A new example of a tree-like continuum with a fixed-point-free self-map

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Fixed-point-free maps on tree-like continua

Continuum $\equiv$ compact, connected, metric space

X is *tree-like* if $X \approx \varprojlim \langle T_n, f_n \rangle$, where each T_n is a tree.

$g : X \to X$ is *fixed-point-free* if $g(x) \neq x$ for all $x \in X$.

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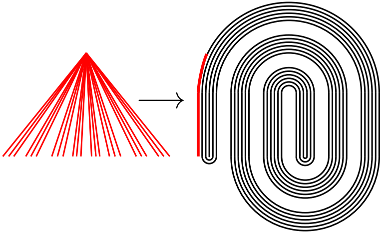

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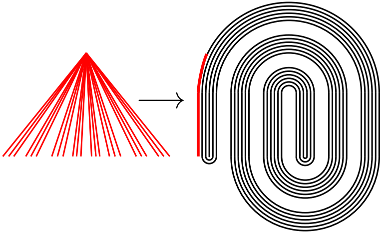

Question

Does there exist a tree-like continuum in the plane $\mathbb{R}^2$ with a fixed-point-free self-map?

Map on inverse limit

Let $X = \varprojlim \langle T_n, f_n \rangle$.

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$$\begin{array}{ccccccccccc}
 & T_{n_1} \longleftarrow & T_{n_2} \longleftarrow & T_{n_3} \longleftarrow & T_{n_4} \longleftarrow \cdots & & X \\
\swarrow g_1 & \swarrow g_2 & \swarrow g_3 & \swarrow g_4 & & & \downarrow g \\
T_1 \longleftarrow & T_2 \longleftarrow & T_3 \longleftarrow & T_4 \longleftarrow \cdots & & & X \\
\varepsilon_1 & \varepsilon_2 & \varepsilon_3 & \varepsilon_4 & & &
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Oversteegen & Rogers 1980/82: An example with commuting diagrams

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- Given $\mathbf{x} = \langle x_1, x_2, x_3, \ldots \rangle \in X$, $\quad g(\mathbf{x}) = \langle g_1(x_2), g_2(x_3), \ldots \rangle \in X$

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- Given $\mathbf{x} = \langle x_1, x_2, x_3, \ldots \rangle \in X$, $\quad g(\mathbf{x}) = \langle g_1(x_2), g_2(x_3), \ldots \rangle \in X$
- $f_1(t) \neq g_1(t)$ for all $t \in T_2 \quad \Rightarrow \quad g$ is fixed-point-free

Comparison

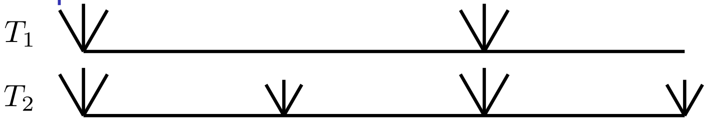
$$\begin{array}{ccccccccccc} & T_2 \longleftarrow & T_3 \longleftarrow & T_4 \longleftarrow & T_5 \longleftarrow \cdots & & X \\ \swarrow g_1 & \swarrow g_2 & \swarrow g_3 & \swarrow g_4 & & & \downarrow g \\ T_1 \longleftarrow & T_2 \longleftarrow & T_3 \longleftarrow & T_4 \longleftarrow \cdots & & & X \end{array}$$

	Oversteegen & Rogers example	**New example**
# Branch points in T_n	3, 9, 33, 129, . . .	2, 4, 7, 13, 25, . . .
Largest degree of branch point in T_n	18, 30, 42, 54, . . .	5 for all n
# Endpoints in T_n	24, 54, 138, 438, . . .	7, 12, 21, 39, 75, . . .
Valence of f_n	24, 96, 384, 1536, . . .	6 for all n
Valence of g_n	48, 192, 768, 3072, . . .	12 for all n

New example: continuum

T_1

New example: continuum



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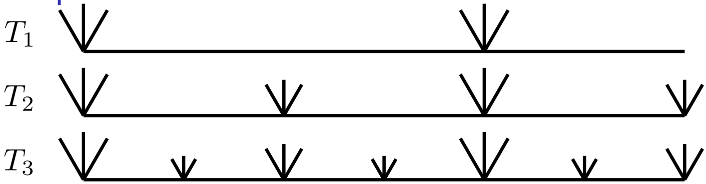

