvis and Franks 2011).

PBL was originally developed by a medical professor, Dr. Howard Barrows, at McMaster University in the 1970s (Barrows and Tamblyn 1980). According to Delisle (1997), Barrows was deeply influenced by educational philosopher and researcher John Dewey, who, decades earlier, had promoted a hands-on, experiential approach to schooling in his Chicago school laboratory. PBL, although not synonymous with 'problem solving' in the literature, often features overlapping principles or characteristics (e.g. learning is based on challenging, open-ended problems; students often work in collaborative groups; teachers often assume the role of 'facilitator' of learning). Edwards and Hammer (2006) argued that PBL can be more readily

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described as a general approach to learning, as opposed to an actual teaching technique:

> The philosophical underpinnings of PBL hold at its core those principles of learning articulated by the constructivist and social-constructivist views of learning and cognitive development. … Here the belief that learning is actively constructed by learners as they interact and engage with other learners (and/or more competent peers) is manifest. In addition, the exploratory nature of PBL is likewise consonant with the constructivist belief that learning occurs most readily when it is supported by opportunities for learners to engage with ideas and materials. (466)

Beyond its social-constructivist roots and hands-on focus, PBL has more recently also been connected with the need for self-regulated learning, as described by English and Kitsantas (2013):

> [T]o effectively engage in PBL, students must become responsible for their learning and actively participate in the processes of constructing knowledge and making meaning. … For many students, this role conflicts with deeply ingrained habits they have developed through more familiar classroom experiences, in which they have been passive recipients of knowledge. … In order for the potential of student-centered, inquiry-based approaches to be realized, students must make the shift to their new role as active learners and develop self-regulated learning skills. (129)

As in the case of Barrows' work, which focused on the preparation of adult medical students using PBL, teacher preparation should also include problem-based mathematical experiences in order to best prepare new teachers in using PBL strategies in their beginning practice (Pourshafie and Murray-Harvey 2013). Problem solving has certainly received an increasing amount of attention within the reform-oriented mathematics education agenda. For example, the NCTM strongly argued for a problem solving focus in its *Principles and Standards for School Mathematics* (2000) document:

> Solving problems is not only a goal of learning mathematics but also a major means of doing so. Problem solving is an integral part of all mathematics learning, and so it should not be an isolated part of the mathematics program. Problem solving in mathematics should involve all the five content areas. … Good problems will integrate multiple topics and will involve significant mathematics. (52)

Further, the *Ontario Curriculum: Mathematics, Revised* (OME 2005a, 2005b, 2007) lists Problem Solving first among its seven core Mathematical Processes.[1] It has been proposed that all seven of these processes (i.e. problem solving, reasoning and proving, reflecting, selecting tools and computational strategies, connecting, representing, and communicating) can be meaningfully practiced within a classroom where PBL is regularly being implemented (Jarvis 2008; MacMath, Wallace, and Chi 2009). According to Van de Walle and Folk (2008), PBL is best conducted with a '3-part lesson,' i.e. the introduction of the activity (verbally and/or via demonstration), the exploration/activity itself (which may occur individually, or with a partner, or, more commonly, in small groups), and the whole-class debriefing/discussion that follows.

In describing her '10 big math ideas,' author Marilyn Burns (2004) noted that 'confusion is part of the process' and that we as teachers should be continually attempting to 'encourage different ways of thinking' (19). In discussing learning contexts that are highly engaging *and* satisfying for students, Seymour Papert (the renowned MIT mathematician, computer scientist, and educator who was one of the first pioneers of artificial intelligence, and the inventor of the Logo programming language) introduced the intriguing notion of ‘hard fun’:

> I have no doubt that this kid called the work fun *because* it was hard rather than *in spite* of being hard. Once I was alerted to the concept of ‘hard fun’ I began listening for it and heard it over and over. It is expressed in many different ways, all of which all boil down to the conclusion that everyone likes hard challenging things to do. But they have to be the right things matched to the individual and to the culture of the times. (Papert 1998, 88)

Canadian author Marian Small (2013), another strong advocate for problem-based mathematics learning, notes that ‘some of the ways teaching through problem solving differs from a traditional approach include: an increased level of mathematical dialogue between students; the teacher’s role as guide or coach more than as a presenter; and, the teacher’s more judicious use of intervention’ (38).

Finally, Michael Fullan (2013) in his role as Special Advisor to the Premier of Ontario, provided six key components for twenty-first century learning in his report entitled, *Great to Excellent: Launching the Next Stage of Ontario’s Education Agenda.* Although his list applies to education broadly speaking, note carefully that three of his ‘Cs’ lie at the heart of problem-based mathematics learning (communication, critical thinking, collaboration):

- Character education: honesty, self-regulation and responsibility, perseverance, empathy for contributing to the safety and benefit of others, self-confidence, personal health and well-being, career and life skills.
- Citizenship: global knowledge, sensitivity to and respect for other cultures, active involvement in addressing issues of human and environmental sustainability.
- Communication: communicate effectively orally, in writing and with a variety of digital tools; listening skills.
- Critical thinking and problem solving: think critically to design and manage projects, solve problems, make effective decisions using a variety of digital tools and resources.
- Collaboration – work in teams, learn from and contribute to the learning of others, social networking skills, empathy in working with diverse others.
- Creativity and imagination – economic and social entrepreneurialism, considering and pursuing novel ideas, and leadership for action.

Teachers who have adopted problem solving or PBL have also begun to implement *newly available technologies* into their classroom repertoires. With the advent of new technologies and powerful software packages (e.g. highspeed Internet, social media, digital cameras, smartphones, mobile devices) – many of which are now freely available via the Internet – and the rapidly growing use of these tools internationally, as a researcher I grew increasingly interested in how these pervasive tools were being used, or how they might be used more effectively by teachers, in terms of supporting student mathematics learning, and enhancing their own professional growth as educators. To that end, I determined to design a research project that would allow me to create a video-based teacher resource that would serve to document exemplary (yet realistic) reform-based classroom practices at the Grade 7 and 8 level; would capitalize on mobile options which are now so much a part of many teachers' and students' lives (e.g. smartphones/tablets); and, would include the use of popular social networking tools *Twitter* and *Facebook*, as a means to update teachers regarding math-related news/resource items.

