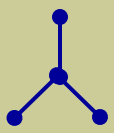
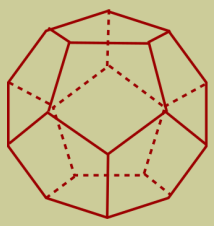
NIPISSING UNIVERSITY - MATHEMATICS

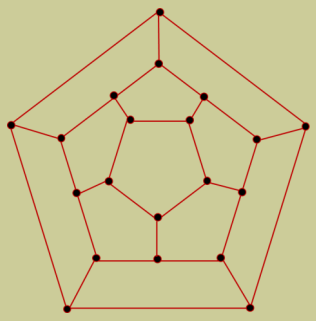
Games on Graphs



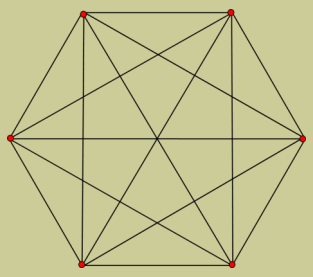
Triod



Dodecahedron



Dodecahedron graph



Complete graph on six vertices

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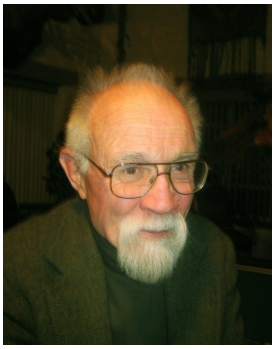
What do a tree, a road map, and the Internet have in common? All of them can be described using mathematical structure called **graph**. A graph is a collection of points, called **vertices**, and segments joining them, called **edges**.

William Rowan Hamilton

Some graphs have a very peculiar property. Namely, starting from some vertex of the graph, you can trace the edges so that each vertex is visited exactly once, and finally return to the initial vertex. Such path in a graph is called a **Hamiltonian cycle** after a famous Irish mathematician Sir William Rowan Hamilton. It is easy to see that the triod graph on the left does not have a Hamiltonian cycle. On the next two pictures, you can see one of the regular polyhedra, called dodecahedron, and the graph representing its edges and vertices. **The Icosian** is a game of finding a Hamiltonian cycle in the dodecahedron. Can you find and trace a Hamiltonian cycle?



Gustavus Simmons

If you draw a line segment joining each pair of n vertices of a regular polygon, the resulting object is called a **complete graph on n vertices**, denoted $\mathbf{K_n}$. On the last picture you see K_6. An interesting game that can be played on K_6 was proposed by mathematician Gustavus Simmons, who specializes in the field of cryptography. This game, **the Game of Sim**, is described next. Two players choose one color each and take turns coloring the edges of K_6. The player who gets the first triangle of the same color loses. In fact, for any coloring of K_6 with two colors there is always a triangle with sides of the same color (exercise: prove it!), while there exists a coloring of K_5 in two colors without triangles of the same color (exercise: find such coloring!). For the game of Sim, it has been verified by computer that second player always has winning strategy. However, no simple such strategy is known at the moment.

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