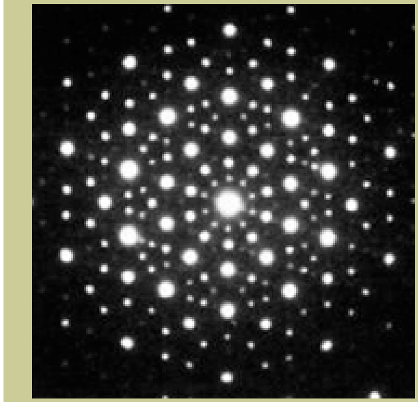
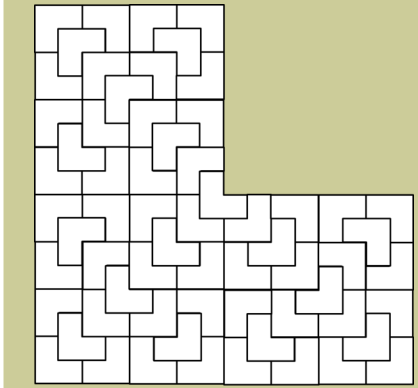
OAME 2016

NIPISSING UNIVERSITY - MATHEMATICS

From aperiodic tilings to quasicrystals



Electron diffraction pattern of the Ho-Mg-Zn quasicrystal (Shechtman et al., 1984)



Aperiodic "armchair" tiling

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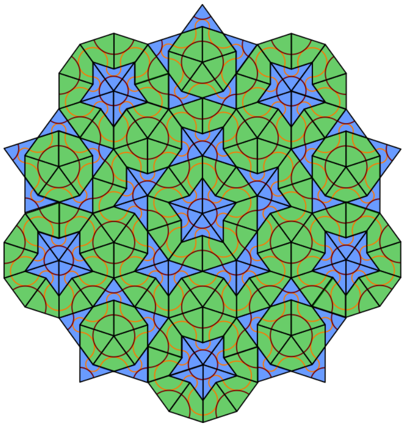

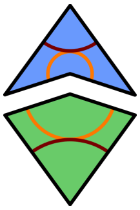

Penrose "Kite and Dart" tiling

A tiling is a cover of the plane by various shapes without gaps and overlaps. Floor tiles and mosaics, wallpaper patterns, brick and stone sidings are familiar examples of tilings.

These tilings are usually periodic. This means that the whole tiling can be shifted in a particular direction to obtain a perfect copy of itself. In other words, these tilings possess translational symmetry. The individual shapes that are used in tilings (triangles, squares etc.) are called prototiles.

In the physical world, periodic tilings (of the space) can be used to model crystals (e.g. metals, quartz, diamond, etc.). Surprisingly, the plane (and space) can be tiled so that the resulting pattern does not have translational symmetry. Such tilings are called aperiodic. One can see an example of (a part of) an aperiodic tiling on the left of this poster (the "armchair" tiling). However, it is not hard to create periodic tilings using the same ("armchair") prototile. Are there any (finite) sets of shapes that admit only aperiodic tilings of the plane? The answer, unexpectedly, is yes! The first such sets of aperiodic prototiles (consisting of over 20,000 elements!) were discovered in 1964. About a decade later, a famous mathematician and physicist, Sir Roger Penrose, found sets of aperiodic prototiles consisting of three and two elements. One of these sets—the "kite and dart" set — and the corresponding tiling are shown at the top of this poster. The arcs on the prototiles describe the "matching rules" between adjacent tiles.

Aperiodic tilings may seem to be a beautiful but otherwise useless mathematical abstraction. However, this turned out to be wrong! In 1984 Dan Shechtman discovered a new type of structures, now called quasicrystals. Quasicrystals are ordered, but not periodic and thus differ substantially from ordinary crystals. Shechtman received the Nobel Prize in Chemistry for this discovery in 2011. Properties of aperiodic tilings are related to properties of quasicrystals. Thus aperiodic tilings can be used to study physical world.

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