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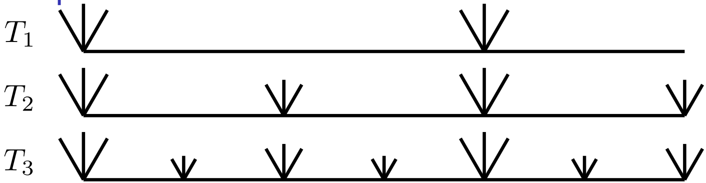

$f_n \sim$ 3-fold tent map from spine of T_{n+1} to spine of T_n

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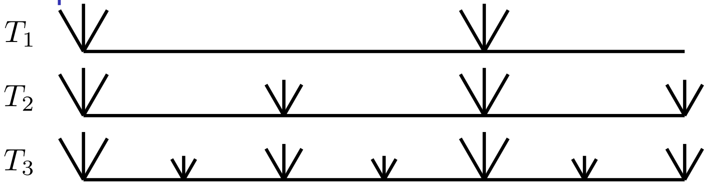

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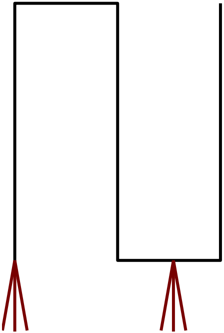

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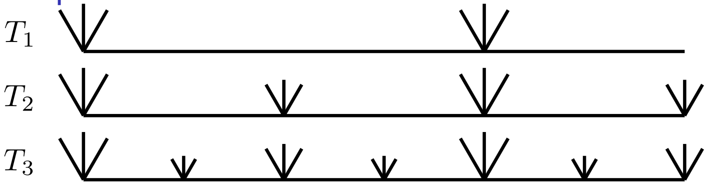

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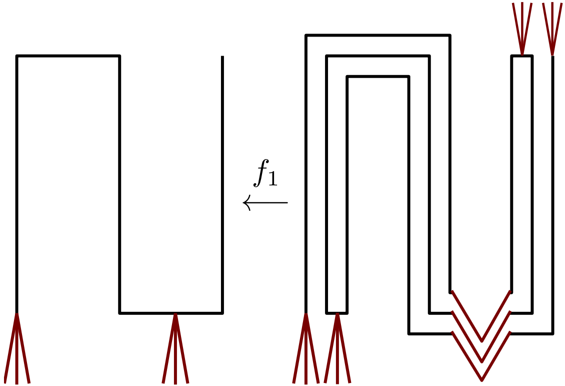

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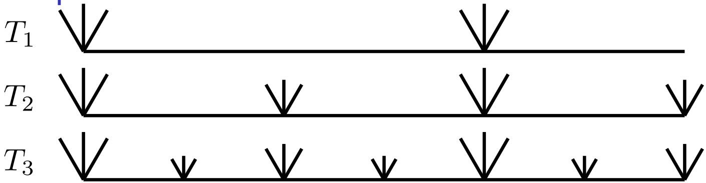

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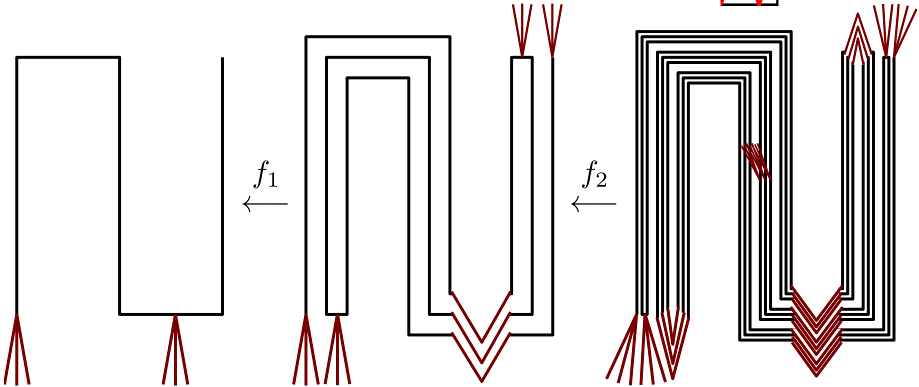

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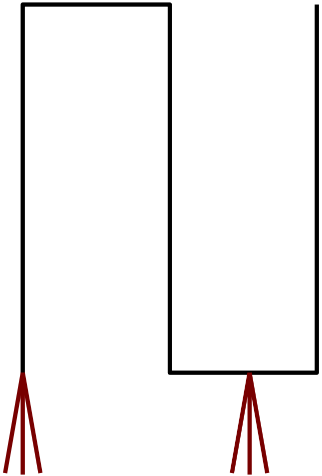

