“Messy Time” Transition to Reform-Based Mathematics Teaching and Learning

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The Challenge of Effective Mathematics Teaching in 2011

In an era of reform-based mathematics education (National Council of Teachers of Mathematics, 2000; Ontario Ministry of Education, 2005a, 2005b, 2007), in which key elements such as problem-based learning (PBL), co-operative group work, manipulatives, technology, and varied types of assessment are being emphasized, it is not difficult to understand why teaching mathematics remains a complex undertaking. The twenty-first-century mathematics educator must possess at least four overlapping areas of specialized knowledge and related competencies (see Figure 1):

- He or she must have a thorough understanding of the mathematics content (e.g., Ontario Curriculum expectations as listed/described within the five mathematics strands).
- He or she must also master, and be able to utilize with confidence, an ever-expanding range of mathematics resources for teaching, such as manipulatives (e.g., linking cubes, GeoBoards, Pattern Blocks, Algebra Tiles) and technologies (e.g., graphing/CAS calculators, computer software, Internet web resources, interactive whiteboards).

- He or she must develop what Deborah Ball and colleagues (Ball & Bass, 2000; Thames & Ball, 2010) have described as unique “mathematics knowledge for teaching” (MKT), or a pedagogical awareness of how mathematical topics are connected, and how students acquire, organize, and communicate their mathematical thinking (including an awareness of common student misconceptions and alternative solution strategies).
- He or she must incorporate balanced instructional planning, featuring both the more traditional methods of teacher-directed lessons and skill-based practice (e.g., notes, textbook questions), combined with regular opportunities for rich, interactive problem-based learning (PBL).

Figure 1. Four components of effective mathematics teaching and learning

While all four of the components described in Figure 1 are significant aspects of teacher practice, for the purposes of this article, and based on the themes emerging from the collected research data, we will focus primarily on Mathematics Content Knowledge and Balanced Instructional Planning.

Problem-Based Learning (PBL)

1 For more information regarding the use of, and research relating to, Mathematics Resources (manipulatives, technology, Internet), the reader may wish to refer to the Canadian Council on Learning’s report entitled Lessons in Learning: Promising Practices in Primary Mathematics Instruction (Canadian Council on Learning, 2009) and to the insightful, regularly occurring Gazette articles/columns (e.g., Abacus activities, Technology Corner). Canadian research in the area of Mathematics Content Knowledge (Kajander & Mason, 2007; Kajander et al., 2010) is also recommended for further reading.
Medical professor Dr. Howard Harrows originally developed problem-based learning at McMaster University in the 1970s. Problem-based learning (PBL), although not synonymous with “problem solving” in the literature, often features overlapping principles or characteristics (e.g., learning is based on challenging, open-ended problems; students often work in collaborative groups; teachers often assume the role of “facilitator” of learning).

Problem solving has certainly received an increasing amount of attention within the reform-oriented mathematics education agenda. For example, The National Council of Teachers of Mathematics (NCTM) strongly argued for a problem-solving focus in its Principles and Standards for School Mathematics (2000) document. Further, the Ontario Reviwed Curriculum: Mathematics (Ontario Ministry of Education, 2005a, 2005b, 2007) lists problem solving first among its seven core mathematical processes. It has been proposed that all seven of these processes (i.e., problem solving, reasoning and proving, reflecting, selecting tools and computational strategies, connecting, representing, and communicating) can be meaningfully practised within a classroom where PBL is regularly being implemented (Consortium of Ontario School Boards, 2005; Jarvis, 2008; MacMath, Wallace, & Chi, 2009). For the purposes of this paper, we would like to suggest that PBL often encompasses the three stages of the popular Van de Walle “3-part lesson.” These three parts are listed as: the introduction of the activity (verbally and/or via demonstration), the exploration/activity itself (which may occur individually, with a partner, or, more commonly, in small groups; also, which may occur during one class period/session, or which may extend over several sessions), and the whole-class debriefing/discussion that follows (Van de Walle & Folk, 2008).

**Collaborative Action Research in Mathematics Education**

In 2008–2010, we were involved in a provincial initiative entitled Teachers Learning Together: The Math Journey, which was implemented by the Elementary Teachers’ Federation of Ontario (ETFO). As such, we were part of one of the research teams who provided support to school-based teacher teams, and who were empowered to create, implement, and report on collaborative action research projects throughout the province. Many of the action research projects focused on problem-based learning in mathematics classrooms. The remainder of this article will highlight two key concepts that have emerged from the project cycle as we have conducted interview conversations with teacher teams, namely, what we’ll refer to as the “messy time” of problem-based teaching transition, and the “balanced instructional programming” with which every teacher must arguably struggle in order to find a personal/professional comfort zone which they feel is effective in their daily work of teaching mathematics (Ontario Ministry of Education, 2004, pp. 7–9).

