Applications of Computer Algebra (ACA)
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ETS, Montreal, Quebec, Canada

Innovative CAS Technology Use in University Mathematics Teaching and Assessment: Findings from a Case Study in Alberta, Canada

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Dr. Michel Huneault (ETS Dir. of Academic Affairs) Opening Remarks

ETS History

- How to attract students who were perhaps weaker in math/science areas, but excited about "working with their hands and heads" … future engineers?
- Critical choice between downplaying or accepting high-power calculating (CAS) machines. They chose to **change their curriculum** [assessment?]
- Mandatory co-op program (experiential learning)

Dr. Gosia Brothers TI-Nspire Remarks

- **CAS-disabled feature** for assessment (e.g., IB)

Dr. Helmut Heugl Remarks

Multiple Representations key to Mental Models

- **Descriptive** (algebraic) + **Depictive** (graphic) representations allow for richer connections
- 2M of 2.5M brain nerve connections are related to the **optic nerves** in children; wired for the **visual**
- **Dynamic linking** is the key to conceptual learning

Dr. Paulina Chin Remarks

Assessment Tools in Maple

- Strong **emphasis on/commitment to education**
- **Maple TA** (DigitalEd + offered via Möbius products)
- Customized grading; student feedback; quiz types
- PracticeSheet; EssayTools; CompanionApp (beta)

Computer Algebra Systems (CAS) in University Instruction: An International Research Study in CAS Usage and Sustainability

Introduction

Social Sciences and Humanities Research Council of Canada, **International Opportunity Fund Development Grant** (2007-08); **Project Title:** *Implementing Computer Algebra Systems (CAS) in University Mathematics Instruction: Developing a Framework for International Research on Professional Development*

Social Science and Humanities Research Council of Canada, **International Opportunity Fund Project Grant** (2008-09 funding; 2009-12 project completion); **Project Title:** *Computer Algebra Systems (CAS) in University Instruction: An International Research Study on CAS Usage and Sustainability*

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Teaching Associate, Educational Research Methods & Mathematics Education
Faculty of Education, University of Cambridge, Cambridge, England

Graduate and Undergraduate Students Involved in Project

2010-11 Erin Brock, Master of Education (Full-Time), Graduate Research Assistant, NU
2010-11 Andrew Skelton, MSc in Mathematics (Full-time), Graduate Research Assistant, Brock U
2007-10 Neil Marshall, Undergraduate (Full-time), Research Assistant, Brock U
2009-10 Emily Kamphuis, Master of Education (Full-time), Graduate Research Assistant, NU
2008-09 Stacey Springall, Master of Education (Full-time), Graduate Research Assistant, NU
2008-09 Antonia Tereshchenko, Master of Education (Full-time), Graduate Research Assistant, Cambridge
2007-08 Trisha Kelley, Master of Education (Full-time), Graduate Research Assistant, NU

http://casresearch.nipissingu.ca/index.htm

Research Team Context

Lavicza's comprehensive study (2008a, b) featured an online survey of 1100 mathematicians as well as interviews with 22 mathematicians in three countries, namely, Hungary, United Kingdom, and United States, which examined **mathematicians' beliefs/conceptions regarding CAS and its instructional potential**.

Lavicza's findings showed some similarities, but also notable differences, between university- and school-level research findings (e.g., use of **CAS in one's research being the strongest factor influencing the use of CAS in one's university level teaching**).

Research Team Context

Jarvis, Buteau, and Lavicza (2008-09) implemented a mixed-methods research study to examine individual and systemic CAS usage in undergraduate mathematics instruction.

- **Extensive literature review (300+ papers)**
- **National survey of Canadian mathematicians** (302/1913; 15.8%)
- **Case study** of **two technology-enhanced mathematics departments** (Brock U, Canada; Sheffield Hallam U, England)
- **Two national workshops** at premier Canadian research institutes in both Quebec (in French, Montreal) and Ontario (in English, Fields)