Much research is presently being done around mobile technologies in classrooms, and related effects on students' learning and engagement (Franklin and Peng 2008; Jarvis and Franks 2012; Keengwe, Pearson, and Smart 2009; Shuler 2009). I was particularly interested in the use of such technologies by *teachers*, as pertaining to the expansion of their own vision for practice (i.e. professional learning), in light of potential gains in student learning.

2. Research context

To 'reformulate' is a late nineteenth-century word meaning to formulate again, or in a different way; to alter or revise; to reformulate ideas or plans (Pearsall 2001, 1560). I choose this title in a revised format, 'RE4MUL8,' in keeping with the overarching theme of changing teacher beliefs and practice, as well as wanting to engage with the generation of 'digital natives' (Prensky 2001), or even their somewhat older 'digital immigrant' colleagues, all of whom would be familiar with texting and the use of such substitutions in shortened titles.

Abrahamson (2004) differentiated between what he called 'creative destruction' and 'creative recombination,' the latter term referring to the repeated use of existing quality initiatives and projects in new configurations, as opposed to the complete abandoning of former ideas simply for the sake of change. The development of a Grade 7 and 8 mathematics video teaching database, drawing upon the reform-oriented lessons found within the *Targeted Implementation and Planning Supports* (Consortium of Ontario School Boards 2005) document (i.e. video-taping instructors implementing standards-based lessons) served to likewise 'creatively recombine' these existing quality resources which had already been used extensively in the field throughout Ontario. The *Curriculum Implementation in Intermediate Math: Research Report* (Suurtamm and Graves 2007), based on focus group interviews, online surveys, and several case studies, offered clear recommendations for further research and development:

> Teachers' responses to the questionnaire also suggest that there is a desire for further professional development in areas connected to new ways of teaching mathematics. … [T]here are specific characteristics of professional development that emerge as very important in supporting changes in classroom practice. One pertains to the importance of dialogue with colleagues and opportunities for collegial collaboration. … Another … was the effectiveness of those professional development initiatives that were connected to classroom practice and sustained over time. (24–27)

Teachers require *voice*, *choice*, and *ongoing support* (i.e. accessibility of resources and training) in terms of professional learning opportunities (Jarvis 2006). The online video database of teacher lessons and interviews/testimonials allowed Intermediate Division mathematics teachers to view reform-based lesson video clips either using an Internet-enabled computer or with mobile devices (smartphones, tablets, laptop technologies), and to receive math-related news and resource updates via popular social networking media (e.g. Twitter, Facebook).

3. Research question, methodology, and methods

The research questions that were being addressed in this research project were the following: (i) What are the perceived challenges and enablers (affordances, benefits) of a reform-based approach to mathematics teaching as shared by Intermediate Division teachers of mathematics?;[2] and (ii) What are the perceived challenges and enablers (affordances, benefits) of a reform-based approach to mathematics teaching as shared by mathematics coordinators via their own observations of, and ongoing classroom and professional development experiences with, teachers?

Unlike quantitative educational research, where the goal is to test an hypothesis, and then if the results are found to be statistically significant, to be able to generalize findings to a larger population, the purpose of qualitative educational research is to understand more deeply a particular individual, group, or educational phenomenon. To do this, qualitative researchers use methods that often involve interviews, observations, artifact analysis, and other data gathering techniques and tools. For this particular research project, a qualitative case study was adopted. Stake (1995), in his text *The Art of Case Study Research*, provides the following description of this type of research:

> A case study is expected to catch the complexity of a single case. … We study a case when it itself is of very special interest. We look for the detail of interaction with its contexts. Case study is the study of the particularity and complexity of a single case, coming to understand its activity within important circumstances. (xi)

Creswell (2013) defines case study research as research within a 'bounded system' (97) involving 'interrelated parts that form a whole' (294) designed to present an in-depth understanding of the case. Yin (2009) notes, 'A case study is an empirical inquiry that investigates a contemporary phenomenon in depth and within its real-life context, especially when the boundaries between phenomenon and context are not clearly evident' (18). The bounded case in this particular research study was the group of seven Intermediate Division teachers of mathematics, along with their respective board-level mathematics coordinators who, at most of the school sites, also took part in the mathematics lessons and subsequent interviews.

The purpose of a case study is to 'provide a detailed account of one or more cases' (Johnson and Christensen 2012), and it can be used to address exploratory, descriptive, and explanatory research questions (49). There are typically three types of case study research: (i) intrinsic case studies – in which the research seeks to understand a specific case; (ii) instrumental case studies – in which the research is seeking to understand something beyond the case study (more universal application of knowledge); and (iii) collective case study – in which the research seeks to gain deeper understanding by studying multiple case studies (396–397). The RE4MUL8 Project case study was arguably instrumental in nature, in that it sought to understand commonly experienced challenges and affordances, as perceived by Grade 7 and 8 teachers of mathematics, as they adopted and implemented reform-based teaching and learning practices.

In each of the participating District School Boards, I asked the mathematics coordinator, or equivalent, to nominate an effective (i.e. using, or moving towards, a reform-based practice as per the vision and Mathematical Process Expectations of the existing provincial curriculum) (OME 2005a) Grade 7 or 8 teacher who might be interested in taking part in this video-based project. Five of the six boards put forward one such teacher name, and in a sixth board, two co-teaching instructors were both nominated. These seven teachers all agreed to take part in the study, and after signing Letters of Informed Consent were then asked to prepare and deliver a reform-based lesson tied to one of the *Targeted Implementation and Planning Supports for Revised Mathematics* (Consortium of Ontario School Boards 2005) topics (i.e. Fractions; Integers; Patterning to Algebraic Modeling; Perimeter, Area, and Volume; Proportional Reasoning; and, Solving Equations and Using Variables as Place Holders). Participating teachers also agreed to take part in a post-lesson debriefing interview (often with the coordinator if s/he was in attendance and wanting to participate) that was audio- and video-recorded.

