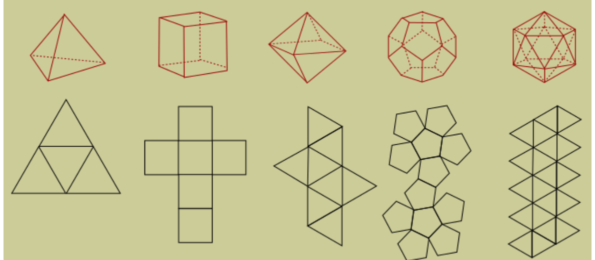
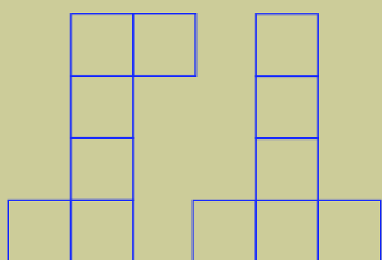
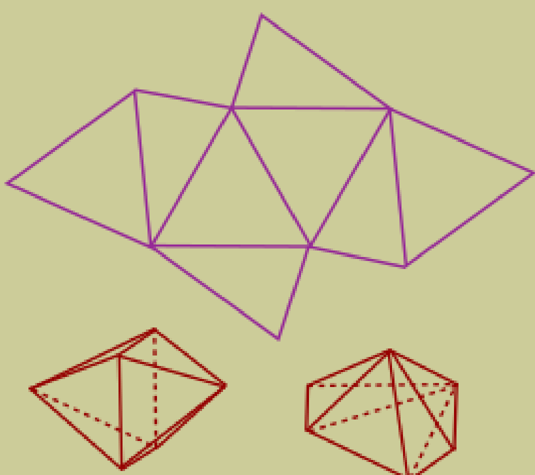
Dürer's conjecture



Regular polyhedra and their nets



Two different nets for a cube



Two polyhedra from the same net

Albrecht Dürer (1471-1528), self-portrait

One of the most fundamental types of three-dimensional objects are polyhedra. Recall that a shape is called **convex** if for every two of its points it contains the line segment joining them.
A **convex polyhedron** is a convex shape whose boundary consists of finitely many of convex polygons, called **faces**.

The most famous polyhedra are **regular polyhedra,** or **Platonic solids**. All faces of a regular polyhedron are regular polygons (i.e. polygons whose sides and angles are equal to each other). Moreover, the same number of faces meet at every vertex. It has been known since antiquity that there are only five regular polyhedra: tetrahedron, cube, octahedron, dodecahedron, and icosahedron (see the first picture).

A natural question about a polyhedron is whether it can be glued from a flat region. More precisely, a **net** of a polyhedron is a polygon in the plane (not necessarily convex) which can be folded without overlaps to assemble the original polyhedron.

The nets for regular polyhedral are shown on the first picture. Note that even simple polyhedra can have several different nets, e.g. a cube has eleven different nets, two of which are shown on the second picture. On the other hand, a single net may produce different polyhedra. An example of such net, due to Hafner and Zitko, is on the third picture. There are examples of non-convex polyhedra for which there is no net. For convex polyhedra, nobody knows whether the following statement is true.

Dürer's conjecture: Every convex polyhedron has a net.

This conjecture (proposed in 1975 by Shephard) is named after famous German artist Albrecht Dürer. In his 1525 book on measurements and geometry, Dürer proposed planar unfoldings of Platonic solids and other polyhedra. In 2014 Mohammad Ghomi proved that every convex polyhedron has a net after a linear transformation.

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