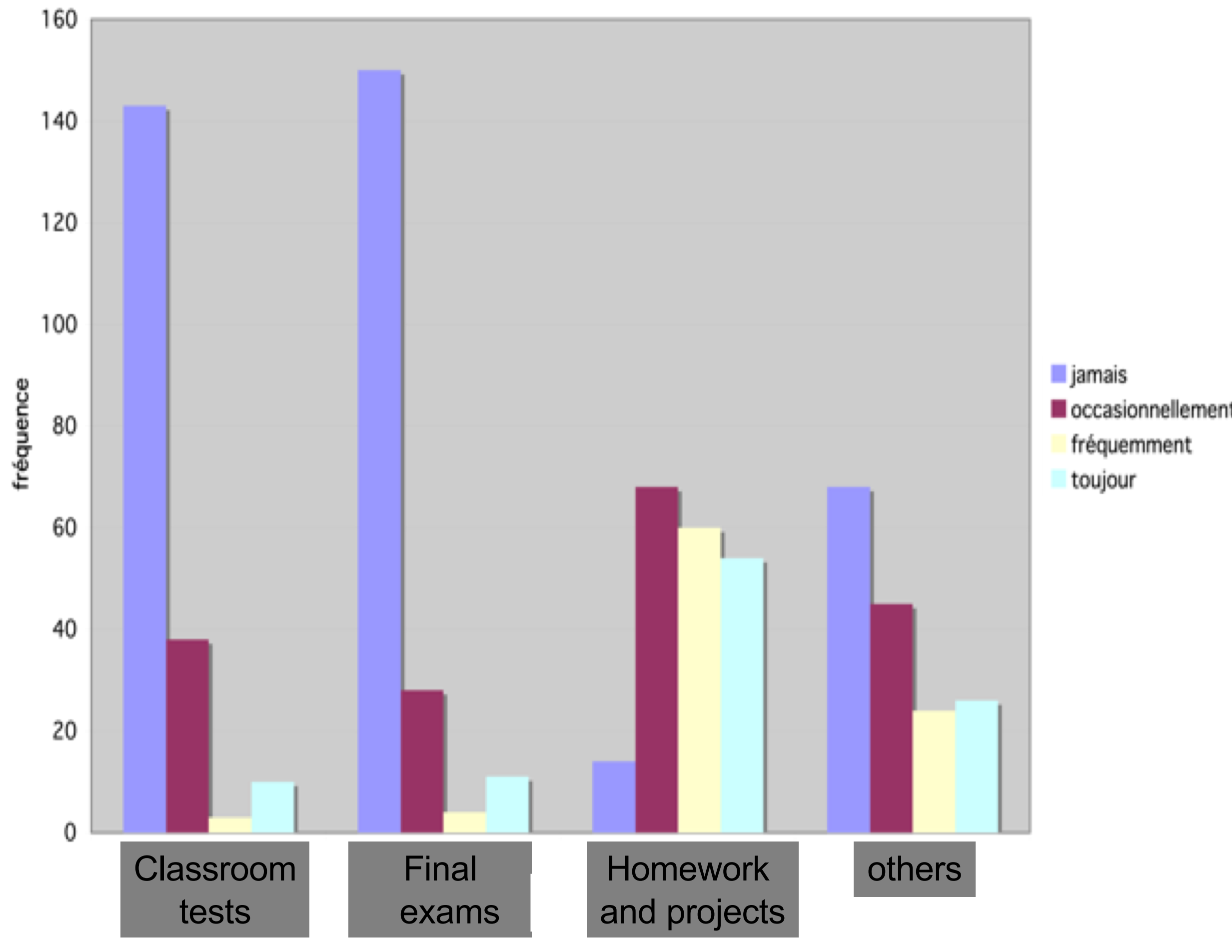
Some Findings:

- **Lit Review:** Categorized types of, affordances/barriers for tech integration; noticed a lack of reported research around tech/assessment at university
- **Survey:** Many of participating professors using CAS in their research (81%) *and* in their instructional practice (69%); reinforced the Lavicza finding that the greatest factor influencing the use of CAS in one's post-secondary mathematics teaching was the use of CAS in one's own research
- **Case Study:** Key factors affecting systemic departmental change
- **Workshops (2010):** Video-recorded keynotes and sharing/posting of ideas

Canadian Survey Findings

CAS in Assessment

Only **22.3%** of all CAS user responders integrate CAS, at least occasionally, in final exams and **26.3%** in classroom tests



Publications

Book Chapters

Buteau, C., & E. Muller. (2013). Teaching roles in a technology intensive core undergraduate mathematics course. In A. Clark-Wilson, O. Robutti, & N. Sinclair (Eds.), *The mathematics teacher in the digital era: An international perspective on technology focused professional development.* Berlin: Springer.

Refereed Journal Publications

Jarvis, D. H., Lavicza, Z., & Buteau, C. (2014, December). Systemic shifts in instructional technology: Findings of a comparative case study of two university mathematics departments. *International Journal for Technology in Mathematics Education, 21*(4), 117-142.

Jarvis, D. H., Buteau, C., & Lavicza, Z. (2014, December). Computer Algebra System (CAS) usage and sustainability in university mathematics instruction: Findings from an international study. *Research Journal of Mathematics and Technology, 3*(2), 126-138.

Buteau, C., Jarvis, D. H., & Lavicza, Z. (2014). On the integration of Computer Algebra Systems (CAS) by Canadian mathematicians: Results of a national survey. *Canadian Journal of Science, Mathematics and Technology Education, 14*(1), 35-57.

Marshall, N., Jarvis, D. H., Lavicza, Z., & Buteau, C. (2012). Do mathematicians integrate Computer Algebra Systems in university teaching? Comparing a literature review to an international survey study. *Computers & Education, 58*(1), 423-434.