These audio/video-taped interviews were then transcribed, using member checking to allow all participants to read the completed transcript and to offer any corrections or modifications. Transcripts were then analyzed using qualitative research software known as *Atlas.ti*, through the use of Thematic Analysis methods, i.e. familiarization with data, generating initial codes, searching for emergent themes among codes, reviewing themes, defining and naming themes, and producing the final report. Three major themes based on teacher/coordinator perceptions emerged from the analyzed data-set: (i) characteristics of PBL in mathematics classes; (ii) the 'messy time' transition experienced by teachers who are en route to reform-based practice; and, (iii) the realities of trying to 'balance instruction' (i.e. to incorporate both PBL and more traditional strategies) within a twenty-first century mathematics classroom.

The online version of the resource was created using an 'edublogs' website (see Figure 1), which would be tied to a registered Canadian domain name (www.re4mul8.ca) for ease of access and searching.

From the home page, the RE4MUL8 resource links out separately to the seven math lesson videos, the seven interview videos, a series of emergent theme videos, a set of prepared group questions for teachers watching the online videos (i.e. for use in Professional Development sessions), two related *Ontario Mathematics Gazette* articles, my own MATHeMATRIX instructional resource (several hundred math-related links), the Ontario Ministry of Education (OME) 'MathGAINS' home page, and information about the Schulich STEM Initiatives Research funding/sponsor.

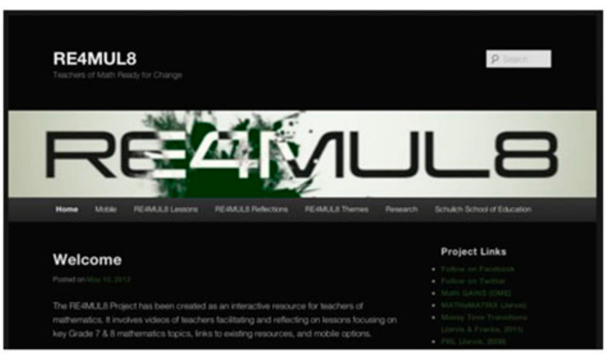

Figure 1. RE4MUL8 Project website (www.re4mul8.ca) featuring videos and related resources.

4. Research findings

A full recounting of the background context for the RE4MUL8 resource, and the process involved in creating this online/mobile resource has been submitted for publication elsewhere (Jarvis 2012). The remainder of this paper will deal specifically with the informative commentary shared by teachers and coordinators within the scheduled interviews, that followed the math lessons proper. In particular, we will look carefully at three major emergent themes based on teacher/coordinator perceptions: (i) characteristics of PBL activities in mathematics classes; (ii) the 'messy time' transition experienced by teachers who are en route to reform-based practice; and, (iii) the realities of trying to 'balance instruction' within a twenty-first century mathematics classroom.

4.1. Problem-based learning

In the RE4MUL8 Project, teachers and coordinators shared valuable insights regarding their perceptions of problem-based learning (PBL, as already described in Section 1 above), particularly when asked if they felt that this method of learning mathematics actually had any effect on a student's attitudes towards mathematics, and/or a student's ability to understand the math content more deeply and to apply it more readily in multiple contexts (non-identical questions). But first some comments regarding the actual characteristics of PBL in classrooms, as perceived by the study participants. What does a PBL lesson look like, in terms of teacher/coordinator perceptions?

4.1.1. Mathematical processes

The *Ontario Revised Curriculum: Mathematics* (OME 2005a, 2005b, 2007) lists Problem Solving first among its seven core Mathematical Processes. It has been proposed that all seven of these processes (i.e. problem solving, reasoning and proving, reflecting, selecting tools and computational strategies, connecting, representing, and communicating) can be meaningfully practiced within a classroom where PBL is regularly being implemented (Jarvis 2008). Ann, a Math Coach, reinforced this idea as she described her class:

> Ann: I've been using problem-based teaching for many, many years and that's the approach, that's what I'm promoting. When I open my curriculum document and I scan the Processes, spelled out, I know that when I'm doing rich problems in my classroom, whether I have one problem or 'parallel task' problems, I know that my students are going to mainly become engaged, I'm going to encourage these manipulatives, they're going to be discussing, defending, reflecting, selecting tools and strategies, all those rich, rich higher-level thinking types of skills they need to move on in math in the higher Grades.

The Mathematical Process expectations of 'selecting tools', 'representing', and 'communicating' lead one to assume that manipulatives, technology, and cooperative groupwork would likely form part of a PBL approach.

4.1.2. Manipulatives, technology, and cooperative groupwork

One common characteristic of a PBL classroom is the existence and regular use of manipulatives, or learning objects that students can 'manipulate' in representing or modeling mathematical problems. Richard, a Secondary Curriculum Consultant, highlights different school scenarios in terms of how manipulatives are commonly stored and shared.

> Richard: That [manipulative storage/use] really varies school to school. You'll go into some schools and they'll have a big central area for all of their grades. Some will have it by division grade, some – there'll be one teacher who uses them a lot and has them in their room, and others – people will come and get them. Some, it's whatever they prefer. There's the occasional school where there's a dusty shelf somewhere where they can be found on.

Jason, a relatively new teacher in a Grade 8 classroom, shared how manipulatives had become a mainstay of his daily routine during the mathematics period. In his particular case, an experienced teaching colleague (with coordinator experience) who was on leave had left him a generous supply of manipulatives in the classroom, and as he began to use them, Jason quickly saw the value in making them a common and accessible part of math learning.

> Jason: I'm definitely trying to use a lot more manipulatives. … The problem-based approach – it's interesting, it's not always a problem that's the focus of the lesson, but I think that I always try to do some sort of an activity within the lesson. … If it's not in the room, then you're not going to use it. … I definitely understand why some schools have a central storage area for manipulatives, and I'm not going to say that there's anything wrong with that, but if you can get it into the room where you're starting to explain a concept and I can grab it and just take it and show it, there's so much power in that.

In terms of technology use, one of the most popular items that was noted in the participating classrooms was that of the Interactive Whiteboard (IWB). During the interviews, a number of the teachers noted that since they had become accustomed to using the IWB for mathematics and other topics/resources, they were not sure that they could even imagine teaching without this particular technology present.

> Jodi: This is my second year with the SMART Board. … I love it, I honestly can't imagine not having it now that I've had it for the last few years. The versatility with it, and the engagement – specifically the boys – asking them a question, if it's done on the SMART Board, they want to come up and give it a try.