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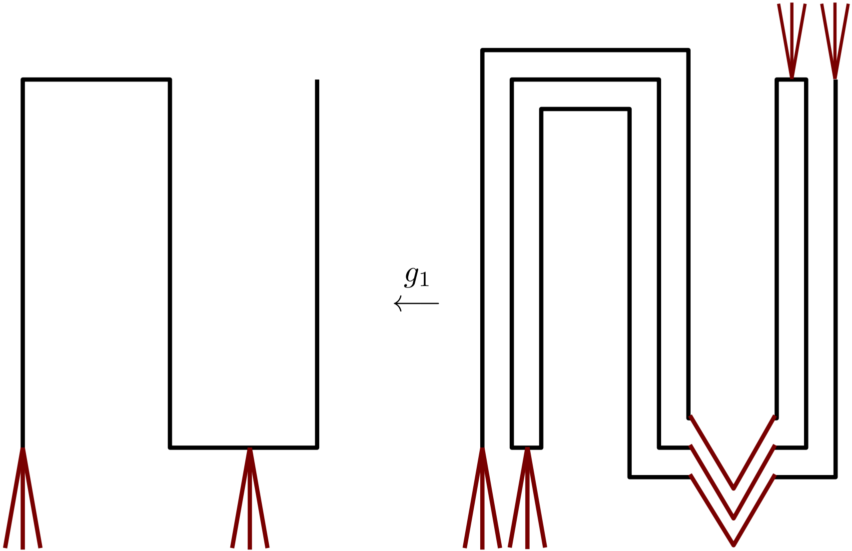

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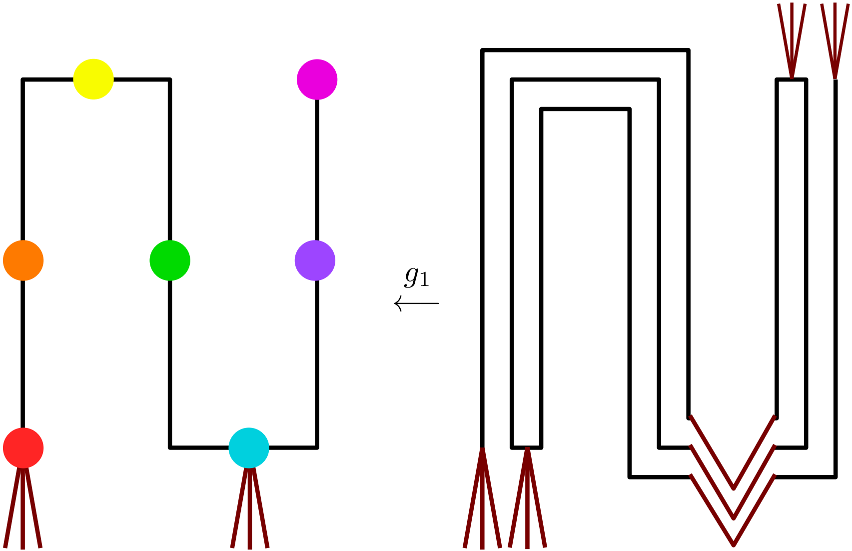

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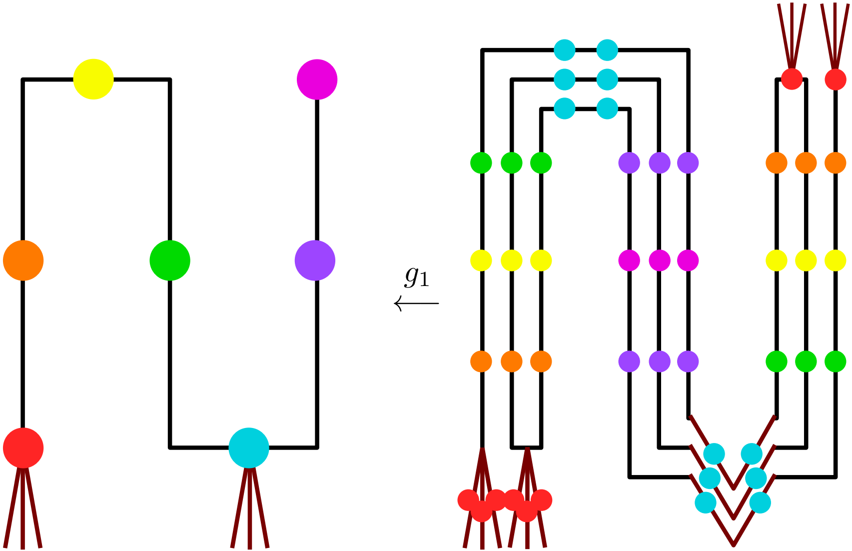