Buteau, C., Marshall, N., Jarvis, D. H., & Lavicza, Z. (2010). Integrating Computer Algebra Systems in post-secondary mathematics education: Preliminary results of a literature review. *International Journal for Technology in Mathematics Education*, *17*(2), 57-68. [Full-text available with permission granted by IJTME]

Dr. Doran, U Alberta: TLEF Project

Dr. Charles Doran, professor in the Department of Mathematical and Statistical Sciences at the University of Alberta (UA) and Site Director of the Pacific Institute for the Mathematical Sciences (PIMS), received internal funding (2013-16) by way of the **Teaching and Learning Enhancement Fund** (TLEF). In conjunction with the TLEF funding, he was also named to the position of McCalla Professor of Science Chair. Dr. Doran had created a new upper level, joint (graduate and advanced undergraduate) computing and mathematics course entitled *Computing in Mathematics: Research via Experimentation* (MATH 497); and the funding also allowed him to focus research on an existing third year ***Mathematical Programming and Optimization*** (MATH 373) course in which SageMath, an open source mathematics software program, was being used by a Post-Doctoral Fellow, **Dr. Andrey Novoseltsev**, in new and creative ways in terms of mathematics teaching and assessment.

Jarvis and Buteau were invited by Dr. Doran to conduct case study research at the University of Alberta (2014-15), with a particular focus on the undergraduate Optimization (MATH 373) course taught by Dr. Novoseltsev.

U Alberta Case Study Research Question (Edmonton, AB, Canada, 2015)

What are the **perceptions** (e.g., benefits, barriers, other observations) of key stakeholders (i.e., project leader, instructors, students) regarding the **implementation of technology-enhanced (SAGE applets) mathematics courses** which involve new forms of **curriculum and assessment** practices?

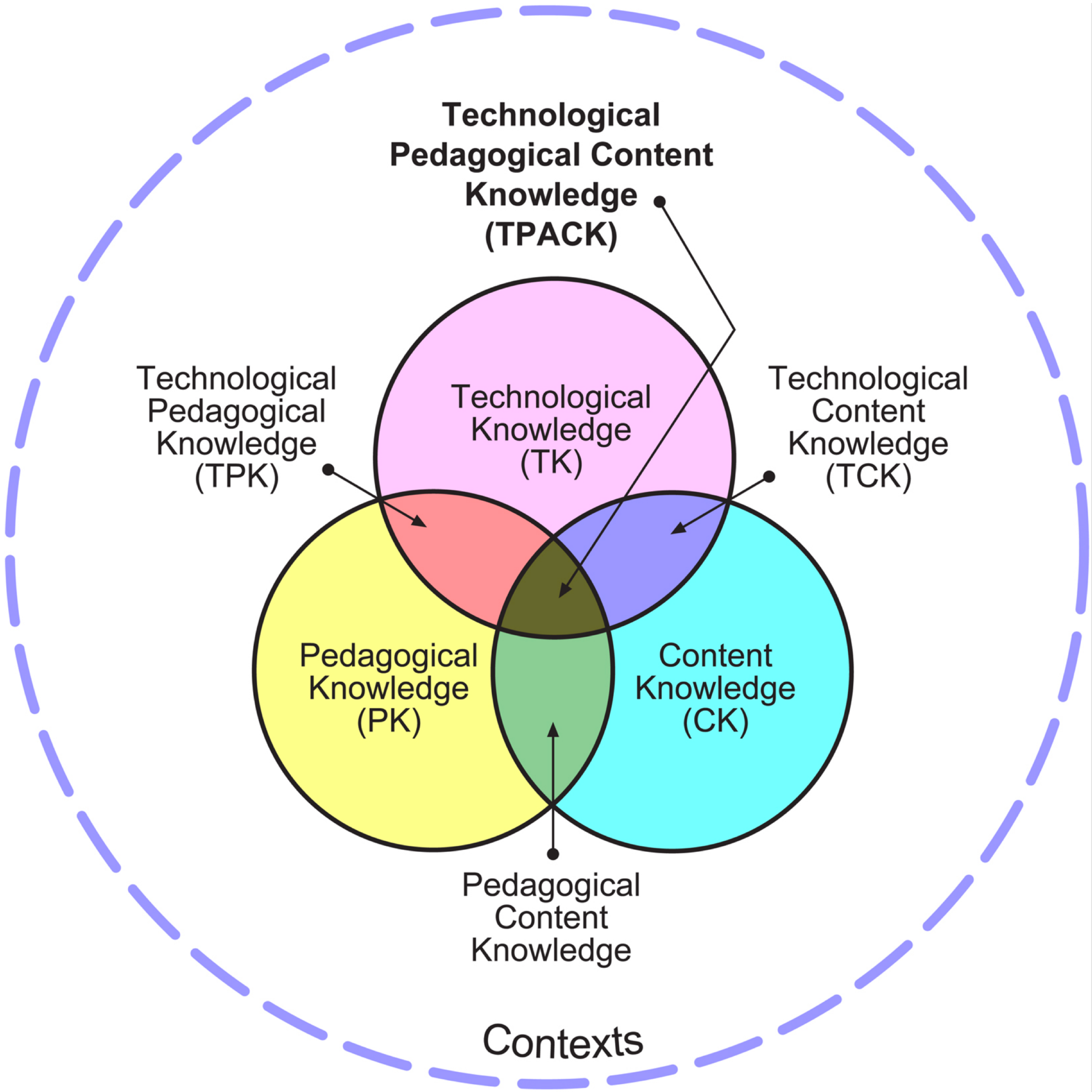
Literature Review

A growing number of international studies have shown that Computer Algebra Systems (CAS)-based instruction has the **potential to positively affect the teaching and learning of mathematics at various levels of the education system**, even though this has not been widely realized in secondary schools and in higher education (Artigue, 2002; Beaudin & Picard, 2010; Bray & Tangney, 2017; Bossé & Nandakumar, 2004; Kendal & Stacey, 2002; Lavicza, 2006; Meagher, 2012; Pierce & Stacey, 2004; Somekh, 2008; Smith Risser, 2011).