According to what was observed in the six locations, all PBL tended to take place in paired or small-group seating arrangements. According to Kathy, a Grade 8 teacher, the predominant use of small groups, with regular rotations, was a natural part of the move towards reform-based teaching:

> Kathy: We're always in groups. … So, the tables just provide for a better use of the space than rows. And then we always make sure that we have single desks around so that if you have people who are distracted from their learning and need to be removed from the group, you have an approach to do that. We move groups about once a month for the regular home-based stuff so that you're not always stuck with the same four people that you're sitting with.

Problem-based classrooms in this study were found to exhibit all of the common elements as portrayed in the literature, i.e. group seating for collaboration, problem-based explorations, readily available manipulatives, and instructional technologies. It is to the perceived effectiveness of these resources and classroom arrangements and strategies on student engagement and mathematical understanding that we now turn.

4.1.3. Student engagement

All 12 participants, teachers and coordinators alike, spoke of the perceived positive effects of the PBL approach on student attitudes in mathematics classrooms. Liz (Grade 8 Teacher) and Anne (her Math Coach at the board level), described the apparent increase in student engagement:

> Liz: For me, what I've seen this year is that they're far more engaged, and there's a culture of acceptance because we'll say, you know, 'There's more than one way to find an answer,' so you need to put it out there. So, kids who normally would sit back will try and say, 'Well what if we did it this way – what if they did it that way?' So, it's less threatening for them, when they know they're in a group.
>
> Anne: I think there's an increasing independence as well, because they're not relying and sitting and waiting and relying on somebody to come and get them going if they're stuck.

Not only were teachers surprised and impressed by the, in some cases quite dramatic, changes in student engagement, the mathematics coordinators whose job it is to travel the district extensively, visiting and observing in many classrooms, were likewise impressed with the things they had been seeing throughout the board.

> Dan: Going into a class and seeing every kid engaged and discussing and talking with the math, and seeing them get up at the end and share with the class their thinking, and having other kids question them about it – you can't deny the value in it.

Jodi, a Grade 7 teacher, asked her students directly how they felt about this form of math learning compared to the more traditional approach (short lesson on the board, textbook practice, taking up homework next day) with which they had been familiar in previous classes. Her description of the students' overall response corroborates the teacher perceptions mentioned above – they expressed a strong desire for social learning:

> Jodi: They do like it [PBL model], and I've asked them … their opinions on the old school math vs. the way that we've been doing math, and their comments on it are great. They don't want to work from the textbook, they like the communication piece. They like hearing what their classmates are saying.

Finally, Deb, a Math Coach who traveled her board extensively working with many teachers in supporting their practice, shared a rather significant observation regarding what she has noticed in mathematics classrooms:

> Deb: Sure, and it's interesting because I'm going to classrooms not knowing who are the strongest students and who are the weakest students. So, I teach a lesson and if it's a problem solving type of approach using manipulatives – I don't know how many times I've heard teachers say to me, 'That student

> never gets up and shares anything in class, they are math phobic.' And because of the way it's structured in a problem solving approach there is more opportunities, or more entry levels for students to be successful, and then we're allowed to see how they're smart, not how they're not getting the right answer – it shifts that thinking. So, I think it speaks to their comfort level, and creating students who are excited about math and willing to take risks, where the procedural approach does not necessarily do that.

Beyond the affective domain and any potential gains in student engagement that may or may not be part of a problem-based approach to mathematics education, a larger and perhaps more important question focuses on the effects of PBL on the actual of learning mathematics concepts and skills. How was PBL perceived in that light?

4.1.4. Mathematical understanding

In terms of cognition, or how students may, or may not, learn the mathematics content 'better,' or in a different or deeper way, or be able to apply the new learning with more facility, several teachers and coordinators addressed this issue. Many of their responses touch on how the social learning aspect plays a key role in this regard.

> Ted: In any classroom environment you're going to run into some children who are functioning at a Level 1 and some kids who are off the Richter Scale at Level 4. When you can get those Level 1 kids to show those Level 4 kids an 'aha moment,' where they're almost the teacher and the Level 4 student is going, 'Oh, I like that idea,' it really breaks down some barriers around the idea that, 'I can't do math.'

In addition to enjoying and academically benefitting from social interactions, students tend to, according to Jason, a Grade 8 teacher, have an uncanny ability to be able to 'scaffold' math learning experiences for their peers, often in more creative and effective ways than the classroom teacher – they, in effect, speak the same language.

> Jason: That's also another neat thing about this style of lesson that I find, is those kids that get it, they often times want to share it – kids are really good at scaffolding for each other, and that's one thing that I've learned this year for sure, that they're amazing at helping their friends. They don't always do it for them, we often think that kids just do it for each other, but I've got several examples where kids are really good at scaffolding, saying, 'Well, ok, I want you to look at this,' and they'll give the little clues.

Richard, a mathematics consultant, concurred that better/deeper learning is, in his opinion, reinforced via PBL: 'And the retention is so much greater this way – the retention of the skills and the understanding. I mean it's that deeper understanding that we're really finding.' Ann, a Math Coach, explained that Intermediate students who do well in terms of math test marks in Grades 7/8, often lack this kind of conceptual understanding in a more traditional style classroom and that this unfortunate truth, regardless of the students' abilities to memorize and apply simple, repetitive algorithms, means that they often end up hitting mathematical barriers at the secondary level when the same skills are applied within new contexts (e.g. fractions/integers within polynomial rational expressions).

> Ann: The students who do not struggle, I really feel, especially in Junior and Intermediate, they get by with high marks because they're really good at memorizing, and they're really good at following my directions. … Even if they are showing me on a test that they're an A student – I think that they still need to go back and demonstrate to me that conceptual understanding.

Participants in the RE4MUL8 lessons and debriefing interviews demonstrated and described, respectively, classroom contexts which very much coincide with the NCTM/OME picture of reform-based classrooms. Teachers and coordinators clearly expressed how they felt that such an approach to mathematics learning, whether relatively new for them or as longstanding modus operandi, not only increased student motivation and engagement, but also helped to deepen the mathematical knowledge of most students regardless of prior achievement or math aptitude.