**Teacher Beliefs**

Each teacher team selected, or had appointed for them, a “Lead Teacher” who would act as facilitator and general organizer for the group in supporting the ongoing research project. During the Lead Teacher meeting in February 2008, one of the participants drew our attention to the notion, based on her own observations, that when teachers begin to move toward a more PBL-based curriculum, they often experience an uncomfortable, second-guessing, “messy time” of sorts. This messy time is presented as natural and necessary, as teachers work through the process of being less directive in their methods, and more open to the structured unpredictability of both the mathematics activity and the debriefing that follows. The original exchange follows:

**Interviewer:** I think you said something earlier along the lines of when you started for the first month or two, it was “messy” or “fuzzy” or did not have a lot of movement? You were thinking that this was going nowhere with your teachers, but in retrospect, this time was necessary?

**Teacher:** You know what? It reflects two things that I said. One thing is that I notice that in my classroom..., when you start this whole problem-solving thing, it’s not very neat and tidy. When we started at the beginning with our team, everybody was a little bit in, and a little bit out. People were happy to talk, but... they’re not sure they’re engaged... That was such a very obvious shift in their belief about the role of the teacher... we could talk the talk, and we could all have that conversation... it might not be every day that they’re doing problem solving in their classrooms, but the shift is massive. So that piece is really exciting.

In subsequent interviews, by way of follow-up, we presented this idea of “messy time” to teachers within the various case study teams, asking for their comments on this idea. One team shared the following thoughts on this particular issue:
Teacher: I think I felt at first like, When am I actually teaching? They’re doing a lot of problem solving, and they’re showing me a lot of things, but when am I really giving the explicit instruction, and how am I actually teaching? At some points, I really felt like, they’re just running the show and why am I here?

Teacher: It takes a lot of time too. You have to give them time to work on the problems, and to talk about it, so there is less of us talking. I don’t think we’re used to that.

Another significant factor contributing to teacher uneasiness with a move toward PBL instructional strategies is that it often reveals how little teachers sometimes know about the mathematical content, in terms of being able to understand student solutions and/or recognizing student misconceptions. For example, in the following rich exchange, several teachers discuss their discomfort with their perceived absence of understanding of math which they feel is lacking in their own experience and yet necessary for successful PBL implementation:

Teacher: We have had this conversation about the content knowledge being something that we lack. We weren’t taught with understanding. We’re being told to teach the problem solving and we’re really moving in that direction, but unlike a lot of other subject areas, we’re missing some content knowledge.

Teacher: So, because we weren’t taught for understanding, we’re missing some of those pieces, and so far, our board hasn’t paid much attention to mathematics at all, so we’re not developing our content knowledge in that way... I’m pretty much able to get my head around Grades 3 and 4 concepts, but I’m still, every day, thinking, “Wow. That makes a lot of sense.” Just realizing after 15 years of teaching it and having obviously passed Grades 3 and 4 that, “I never thought of that before.”

Teacher: Yes. I’m sitting there thinking, “I’m not smart enough to do this myself, so how can I expect these kids to do it?”

Regardless of the frustrations regarding weak content knowledge, awkward classroom implementation, and a perceived lack of control, all of which were shared as part-and-parcel of the reform-based mathematics teaching journey, several of the teachers in the study indicated that, in retrospect, their beliefs had indeed shifted throughout the year. The following excerpt notes a teacher’s changing beliefs regarding both mathematics teaching and math education research, in which he had now played a part:

Teacher: People are doing research all the time. I’ve never thought of ever participating in anything like that before until I had this experience. Obviously now, I think I understand why people are doing it... Because we have been focused and concentrating on our problem-solving situation, my whole opinion has changed about math, about teaching, about what children can do, about what the expectations are, where we’re going. I think for that reason in itself, it’s given me some indication that research is really important and it’s a good thing that people are doing it other than from the personal point of view... We do have something to share with people.

Not only are teacher beliefs crucial in the examination of “messy” time, but also those of students.