> "In addition to its computational power, modern technologies can help increase **collaboration** and bring about more of an emphasis on **practical applications of mathematics**, through **modelling, visualisation, manipulation and the introduction of more complex scenarios**. . . . For these reasons, the use of technology in mathematics education is becoming **increasingly prioritised** in international policy and curricula (Bray & Tangley, 2017, p. 256)."

Literature Review

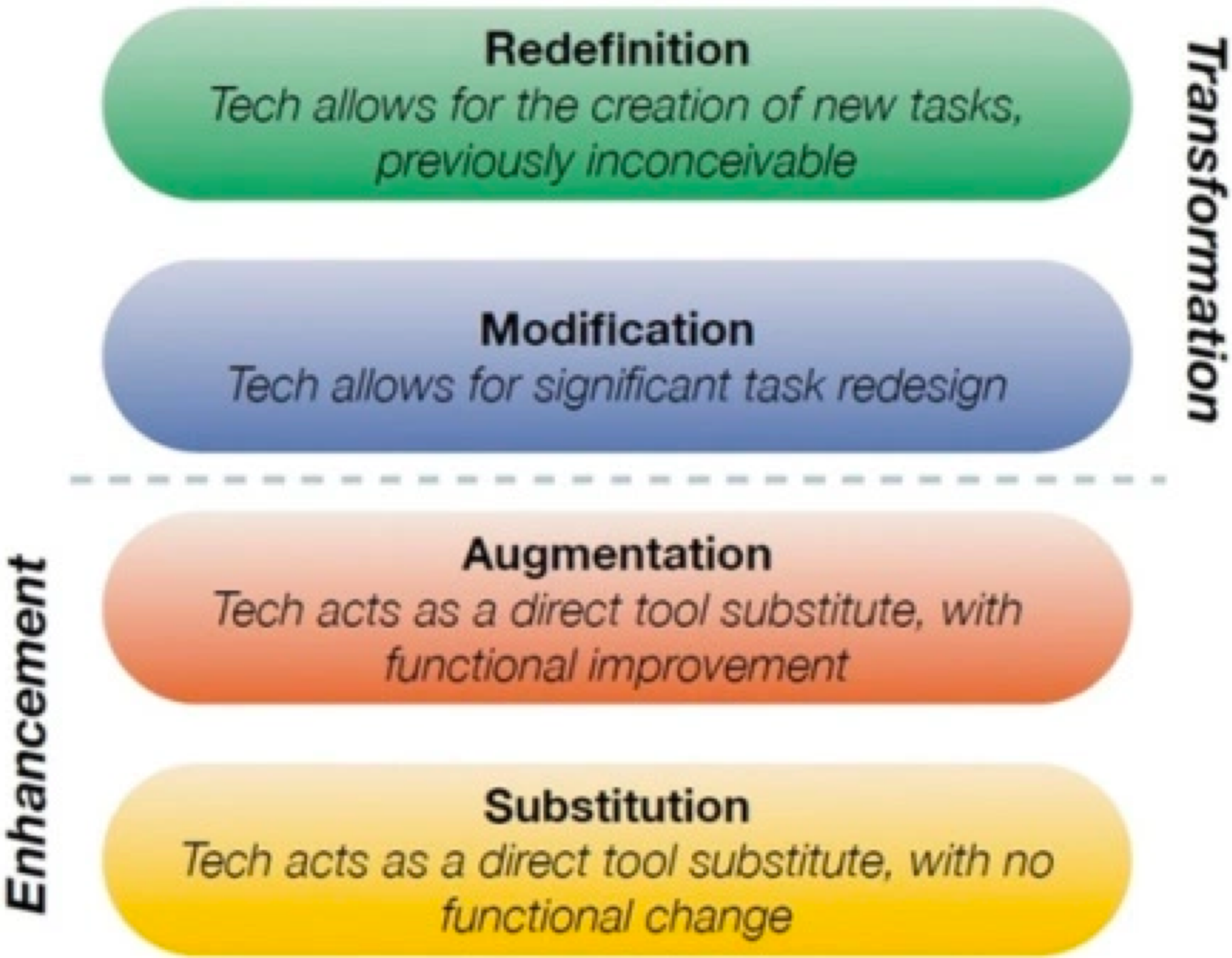
Koehler and Mishra (2009) have developed their own **Technology, Pedagogy, and Content Knowledge (TPACK)** model for the analysis of teacher practice: TPACK is the basis of effective teaching with technology, requiring an understanding of the representation of concepts using technologies; pedagogical techniques that use technologies in constructive ways to teach content; knowledge of what makes concepts difficult or easy to learn and how technology can help redress some of the problems that students face; knowledge of students' prior knowledge and theories of epistemology; and knowledge of how technologies can be used to build on existing knowledge to develop new epistemologies or strengthen old ones. . . . Teaching successfully with technology requires continually creating, maintaining, and re-establishing a dynamic equilibrium among all components. (Koehler & Mishra, 2009, pp. 66-67)