If PBL truly is an effective form of mathematics learning, as these participants seem to have claimed, why then is it so difficult for existing teachers (or even for those completing teacher certification in pre-service programs) to move towards this pedagogical paradigm? One of the main, overarching themes of the RE4MUL8 Project is this notion of 'messy time' transition; purposefully captured in the project title (i.e. to reformulate, or change), paint splatter animation graphics, and Twitter/Facebook icons. In a previous publication, Jarvis, Mgombelo, and Franks (2012) have mined this notion within the parameters of another study dealing with Collaborative Action Research amongst K-6 elementary teachers. I was interested to know how the Grade 7 and 8 teachers, as well as their board-level support personnel (i.e. coordinators/coaches), would describe this journey. Would the 'messy time' transition hold for them the same sorts of not insignificant obstacles, yet also rich opportunities for growth and professional learning?

4.2. Messy time transition to reform-based pedagogy

The concept of 'messy time transition' was born out of a number of research projects in which teachers described the journey of moving towards reform-based pedagogy as being strewn with obstacles, some of which were more predictable and external in nature, others being more subconscious and surprising. The twelve participants in the RE4MUL8 study were asked during the interviews to comment directly on this idea of 'messy time' transition for teachers at the Intermediate Division level. The variety and honesty of their responses, which we will now explore in some detail, provides yet further insight into this pedagogical phenomenon.

The range of teacher experience represented in our participant sample was noticeably large: one teacher nearing retirement, several others in mid-career, and one neophyte Grade 8 teacher who had only just begun his teaching journey. Five experienced mathematics coordinators, themselves all former teachers, also added a remarkable depth to the interview discussions. From these very different perspectives, then, much was to be learned about teacher perceptions and sustainable change. In what follows, the reader is presented with some of the highlights regarding barriers and enablers relating to the messiness of transition to reform-based practice.

4.2.1. Perceived barriers to transition

Based on the data from all nine interviews with teachers/coordinators, the three most frequently mentioned perceived barriers to transition for Grade 7 and 8 teachers were: the *lack of expertise* in mathematics content knowledge; a sense of *urgency* in covering curriculum expectations, coupled with a perceived *lack of efficiency* relating to PBL activities; and, the fear of increased *classroom management* issues.

Anne, a veteran classroom teacher and, at the time of the interviews, Math Coach in her board, expressed how a generalist's lack of deep content knowledge in mathematics is often tied to apprehension around 'leaving the script:' 'We mentioned before that we're generalists, so if you don't have a content knowledge you're not really sure what kinds of questions to ask, or how to set up a problem, how to make it richer. I think that's one of probably the biggest challenges, is content knowledge.' Liz, the Grade 8 teacher who had worked closely with Anne in the filmed lesson and on previous occasions, shared openly from her recent transition experiences:

> Liz: [In the past] I've done, 'Ok, here's the formula and here's some questions to practice, ok, now you know it.'… So, now I'm trying to let go, and if I get really flustered then sometimes I revert back, but I'm trying not to. It is scary – it's really scary, and I think it's scary because you think that you need to know everything, and the kids expect you to know everything. But they need to know that it's ok that you don't know everything, and together you can work it out. … So, it is scary but it certainly is worth it. I think you get more respect from the kids as well.

Dan, a mathematics coordinator in the K-6 panel, agrees that lack of content knowledge is one of the greatest deterrents for teachers of mathematics to move towards a reform-based practice that is inherently unpredictable.

> Dan: Also, their own level of confidence with the math. It's hard to sort of anticipate student responses when they themselves, just through experiences, or lack thereof, don't really understand the math themselves. … Being apprehensive about it has sort of been a roadblock, so we've been trying to do a lot of work on building their own capacities so that they do have that comfort level of letting kids explore. … I think that's been one of the most positive things this year – seeing that fear [go away].

Richard takes this perspective one step further, arguing that not only are non-specialists hesitant to try PBL due to their lack of content knowledge, but also noting that math specialist teachers – those with actual degrees in mathematics at the undergraduate and perhaps even graduate levels – often are no better prepared, or willing, to adopt a PBL approach, but for quite another reason.

> Richard: I think that some of our teachers were not necessarily that strong in mathematics. This type of teaching opens up a lot of exploration. You saw the different ways that students were looking at questions. Suddenly it's not the one nice answer. Students can ask a lot of questions and go a lot of ways that teachers don't expect, and they could be a little hesitant about their own background. … The ones that are very strong in math often have quite a set view of what mathematics is. Mathematics instruction, when they went through, was very much a teacher-driven, 'Here are some examples, try some more questions,' and then, 'Here are 20 more examples,' or whatever. So, they stay with that routine.

Tied to this lack of mathematical confidence is the practice among many teachers of slavishly following a textbook as the primary driver of their math curriculum, rather than looking to the Ontario Curriculum itself as a guide and repository of helpful sample problems and teacher questions. As Dan noted, 'One of the biggest things that I see is the textbook – for a lot of teachers it is the curriculum, and that's been one of the biggest barriers to getting into teaching through problem solving, this reliance on the textbook as opposed to the curriculum.'

Not only do teachers feel that they lack the mathematics content knowledge required to fully adopt a PBL approach, but they also feel strongly about their desire to cover all of the Ontario Curriculum expectations for their grade level, and hence often view reform-based practice as an obstacle to that goal. Richard sympathizes with teachers' desire to cover curriculum, but he also explains that in setting problem-based activities, often multiple expectations, from across the five Ontario Curriculum math strands, can be addressed simultaneously.

> Richard: Getting through the curriculum expectations – that's always a challenge, and so people [say], 'So I don't have time for this.' It's been great to be able to do a lesson like this, that you saw today. Besides all of the algebra patterning, Jodi then threw in equivalent fractions, and suddenly you can go through a lesson and [say], 'Well, you did some graphing here, you did some of this, you did some of that.' We did it with one group afterwards, and sat down and said, 'Look, you've covered a third of your expectations this year in this week's work.' And they were like, 'Well, we never saw it that way before.'

Connected directly to the perceived sense of urgency in covering curriculum, was a perceived lack of efficiency in terms of the apparent use of class time during PBL sessions. Anne describes a real moment of awakening in her own career when she realized that by helping students do everything – even though this seemed more 'efficient' at the time – she was in fact robbing them of an authentic opportunity for real learning.