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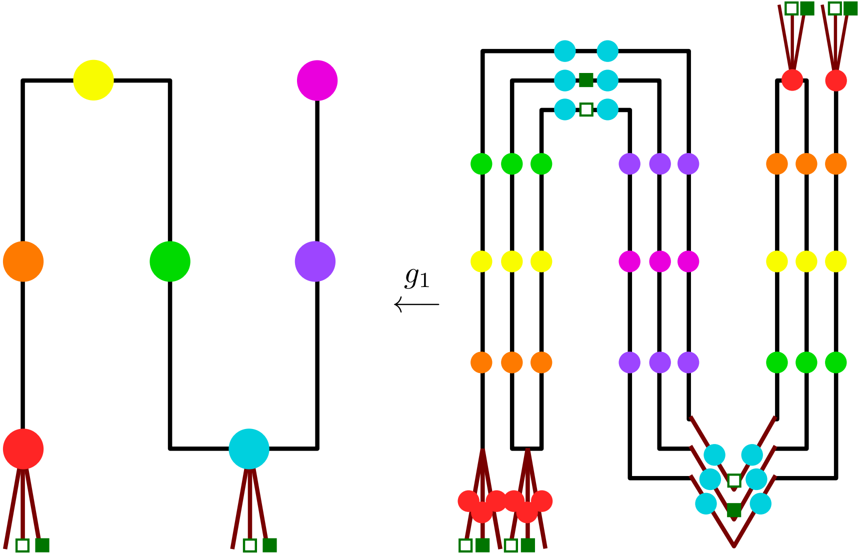

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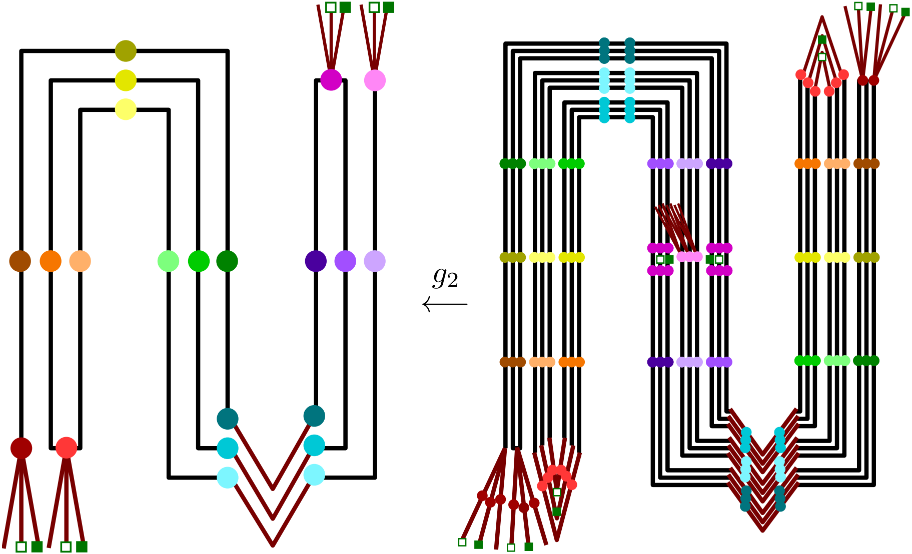

Questions

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Is there a coincidence-point-free commuting system

$$
\begin{array}{ccccccccccc}
 & & T_2 & \xleftarrow{f_2} & T_3 & \xleftarrow{f_3} & T_4 & \xleftarrow{f_4} & T_5 & \xleftarrow{f_5} & \cdots \\
 & \swarrow g_1 & & \swarrow g_2 & & \swarrow g_3 & & \swarrow g_4 & & & \\
T_1 & \xleftarrow[f_1]{} & T_2 & \xleftarrow[f_2]{} & T_3 & \xleftarrow[f_3]{} & T_4 & \xleftarrow[f_4]{} & \cdots & &
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such that...

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Does there exist a pair of disjoint commuting maps on a tree? on the simple triod? on a dendrite?

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Thank you!