Technology, Pedagogy, and Content Knowledge (TPACK) model; Koehler & Mishra (2009)

Literature Review

Another informative taxonomy for understanding the different uses of technology in mathematics instruction is the **Substitution Augmentation Modification Redefinition (SAMR)** model created by **Puentedura** (2006, 2014). Although the SAMR model has been criticized on several points (diverse interpretation/application of the model; an absence of context; an overly rigid structure; and emphasizing product over process), it remains an increasingly popular tool for practitioner reflection and planning (Hamilton, Rosenberg, & Akeaoglu, 2016).



Substitution Augmentation Modification Redefinition (SAMR) model; Puentedura (2006)

Literature Review

Somekh (2008) described this difficult yet required paradigm shift relating to instructional technology as follows:

> "The pedagogical adoption of ICT is complex and requires an integration of vision, system-wide experimentation and new roles and relationships for teachers and students. . . . The affordances of the Internet, digital photography and cyberspace are **radically changing how knowledge is constructed, represented and accessed in the world** outside school, and policy-makers need to acknowledge this and **restructure the systems of curriculum, assessment and school organisation** (p. 458)."

Literature Review

In categorizing **barrier types** as either **first-order (external)** or **second-order (internal)** in nature, **Ertmer et al.** (2012) have provided a helpful set of related definitions:

> "First-order barriers were defined as those that were external to the teacher and included resources (both hardware and software), training, and support. Second-order barriers comprised those that were internal to the teacher and included teachers' confidence, beliefs about how students learned, as well as the perceived value of technology to the teaching/learning process. Although first-order barriers had been documented as posing significant obstacles to achieving technology integration . . . , underlying second-order barriers were thought to pose the greater challenge (Ertmer, Ottenbreit-Leftwich, Sadik, Sendurur, & Sendurer, 2012, p. 421)."

Literature Review

Greg Oates, U Auckland

“Effective integration of technology into the teaching and learning of mathematics presents a **significant challenge** to tertiary mathematics educators. **Assessment issues** in particular are widely considered in the literature as a critical factor in technology implementation. . . . With respect to assessment, both **pedagogical consistency**, and the **impact of CAS on examination questions**, are seen as particularly significant issues. . . . For content, the findings reported here support the complexity of assessing the values of topics, and support the overall conclusion that a re-examination of the changing pragmatic and epistemic values of specific topics, and **the goals of mathematics education, within a rapidly evolving technological environment**, remains a pressing challenge for undergraduate mathematics educators.” (2011)

Taxonomy Component	Characteristic Survey Response for Taxonomy Component
Access	"It has many benefits if all the students can reach almost the same technology; otherwise it creates important differences between them. I would like to see all my students using laptops, as in the private universities." (Uruguay)
Assessment	"Students may use any hand held calculator, but in exams they must show full written working to reach the answer. Calculators are often used to check results". (Australia)
Organisational Factors	"Bureaucracy slow to change. Use often isolated to single course." (South Africa)
Mathematical Factors	"Less emphasis on techniques, more powerful visualisation." (New Zealand)
Staff Factors	"Technology should be integrated only by staff who believe it is useful. Imposition of technology seems to have a negative effect on all involved." (Australia)
Student Factors	"It's difficult (for students) to make sense of the use of technology, especially those who had High School maths teachers with strong opinions against the use of technology." (Canada)