> Anne: I mean this is my 23rd year of teaching and my past practice – you think you're helping the kids by helping them along and showing, you know, starting to show them and perhaps even grabbing the pencil from time to time and saying, 'Well, you just do this,' and you think that you're really teaching them. And then you would go back and they would have no idea because you just did it for them. … Wait time is huge, and so asking those questions and waiting to see what they're going to do. … It's risky though, because you don't' know where it's going to go. … It's not risky just to tell them, 'Hey, this is how you do it.' It's a nice shift, that the kids own the knowledge instead of us owning the knowledge.

Ross, also a coordinator, explained how he himself felt impatient during the lesson on patterning and algebraic thinking. Was the strategy of having Grade 8 students complete the graph without step-by-step instructions a wise investment of time? What did they learn through this period of negotiation and false starts that may have been lost if the instructor had given them a completed coordinate system in preparation for the graphing exercise that formed the main event? Ross effectively summarizes this tension:

> Ross: So, I think there's a really strong feeling that it's much more efficient – that you can tell students an awful lot of stuff a lot quicker than you can have them do activities. And that's correct, right? But the downside is what they retain of that, or what they actually learn about learning, and what they learn about acting like a mathematician, or any of those kinds of things. I think that's what's hard, time is precious.

A third obstacle to reform-based practice that was commonly shared by teachers was a perceived sense of impending chaos if they were to attempt this kind of mathematics learning in class. Behavior management concerns were described as applying to new and more experienced teachers alike, often depending on the particular group of students involved. One Math Coach explained how she had dealt directly with this kind of teacher apprehension.

> Ann: I've heard teachers say, 'Well I can't use manipulatives, they'll just throw them around the room, it will be chaos.' … I don't understand how we get from, 'I want to effectively use manipulatives in my classroom to teach a conceptual understanding of math,' to, 'It will be chaos.' Somewhere in the middle there has to be a teacher. And I say to the teachers, 'How about if you … model how you would use the manipulatives, model how you put them away, model the respect, and then start using manipulatives.' That giving up of control is huge for teachers.

According to several of the coordinators interviewed, teachers who agree to have the Math Coach come in and help them with a problem-based lesson often express surprise with the types of thinking and communication that their students demonstrate in these sessions, particularly among those students who traditionally dislike mathematics.

> Anne: Some of them are still skeptical, I think, a little bit about just the – 'My kids won't work in groups,' those kinds of things. … Because I think they like some of the ideas that are happening, and they're surprised when we do a lesson like we did today – that the kids can come up with the solutions they come up with, without being spoon-fed and prompted to get there. I think that's been a real eye-opener for some teachers, and we just have to keep working from there.

In summary, obstacles that were perceived among teachers as being the most pronounced in terms of not encouraging the adoption of a reform-based practice include: lack of confidence with the mathematics content; a perceived sense of urgency around expectation coverage, coupled with a sense of diminished efficiency in terms of use of time; and, a foreboding sense of chaos and unmanageable student behavior as a direct result of PBL lessons. In the next section, we will explore some of the perceived enablers that have served to counter the above concerns.

4.2.2. Perceived enablers for transition

The interview data also revealed a number of perceived enablers for transition to reform-based practice. These included the following: *school administration and board-level support*; a sense of *shared adventure* with students as teachers take risks and model life-long learning; and, a *focus on changed practice vs. changed beliefs*.

As teachers contemplate the risk-taking involved in changing mathematics teaching practice, it is very important that they feel well-supported in this endeavor by both their school Principal and by those at the board level (e.g. coordinators, Math Coaches, curriculum consultants) who are responsible for providing guidance. Ted shared his appreciation for the kind of support he felt had been made available to him on an ongoing basis.

Ted: I consider myself super fortunate that we have had tons of support within our board and within our building here. The principal this year, she's phenomenal, she supports us in every avenue we want to take – any technology stuff where we need to spend money. The board math consultant is instrumental in making sure that we have all of the math tools and manipulatives at our disposal. She's an email away, she's a phone call away, she will help you with anything.

Amy, like most other teachers in the study, indicated how greatly she had benefitted from local professional development initiatives that had focused on differentiated instruction and PBL in mathematics.

Amy: I would say that definitely through that seven years, in terms of some of the PD that our board has been provided through some of the Ministry stuff, that I have been fortunate to participate in that reform-based, or that more hands-on approach, looking at critical thinking and different approaches to solving math work, as opposed to just sitting and doing calculations at a desk.

Some local boards went so far as to provide release time for teachers to travel to other schools to observe exceptionally gifted colleagues who would demonstrate problem-based lessons and offer to share ideas and resources. In one case, Richard described an open house lesson where parents/guardians were actually invited in to watch the lesson being taught and to see for themselves why their children were becoming engaged in mathematics:

Richard: We had a number of teachers who, as she said, came to Jodi's class, viewed what she did, and a few weeks ago they had a math afternoon. They opened their school up in the afternoon, and they had 3-part lessons taught to Grades 4–8 and parents were invited. So, parents were circulating through the classes, and I got to drop in on every class, and I was talking to some of the parents and they were saying, 'This doesn't look like my math class,' and she said, 'They're not distracted by us because they're working – they're all on task!' … A couple of parents said, 'My children come home excited about math.'

Beyond school and board support, another perceived enabler for change was the sense of fun and adventure teachers claimed to have experienced while moving towards the PBL classroom. Jodi had taught mathematics in a more traditional, teacher-directed manner for over a decade, and had only recently moved towards a reform-based approach over the past several years. She described the joy of this major transition:

Jodi: That was not my approach when I first started. I've been teaching for 14 years – this approach probably about 3 years ago … we started talking about this 3-part lesson and the importance of exploration and integrating the strands, and I just jumped all over it. It sounded right and it made sense, and I haven't turned back. I find it so much more fun to teach that way, and the things that the kids come up with I would never have thought of.

Coordinators in the study spoke at length about how teacher beliefs relate to practice, and vice versa. For example, in two separate interviews (different boards/ schools), two coordinators both mentioned the idea of trying to change teacher practices first (e.g. having them 'jump in' and try a problem-based lesson, often with coordinator or peer teacher support), which often then led to a change in belief structure regarding mathematics pedagogy.