Student Attitudes

The “messy time” of adjustment, as described in the last section, also affects and challenges students in different ways, regardless of their prior achievement in the more traditional classroom. For example, weaker-performing students can often rise to the occasion of creative problem solving and ultimately find themselves in roles of leadership and receiving peer admiration and positive feedback for their PBL contributions. Other students, who have perhaps been the highest achieving within a more directed approach, often find themselves deeply confused and upset, as not only the classroom experience has drastically changed, but different aspects of learning are now being valued in private conversations (teacher/student) and in public (carpet time) discussions.

As one teacher explained, “It’s usually hardest for the ones who have traditionally done well. They’re not accustomed to understanding. They have no need for that, and they want that direction from you. They want to follow your instructions.” Another teacher noted, “I have children who have done extremely well in math... the way I was teaching math last year. A student was getting Level 4’s in absolutely everything. Her test papers were perfect scores—even the bonus question, and she is struggling with this kind of work.” A third teacher shared similar observations, “The smart ones—the ones who are traditional math students, who are good at memorizing and following the procedure, those are the ones who object the most, even in Grade 3.”

The following teacher documents a perceived progression in student attitudes toward PBL:
Teacher: [T]he kids were feeling discombobulated at first because I wasn't showing them how to do something. As a result, I had some students a couple months ago saying, "Are we going to use the textbooks soon?" That's what they were comfortable with, which was really difficult for me to hear. But after, I would say a four-month period, boom! We're sliding and we're just speeding along and everybody is happy, and we're content. I would say it was kind of a steep learning curve at first. They didn't like the fact that... I wasn't telling them how to do things, and that made them very uncomfortable.

One teacher explained how PBL with very young children sometimes led to emotional responses, even among the stronger students within the class:

Teacher: I teach kids that are much younger than the rest, and I just found that as soon as I asked them to explain what they did, they started erasing. They immediately thought, "What I've done is wrong; otherwise why would she ask me to explain it?"... Ironically with my stronger students in my class, I had a lot of tears... They're seven years old, so there are a lot of tears in Grade 2. I found it wasn't the weaker kids who you might expect are lost... It was the older kids who are used to getting everything, and they're used to having A's, and suddenly you're not telling them what to do, so they have to just figure it out. When they start finding it difficult, or the solution isn't obvious to them, they aren't used to having to just power through. It took... about the same—probably about the end of November [until] it was starting to get better.

Students' reaction to this very different way of experiencing mathematics learning was not entirely unexpected. As one teacher noted, if a student is "not used to being in a classroom where student thinking is encouraged and respected, then that is something that has to be a part of the training process for any children, in any grade really. If they're not used to having that kind of approach, then there is a lot of retraining and reteaching them that."

Notwithstanding the above frustrations for students, a number of teachers attributed increased overall engagement in mathematics learning to the problem-based approach:

Teacher: I find they're much more engaged, and for us it's been very collaborative—often they're working on problems together, so they're exchanging thinking... They've had to shift how they think about math, but it hasn't been a difficult change for them. I'd say that they're enjoying math and they think of themselves as good problem-solvers.

For some teachers, what most stood out was the delight in seeing individual students who had not met with mathematical success in the past really starting to shine in the new environment. One teacher described a student who was new to the school and told her at the start of the year the he did not like math. But things changed during the year: "It's very interesting because he's doing extremely well with [problem-based mathematics], and he has all the inventive strategies, so it's really nice to see him moved around to different groups because he's helping the thinking of others."

And a teacher with another case study team noted: "[I]t's quite amazing. She's really come a long way. Her confidence—you can see it in the classroom. So, kids will go to her [for support]."

Further, one participant asserted that PBL is advantageous for all students, even those with special needs, as it allows multiple entry points and ways of thinking, representing, and communicating:

Teacher: Even our special education people who are the advocates of traditional [math]... they've been part of the process of learning and going to different locations to see problem solving in action, and they're slowly converting to the fact that quite the opposite is true. People definitely need... to know number facts because you don't see relationships between numbers if you don't know those number facts. [But] if all your problem-solving energy goes into a basic fact, you have no brainpower left to do the real mathematics, the real problem solving at those levels. Everybody has an entry point. Those kids who typically struggle aren't disengaged anymore because they're all involved... I think when students didn't do well, in the back of my mind, I heard that comforting expression, "Oh, well, this doesn't work for some kids." I wasn't doing it right, that's why—but I don't believe that anymore at all. I haven't seen a student who doesn't benefit, although the ones who typically do well, do resist. They don't respect other people's ideas. They think they know more, but when they come around, they do well... It's a real life skill to listen to other people.

A move to PBL not only affected the teachers involved, but also their students who daily observe their teachers' reading, planning, experimenting, and communicating with colleagues.