Taxonomy of Integrated Technology (2011); Oates

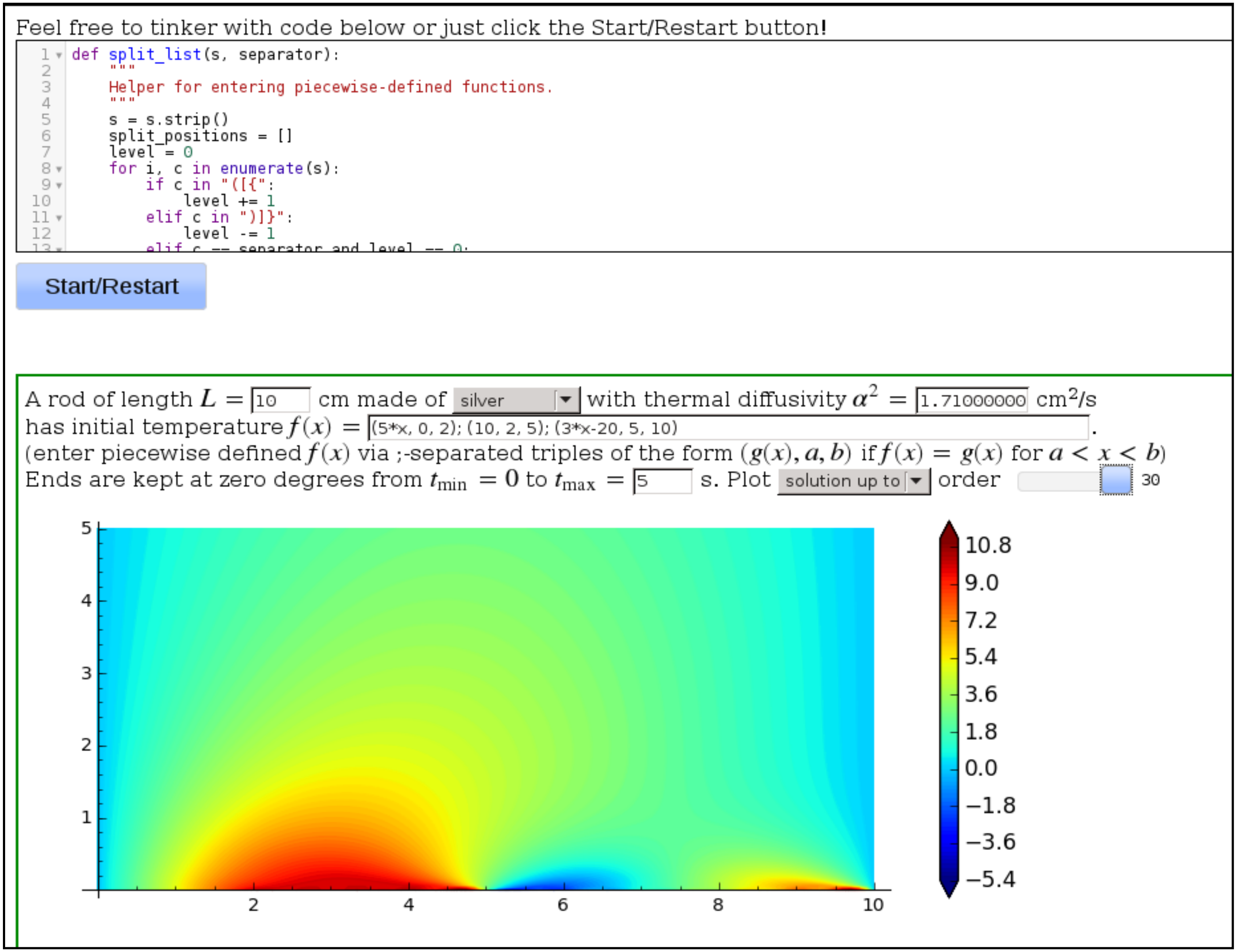
Literature Review

In contrast to the growing body of research focusing on **CAS technology use at the secondary school level** (Connors & Snook, 2001; Fey, Cuoco, Kieran, & McMullin, 2003; Haapasalo, 2013; Kieran & Drijvers, 2006), there is **relatively little parallel research at the post-secondary level** (Buteau & Muller, 2014; Decker, 2011; Oates, 2011; Rosenzweig, 2007; Stewart, Thomas, & Hannah, 2005; Tobin & Weiss, 2016; Thompson, Ashbrook, & Musgrave, 2015).

This is particularly true in the area of **student assessment**, where powerful technology tools such as CAS computer software and CAS-enabled calculators have rarely played a part in formal evaluation in undergraduate mathematics courses (Heidenberg & Huber, 2006; Sevimli, 2016).

SageMath

Created by **Dr. William Stein**, SageMath (http://www.Sagemath.org/) originated at the University of Washington, but now represents an international project with many developers in dozens of different countries. SageMath is a freely available, open-source mathematics software system licensed under the General Public License (GPL).



Heat equation applet created by Novoseltsev using Sage software.

Question

Assuming you would like to begin to use (or have begun to use) CAS-based technology in your own math course assessment practices (assignments, examinations), what are some of the common **problems/challenges** that might be encountered?

- ❑ Technical
- ❑ Financial
- ❑ Pedagogical
- ❑ Philosophical/Beliefs

7 Iterations of Novoseltsev's Optimization Course

Installment	Sage Technology Used in Instruction	Sage Technology Used in Assessment
1. Fall Term 2011	▪ Wrote simple code to check his own lengthy calculations during the course ▪ Minimal use for class demonstrations ▪ Gave students access to it; very few used it	▪ Used applets to prevent exam question calculation errors in marking written papers ▪ Grader used to mark student work
2. Fall Term 2012	▪ Sage applets written for most topics ▪ Demonstration: Frequently used Sage in class lectures to incorporate student suggestions and to provide immediate feedback on overhead screen	▪ Sage worksheets with additional related commentary created and made available to students via the university LMS for completing assignments and for review ▪ No grader used to mark student work; cheating noticed on submitted assignments
3. Spring Term 2013	▪ Student-generated linear programming problems are required, and are to be used throughout the course in various modules ▪ Sage used to explore these problems	▪ Students read/respond to fellow student problems online in assigned peer groups ▪ Sage optional for assignments (could also do by hand, or using other CAS software)