Richard: This whole idea of realizing that teachers – if we can get them to change their practice, then they'll change their beliefs. We used to approach it by trying to get them to change their beliefs, and then get the practice to follow. … And we stress perseverance as well – it's going to be messy. … We find that more and more throughout the board, as we go around – [teachers] that have said, 'I wasn't so sure, but the kids' reactions really sold me on it, when the students became more engaged.' … Once we get a teacher going down this road, they want to have more.

Deb, a Math Coach, described the radical change in attitudes among teachers in her board as they themselves became more engaged in mathematics learning, leading to professional enablement and a desire to learn more.

Deb: And I think that it speaks to belief systems too – they're teaching the way that they believe children learn best, and so to switch their belief systems, you've got to work on changing their practices. And if you don't have supports to change those practices it becomes problematic.

In summary, participants in the study revealed that some of the major enablers for a positive, though understandably 'messy' transition to reform-based practice included: administrative and board-level support for risk-taking and professional learning; a renewed sense of creativity and anticipation as teachers and students learned new things together through interesting problems and the sharing of multiple solution strategies; and, the importance of focusing on helping teachers to change practice (i.e. experiment) rather than on trying to first modify beliefs.

4.3. Balancing instructional planning and practice

Thus far we have examined the two major themes of PBL classrooms, and 'messy time' transition to reform-based practice. Participants in the RE4MUL8 interviews were also directly asked what kind of advice they would give to other teachers or coordinators who may be contemplating a similar change. Within a *standards-based* (NCTM 2000; OME 2005a), yet also *standardized* (i.e. Educational Quality and Accountability Office conducting provincial assessments of mathematics at Grades 3, 6, and 9 in Ontario schools) era (Jarvis 2006), what kind of guidance or advice would participants offer to their peers to help them successfully balance instructional planning and practice given the realities of the twenty-first century school workplace?

Firstly, Kathy, as well as several others, noted that certain strands or topics within the math curriculum lend themselves more easily than others to hands-on, constructivist approaches to math learning, and so these areas might be a good starting point for a teacher first wanting to try something different. She noted, 'I'm moving more to the problem solving. … I'm moving to that direction and enjoying it, but I can't do it all the time. Certain unit strands in math lend themselves more readily to it, as well.' Amy shared similar thoughts in terms of the various strands:

Amy: I think for me personally a lot of it depends on what unit we're going through. Something like integers while we're using the number tiles to represent positive and negative, and using number lines and that kind of thing. Sometimes there's still a lot of paper-and-pencil work. Whereas something with geometry lends itself much more to manipulatives – or to using the computer.

Deb explained that reducing the number of practice questions and focusing on a fewer number of well-selected rich problems/questions can serve a teacher well: 'Valuing the switch from a bunch of questions to one really good question, and understanding the growth of strategies, and how problems could be solved in many different ways, all of that was a big piece for me.' The frequency and duration of problem-based lessons are also significant factors that need to be considered. Richard explained the wide variety of teacher/school practices that he had noticed as he travelled the board in his role as Secondary Curriculum Consultant:

Richard: Not everybody does it every day. Some schools decide, 'Here's where I'm going to go, and I'll teach a few days of more traditional lessons, then do problem solving, and then on Friday it's, "Ok, we've worked as groups, now individually show me what you have learned."' Some people will start with the big question – use it as a diagnostic for, 'What skills do I need to teach?' then two days of more traditional skill-based work, and then more problem solving at the end. Then some people have jumped in wholeheartedly. … We've had people realize that it's really about working smarter – it's not adding on.

Allowing for multiple solution strategies requires some getting used to. This is not to say that there is not often still one correct numerical answer to a given problem, but rather that there are often many ways of conceptualizing, modeling/representing, and solving the problem in ways that make sense to students. As Ann highlighted in the following quotation, teachers are well-advised to learn how to balance open-ended problem solving activities with more traditional memorization (e.g. multiplication facts), estimation (e.g. answer checking for reasonableness), and computation (e.g. following efficient algorithms) skills. The result is a flexible learner who enjoys math and invites challenge.

Ann: If they start to talk a little more … the more they take ownership, the more they have to defend what they're saying. If they can teach what they're doing to someone else, now we've got that retention piece happening. … I am not negating the fact that somewhere along the line you need to know, or have a good grasp of the basics as well. … My advice would be to start small – you model, you guide, and you release to independent work, so give yourself that respect too, slow release. Just a small activity and you bring them back to where it's safe for you to teach – because ultimately you have to be comfortable as a teacher.

Ted highlighted the fact that traditional vs. reform/activity-based approaches are not only to be balanced within the mathematics program, but also across the disciplines within any given day/week of the generalist teacher. I believe that this may be a very significant observation and one that is rarely pointed out to elementary teachers. From the student's perspective, the entire school day is a series of related experiences, and so in planning a balanced program, an effective teacher will take into account that even on days where one may focus more on traditional type skill practice in math class, for example, one may purposefully plan for more constructivist activities in other areas.

Ted: There might be another time where you're going to do less activity, where the kids might go back to traditional style and go back into the textbook because it's information they need to know. On those days, that might be where that rich learning is going on possibly in another subject area. It might be a great science lesson where you have a lab, or it might have to do with language where the kids begin to realize each subject has its own merit. Some days there's going to be a riveting one in math, just like there might be something in terms of language or science or history or geography.

Jodi reassured teachers that perfection is not the ultimate goal, and that risk-taking inherently involves messiness.

Jodi: [Teachers] should put up a question and give it to the kids and see how it works. Don't expect it to be perfect – just give it a try and that's how you learn. We expect the kids to try all the time – we're always telling them to go for it, so just to try it. If you have a class that, off the top of your head, you're thinking, 'There's no way I could put them in groups,' then put them in partners – start small. … Definitely, if you can get a teaching partner … to try it together, and if you're able to get some release time, or after school – plan it out together, and then be able to debrief, and see where to go from there.

Many of the study participants mentioned the fact that within the '3-part lesson' (explain, explore, express), often the most challenging part for teachers is the third component, the consolidation or debriefing part of the lesson. While this is arguably the most important part of reform-based teaching, wherein key concepts are discussed, gaps in student learning are filled, and misconceptions addressed, for many teachers this represents an overwhelming task, particularly in light of the lack of math content knowledge that we discussed earlier. According to participants, comfort and effectiveness in leading consolidation conversations does, in many ways, improve simply by virtue of practice, repetition, and careful listening. Dan noted that in one particular school in his board, teachers would proactively postpone the consolidation piece until later in the day, or even on subsequent days, after they had had the time to analyze student work either individually or in small teacher groups.