Teacher: Even my students, and their [colleagues'] students, knowing that we do this together on a regular
basis has a huge impact on kids' understanding—that learning is real and authentic. Not just something that you do at school. It’s lifelong and teachers don’t have all the answers. They work together and they know exactly what we talk about because we go into each other’s classrooms and we share that with the kids. That makes a big difference. There’s a real sense of community value to this.

Transitioning to a curriculum delivery model that incorporates PBL experiences not only affects the beliefs and attitudes of teachers and students involved, it also includes a very basic, pragmatic question: How often do I plan for PBL-type activities and how does this relate to overall instructional planning?

Striking a Balance in Instructional Programming

Problem-based learning is not new for many teachers in Ontario. Recently, a series of professional development activities have focused specifically on this teaching strategy. Boards are at different stages of implementation and teacher readiness. As one teacher described, the expectations from the Ministry of Education are now clearly along these lines, and, in her particular case, she felt that the head start that her school had experienced regarding PBL methods, had placed them, as a result, in a relatively good position in terms of the encouraged reform.

Teacher: One of the things we had at the end of the year... all of the schools in the board sent one teacher to one of two sites, to a session where they were introduced to the problem-solving model. And the message at that session—so it was board-wide—was, this document—depending on whether you’re K–3 or Grades 4, 5, 6—is now four to six years old. So now it’s not a matter of, “Are you comfortable with it?,” it’s an expectation in Ontario that you will teach through problem solving—not occasionally do problem solving — and that the three-part lesson is to be happening in all classrooms... We’re in an advantageous situation now because we’re not going to have that same awkward, uncomfortable feeling about having to put into practice something that now is going to be a board expectation, so that’s a really nice place for us to be.

One of the most difficult parts of implementing a PBL approach to mathematics teaching and learning is to know how to find “balance” in one’s planning. Teachers struggle with questions of how often, when, and to what degree PBL should be used in their classroom. Does drill-and-practice have any place within a reform-based curriculum? How should difficult questions from parents/guardians be handled when the topic is the content and flow of the mathematics curriculum, particularly in light of Grades 3 and 6 EQAO assessment? How does one plan for split-grade classes? What about students with special needs and ESL/ELD students—do the proposed advantages of PBL apply to all student populations?

Interviewer: Is every day a problem-solving-oriented lesson, or do you have a couple of times a week where... it’s going to be standard skills development?

Teacher: I’m in both places... [M]y last actual test that I did, the whole test was just problem solving... Basically, in our long-range plans, we’ve always gone with the expectations and checked them off. I’m bouncing around. I’m doing both. Where I find that the children aren’t really that knowledgeable, I have gone back to doing the standard algorithms for multiplication. We’ve done a lot of inventive strategies, but the document states that they do need to be able to use it with manipulatives, or whatever, and do a four-digit by a two digit. We’ve been struggling with that, but I’ve done a lot of the hands-on and a lot of the manipulatives. I have found that for those who didn’t have the understanding... I’ve been using words like “groups of” and “sharing.” I’ve been doing that for years, so I don’t understand when the kids don’t get that. I guess I’m just bouncing back and forth, but I’m doing probably more so now—more problem-solving approach most of the time.

When we use the term “balance,” we would like to be clear that we do not necessarily mean equal division of time between traditional and reform strategies/activities; nor do we wish to imply that an achieved focal “balance” would be fixed in the case of each individual teacher—rather, it would likely be more variable in nature, depending on other significant factors such as students’ abilities/interests, the teaching context, and available resources. Whereas some teachers are convinced that all learning should be done using a PBL model (no doubt a minority of teachers), others argue that striking a “balance” between direct teaching and PBL activities requires ongoing classroom experimentation and reflection. Ideally, a reform-based teaching approach would be implemented by all teachers of mathematics on a continual basis in some form or another, recognizing that each teacher needs to regularly (each year/course) establish her or his own “reality” of what is working best for students and himself or herself.
Teacher: You can incorporate that [practising basic skills] into your problem-based learning to allow you to do some more guided math, so that they’re... practising it in the class.

Teacher: Exactly. I think it’s back to... Do you have to do a problem every day to be teaching through problem solving? No. You don’t have to be doing a problem every single day, but I think it’s hard to feel like you’re in between. I think people feel like you’re either here or there.

Time required to prepare PBL lessons, particularly when one is first beginning to use this approach, was cited as being one of the main challenges in striking the curricular “balance.”