Iterations of Novoseltsev Optimization Course

4. Fall Term 2013	▪ Sage moved to online platform with secure passcodes making it more accessible ▪ Sage now required for some assignments	▪ Sage required for three course tests, which led to implementation of strategies to prevent cheating during test writing in computer lab ▪ Required Sage use on final exam in lab
5. Spring Term 2014	▪ Extra, optional computer lab tutorials offered to small groups of students to familiarize them with Sage and LMS ▪ Created a step-by-step video for this also	▪ Three term tests with Sage replaced with one mid-term examination using Sage ▪ Mid-term/Final exams feature hand-written, optional Sage, required Sage use questions
6. Spring Term 2015	▪ Created more and improved online videos for LMS and Sage technical instructions	▪ Short exercises for students to explore included in his LMS posted lecture notes ▪ Mid-term/Final exams feature hand-written, optional Sage, required Sage use questions
7. Fall Term 2015	▪ Created additional applets for demonstration and teaching purposes	▪ Mathematicians at other universities become interested in Simplex Method applet ▪ Mid-term/Final exams feature hand-written, optional Sage, required Sage use questions

1. CAS Use in Optimization Problem Assessment

Student-Generated, Revisited Optimization Problems

<u>Chapter 2 Assignment: Problem Formulation</u>

1. Word Problem: Compose a word problem. You should start by practicing on some other problems (e.g. from course notes) and you may use them for inspiration, but please formulate your own problem in your own words and with your own numbers - do not look at other problems while writing yours. Taking "Dog Food" problem and replacing "Dog" with "Cat" throughout the text does NOT count at your own problem! Refrain from looking at others' submissions until you have posted your own. Your word problem should "make sense" to people who have never heard of Math 373 and optimization and be written in proper English, ask someone to proof read it, especially if English is not your native language. Points will be taken off for typos/mistakes/unclear sentences! While it may seem harsh, this is the problem that you will be working on for the rest of the term and that means your group members will have to deal with it as well—be kind to them and write accurately.

2. Linear Programming Problem: Formulate an LP problem corresponding to your word problem. Make sure to explicitly describe all involved decision variables: what do they represent and in what units are they measured. Explain the physical meaning of each constraint and how do you derive it (e.g., $3C + B \leq 1500$ means that the total amount of fertilizer used cannot exceed the available amount).

Once you are done, input your problem into Sage.

```
A = ([2, -1], [1, -1], [1, 0],)
b = (-1, 2, 1)
c = (1, 5)
P = InteractiveLPProblem(A, b, c)
P
```

1. CAS Use in Optimization Problem Assessment

3. Standard Form: Convert your problem to standard form. You are free to use Sage to do it automatically in a single step, but please explain in words what has to be done (not "use this command" but rather "multiply the second inequality by -1, replace the third equation with two inequalities, etc."). In standard form your LP problem must involve at least 5 decision variables and at least 4 constraints (not counting sign restrictions or constraints involving a single variable only). There is no upper limit: if you are so adventurous that it gets difficult to enter all the coefficient or display output, talk to me and we'll try to figure out how to deal with such a problem. If your problem has too few variables/constraints, go back to the beginning and make it more interesting! Of course, if you change your word problem, you have to adjust its conversion to an LP problem accordingly. There is no need to keep the "old" problem around. In addition, it would be nice if the numbers of constraints and variables are different for your problem. (This will help you to avoid confusion in duality theory.)

These commands have to return "True":

```
P.n_variables() >= 5
```

```
[r.nonzero_positions() >= 2 for r in P.A().rows()].count(True)
>= 4
```

It is not strictly required, but it would be better if this command returns "True" as well:

```
P.n_variables() != P.n_constraints()
```

4. Feasible Set: Adjust your problem (both words and formulas!), if necessary, to make sure that the feasible set is non-empty (i.e. the problem is feasible) and has at least 4 vertices, i.e. the following command should give "True":

```
P.feasible_set().n_vertices() >= 4
```

5. Solution: Use Sage to find the optimal value and an optimal solution for your problem.

What do these numbers mean in terms of your original word problem?

1. CAS Use in Optimization Problem Assessment

<u>**Chapter 3 Assignment: Simplex Method**</u>

1. Word Problem: Start with the word problem you have composed last time: your submission should include the word problem, description of decision variables, and formulation as an LP problem. (No need to keep derivation of each constraint or explanation of conversion to standard form.) If the initial dictionary of your problem is feasible, tweak the problem (both the word and formula versions, so that they continue to match) a little to make the initial dictionary infeasible and force you to go through the auxiliary problem phase! You still should make sure that your problem has a feasible set with at least four vertices, the number of decision variables is at least five, and the number of constraints involving two or more variables is at least four. Make sure also that your problem is bounded, so that you do have the optimal value and at least one optimal solution!

2. Simplex Method—Feasible Problem: Use the Simplex Method to find ALL optimal solutions and ALL BASIC optimal solutions of your problem! (If your problem has a lot of basic optimal solutions, find at least 3 of them.) You may want to watch the "Recovering from Wrong Choices" screencast on eClass for how to "fix mistakes." It is OK to use decimal approximations if precise computations look too ugly, but you need to keep at least 5 digits for each number. i.e. you should use RealField(20) or higher. Beware of approximations issues, however! See "Approximation Issues" worksheet. You are NOT allowed to submit work invoking run_simplex_method() command and I do not recommend using it at all until you have solved the problem yourself.