Dan: Doing it on the fly, and looking at the student work they found sometimes wasn't always the easiest, so they would plan in a break before the third part of the lesson, like the consolidation piece, so they had a chance to actually sit down and look a little more in-depth at the kids' work so they can look at misconceptions and thinking. Sometimes it might even have been a recess break, a prep break, and some teachers have even said, 'I find that I'll do it, and then the next day I'll do the consolidation, and that consolidation could become part of your getting started, right?' So, still following the 3-part lesson format, but realizing that the five minutes, twenty minutes – it has to be a flexible sort of thing.

Finally, in a reform-based paradigm, assessment of student learning and achievement is usually characterized by a combination of different kinds of tools and practices. The importance of *formative assessment* or what has commonly been referred to as *assessment for learning,* is discussed, among other forms, by a Math Coach:

Ann: Formative assessment for sure, when you're doing guided groups as we were this morning. In my classroom I give a little quiz every week, and the quiz is an independent piece. … Sometimes I give a unit culminating activity, sometimes it's a unit test, sometimes I'll do a test question a day, for a week. The students think that's great, and that way I'm moving on to another unit, and it's not as much stress as sitting down to an hour-long exam. Not that that process isn't important too, because when they get to Grade 9 they need to

> know how to take an exam. So, lots of ways to get their marks. … I also leave it open and say, 'If you have a great idea … come and negotiate with me.'

In one of the most delightful comments made by a participant in the study, Richard described how some teachers struggled with how reform-based practice actually turned their traditional assessment practice on its head:

> Richard: We had one teacher who said, 'My biggest problem was getting over the mindset that when we do these problems, [where students] help each other … suddenly, I had too many kids who were at [Level] 3's or 4's.' She said, 'It was a real mindset change for me because it [assessment] was always about ranking, and you sort of knew who your top kids were.' She said, 'Suddenly, the majority of my class is producing Level 3 and 4 work because they know what it looks like – they've had a chance to practice, they've been supported through it, and they're interested and engaged.'

The RE4MUL8 Project lessons and interviews provided a wealth of audio-visual data. Themes emerging from the transcribed interviews have been here dealt with in some detail, and likewise, the video vignettes located in the online/mobile resource will hopefully offer meaningful insights for teachers, coordinators, and school administrators who are considering how to introduce, or further support, reform efforts in classrooms.

5. Concluding thoughts

Teachers and coordinators in the RE4MUL8 Project demonstrated and described PBL classroom contexts that closely adhere to the model being presented by NCTM and OME. This kind of classroom provides a safe and positive space for learners to take risks and to feel supported in their mathematical thinking and communication (Small 2013; Van de Walle and Folk 2008). Students in such classrooms are free to use manipulatives and technology as powerful tools for modeling difficult and challenging problems, and further, they are provided with opportunities for varied assessment for which they can still choose (or not) to use these hands-on or digital tools to demonstrate their thinking or to solve rich problems. PBL allows students to learn socially while in pairs or small groups, practicing daily the art of collaboration, respectful critique, and the ability to persevere as a team member.

The route to problem-based teaching and learning is indeed fraught with obstacles, some external in nature, but a number of which, as we have seen, being more internal – belief structures regarding math pedagogy that make it difficult and dangerous to venture far from the nest of past practice. Teachers have shared several of these very personal and real struggles with us, and coordinators have offered helpful strategies for providing moral, fiscal, and mathematical support to those teachers presently involved in the 'messy time' transition to reform-based practice.

Limitations to this study were the relatively small number of participants (7 teachers and 5 coordinators). Although this is not unusual for an in-depth case study research project, the interviewing of more teachers and coordinators would allow for further corroboration and comparison of ideas. The research was also based on single visits to classrooms for the lesson that was to be observed and discussed. Ideally, the case study would have involved numerous visits and lesson observations, a reality that is simply not affordable or realistic within the confines of the research funding and timeframe. Related recommendations for future research would therefore include the video-taping of multiple lessons over time for each of these, or other teacher participants. Also, the interviewing of students would add another rich layer of analysis in terms of what is being perceived by the learners in these classes.

The mobile device web app for the RE4MUL8 Project (www.re4mul8.ca) has been made available through the main project website, allowing teachers and other interested stakeholders the ability to view and search the existing video documentaries on tablets and smartphones.

When asked what kind of benefit, if any, such a resource may offer to teachers of mathematics, one of the more experienced teacher participants explained the following:

> Ted: I think that it would be very beneficial for Grade 7 and 8 teachers, and when they see one of their real world colleagues who have said, 'You have to try it, you have to take some risks, you have to break down some of those barriers.' … I think that they'll say, 'I can do that – I could try that,' it might be a case that they really see what is the objective at the end of the day – do the kids understand?

The RE4MUL8 resource was designed to assist classroom teachers of mathematics as they navigate the messy yet meaningful transition to reform-based mathematics pedagogy. For those who have already made this initial journey, may they find comfort in the idea that the cyclical risk-taking involved in reform-based mathematics, and in lifelong learning in general, is *perpetually uncomfortable* and that this is, in fact, a very good thing. Further, may the themes that have emerged from the case study findings, as described in this paper, be of assistance to mathematics coordinators and board-level planners as they continue to support teachers of mathematics who, despite the fact that they may not possess specialist mathematics training, are nonetheless passionate about their profession and hence willing and ready to take steps towards 'reformulating' practice – teachers of math ready for change.

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Notes

1. Within the *Ontario Curriculum: Mathematics, Revised* (2005–07), the following seven core 'Mathematical Process' expectations are highlighted: problem solving, reasoning and proving, reflecting, selecting tools and computational strategies, connecting, representing, and communicating.
2. I use the purposefully use the term 'teachers of mathematics' rather than 'mathematics teachers,' with the idea of distinguishing elementary non-math specialists (i.e. no/little undergraduate experience in mathematics) from math specialist teachers. Teachers of mathematics are required to teach this subject among many others.

Notes on contributor

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