Teacher: I find that it’s taking me more time to prepare because I have to rethink the way I did stuff. So, I’m trying to do more stuff where it’s more verbal and fewer paper-pencil tests. So, it’s because it’s new—I know the math program, but to find new ways to do it. We have a lot of good resources, but it’s to go through the resources and find what I’m looking for, for what I want to teach, in a different way—I find that quite time-consuming.

Sharing of developed resources within schools, or across schools within District School Boards, was therefore seen as highly beneficial, something that could be facilitated within existing or targeted professional development initiatives.

Concluding Thoughts

In using the term “messy time,” we are not specifically referring to the messiness of the classroom during a problem-based learning activity, although this may very well be the case when compared to the tidiness of the direct teaching classroom. Nor are we referring to the “ordered chaos” that may also characterize a classroom in which PBL activities are happening—the active communication of students and teachers during such a session may indeed seem relatively noisy to the observer. Rather, we refer specifically to the “messy” transition—the perceived frustration and internal questioning—that we believe often takes place within the minds of teachers who are brave enough to begin to implement these changes in their classrooms. Having observed instances of this transition in schools, and having talked to teachers who are experiencing both the excitement and discomfort of this transition process, we believe that “messy time” may represent an essential, and ultimately beneficial, aspect of teacher instructional transition. This disconcerting, and perhaps even painful, process serves to: (i) solidify teacher beliefs/attitudes regarding reform-oriented practice; (ii) echo the intellectual discomfort (i.e., “cognitive dissonance”) of students experiencing PBL in their classes; and (iii) reify the PBL claims and rhetoric found in the mathematics education literature through actual classroom-based cycles of implementation and reflection.

How can administrators and coordinators help mediate the messiness? The teachers that took part in the collaborative action research project that has been described in this paper would no doubt recommend some form of ongoing peer support mechanism, whether funded (release time) or at least encouraged by the administrator and local board. Reassuring teachers that the first few months of PBL implementation may indeed be accompanied by feelings of frustration and doubt, but that these are normal, beneficial, and often accompanied by “breakthrough” moments of student insight and overall increased student engagement, represents a simple yet perhaps highly significant method of bolstering teacher risk taking:

Teacher: I think that’s the huge shift in moving from very teacher-centred instruction to... more focused on the needs of individual children. Rather than just having a blanket lesson... [wherein] you’re making the assumption that everybody is entering it at the same point, and you lose a lot of children that way, in the problem-solving approach, you can meet the needs of individual children. You may be doing it in your explicit instruction one-on-one. You may pull a little group together when you see they’re having a little bit of a problem. I think that the big shift for us as teachers is to move away from that teacher-centric kind of approach.

Helping teachers who have begun the PBL journey to strike the elusive “balance” in their daily instructional planning is perhaps a much more difficult endeavour—one that will no doubt require an equivalent “messy time” transition experience for those in administration and teacher support positions.

References

Teaching Through Problem Solving in Grade 9

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Rob has been a high school math teacher for the past 10 years and is currently a math coach and department head at St. John's College in the Brant Haldimand Norfolk Catholic District School Board in Brantford, Ontario.

The first thing I want to address in this article is what the phrase teaching math through problem solving means. The answer is... I don't really know what other people mean, but in this article, I would like to share some of the things I have learned about what it looks like and feels like to me. I want to talk about the issues that require attention, and concerns that people have that might not really be problems after all. I want to share with you some of the problems that I used most successfully. Finally, I want to convince you to take the plunge and try it.

The first thing I want to share is that I was scared to death of teaching this way when I started. At one point this past semester, I was more than a week behind the other Grade 9 academic classes. I stopped asking where everyone was, as I was getting worried—kind of like jumping off the scales before it gets to the weight you don't want to be. Ignorance is bliss. When I realized how far behind I was getting, I almost gave up.

Jumping ahead, that's probably the day I learned the most about teaching through problem solving. I remember clearly thinking, "enough of the fun and exploring and getting to the core of the math, we have to start zipping through this stuff so that they see it all." At that moment, part of my brain yelled, "NO! Getting to the core of the math is what I want to do."

(Are you worried that I am advocating not covering some of the curriculum? Well, I'm not. Nor was I advocating it on that day. I had just decided I was committed to seeing my experiment through.)

Anyhow, I am happy to say that it has all worked out very nicely. I am extremely proud of what the students learned, and I taught through problem solving whenever I could. Let me explain that statement. In no way am I claiming I did this every day; sometimes I went a week or
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