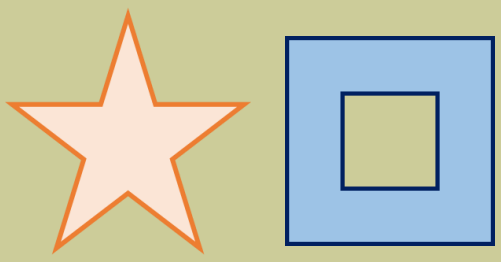
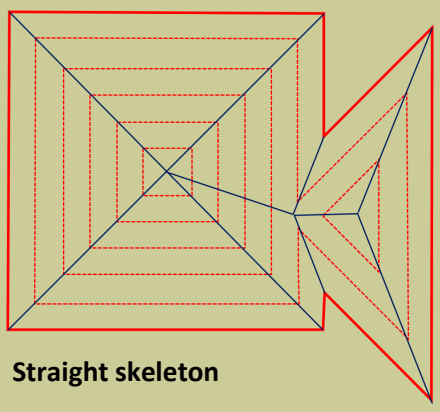
NIPISSING UNIVERSITY - MATHEMATICS

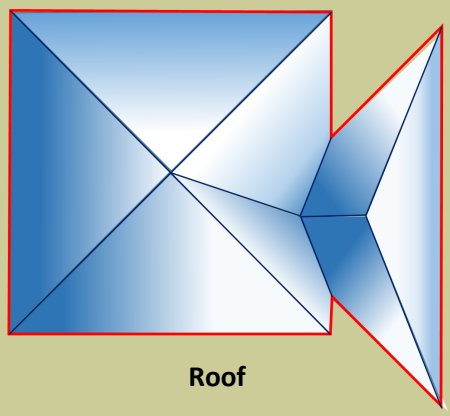
Magic or Mathematics?



Star and square with hole



Straight skeleton



Roof

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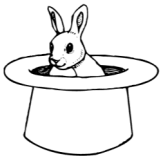

Can you cut a star from a piece of paper, using just **one straight cut**? Surprisingly, the answer is yes, but you need to first fold the paper several times!

The question of whether an arbitrary shape with a boundary formed by straight sides (such shapes are called **polygons**) can be cut from a piece of paper with one straight cut if the paper is folded first, was puzzling people for long time. It has been conjectured that in fact any polygonal shape (even with holes) can be obtained in this way. This conjecture first appeared in "*Wakoku Chiyekurabe*" ("*Mathematical Contests*"), a book that was published in 1721 by Kan Chu Sen in Japan.

Harry Houdini

Interestingly, famous magician Harry Houdini published a book in 1922 describing examples of cutting complicated shapes with one straight cut.

The problem was investigated further by popular mathematics writer Martin Gardner in his 1960 publication in *Scientific American*. It took almost three hundred years, however, to give the rigorous proof of the conjecture. The first proof of what is now known as **Fold-and-Cut Theorem**, was published in 1999 by Erik Demaine, Martin Demaine, and Anna Lubiw.

One way of proving the Fold-and-Cut Theorem and also solving the respective problems for polygonal shapes is related to roof design. Suppose that you need to construct a roof for a house that has a polygonal perimeter. How should you proceed? The idea of solution is based on so-called **straight skeleton** of a polygon, which can be used to determine the locations of the roof ridges. More precisely, imagine that you continuously shrink the polygon, so that the edges are moving inwards parallel to themselves at a constant speed. This obviously will cause the vertices also to move inwards. The speeds of vertices are not the same, and depend on the angles at each vertex. Moreover, in this process some vertices may collide with a nonadjacent edge. This results in splitting the polygon in two parts, after which the process continuous for individual parts. In the case of a polygon without holes, the straight skeleton is formed by the lines traced out by the moving vertices in this process. If the polygon has holes, the process will stop before all vertices (and edges) contract to one or several points, and the skeleton also consists of the final positions of the remaining edges. Straight skeleton is used to determine the fold lines. As an exercise, try to construct the straight skeleton for the square with hole and the star, and then use it to fold-and-cut the shapes.

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