If your solution uses less than 4 iterations of enter-leave-update steps (combined for auxiliary and original problem), go back to your word problem, make it more complicated, and adjust all other steps as necessary. No need to include the old "simple" version in your submission.

1. CAS Use in Optimization Problem Assessment

3. Simplex Method—Infeasible Problem: Take the word problem you have been working on and slightly modify its constraints/parameters in such a way that the problem becomes infeasible (e.g., for Corn and Barley the requirement to grow at least 2000 acres of corn would do the trick.). Provide below your modified word problem and its formulation as a LP problem.

You can quickly check if your modified problem is indeed infeasible via

```
P.is_feasible()
```

Apply the Simplex Method to this problem to prove that it is infeasible. (You cannot just invoke run_simplex_method().)

4. Degenerate Dictionaries: Have you encountered any degenerate dictionaries while working on this assignment? If yes, give a clear reference to it. If no, explain whether it is possible for your problem to have degenerate dictionaries.

1. CAS Use in Optimization Problem Assessment

Student-Generated, Revisited Optimization Problems

Felix: I basically just took the framework of what he wanted, like just a linear problem. . . . It was about Jurassic Park II, and you had to pick which dinosaurs you wanted. Well, it was really simplistic. I had different types of dinosaurs and I said, well, the guys in Finance figured out which dinosaurs make how much money . . . from the increase in attendance from this. So, like, if you have a T-Rex, you get this much money per year. . . . Then you have space and land requirements and water—so, you have a limited amount of space, so you can't just have any number of dinosaurs in the park. And then, I imposed various constraints, so you have to have certain amount of land type for the walking ones, and then you have the water ones, and then the flying ones. . . . The thing I like about that is that you build familiarity with the numbers and the solutions, so every time you take the right steps, you reach something that's recognizable to you, instead of doing all new problems with new numbers each time.

1. CAS Use in Optimization Problem Assessment

Student-Generated, Revisited Optimization Problems

Dawn: Our professor actually gave us a little bit of a warning and said, “Make sure your problem is interesting because you’re going to be working with this for the rest of the semester.” . . . I really loved formulating my own problem. . . . It was about refrigerated coffee beverages. So, one is going to be decaf or light caffeine, and then one was going to be heavy caffeine. It was pretty generic. My different constraints involved the amounts of caffeine to milk ratio that you have to do, and how the sugar should be equal in both amounts. I had so much fun making it up. I remember at the end of it, I ran downstairs and showed my dad. He was like, “Okay, great—go away!” It was a fun assignment—I think it was my favourite.

2. CAS Use in Mid-Term Assessment

Appendix C: MATH 373 Mid-Term Examination (Spring 2015)

Instructions: Points WILL be taken off if you deviate from any of the following instructions:

1. Fill in the information above, including "Desk:" from a plaque along its top side.
2. Authenticate on the lab computer (you are not allowed to use your own device).
3. Start Mozilla Firefox web browser (not Chrome or Internet Explorer).
4. Go to [University of Alberta based url] (https:// is important!)
5. Press F11 to switch to full screen.
6. Log in to your account. You will see no worksheets—do not make any!
7. Once the test starts, make your own single copy of the published test worksheet.
8. You are not allowed to start or use any other program, access any other web site, create any other worksheets, or share/publish your test worksheet.
9. You are not allowed to use any run_... or possible_... commands.

This exam consists of 4 pages (including this title page) with 3 question(s). You can use any calculator without wireless capabilities. TURN OFF AND PUT AWAY ALL OTHER ELECTRONIC DEVICES. You can use one 2-sided sheet of notes in your own handwriting (do not submit it). You may not use any other notes or your own scratch paper. If you run out of space on the problem page, please use the back of the previous problem (which should be conveniently located on your right). Ask for more paper if it is still not enough. You must show your work on the exam paper with explanations in plain English. If a problem asks you to use a specific method, you MUST use this method. You may get zero credit for any other solution, even if it is correct. Each of the 3 questions are worth 10 marks, for a total of 30 possible marks. Good luck!

2. CAS Use in Mid-Term Assessment

1. Consider the following LP problem with a "mystery" constraint:

$$\begin{aligned} \max \quad & x_1 + x_2 \\ & 3x_1 - x_2 \geq 3 \\ & -x_1 + x_2 \geq 1 \\ & Dx_1 + Ex_2 \leq F \\ & x_1, x_2 \geq 0 \end{aligned}$$

(a) Give an example of the last constraint for which the problem is feasible, but there are no optimal solutions or explain why it does not exist.
(b) Give an example of the last constraint for which there are no feasible solutions or explain why it does not exist.
(c) Give an example of the last constraint for which (3; 5) is an optimal solution or explain why it does not exist.

2. Your company produces 4 types of fertilizer: A, B, C, and D. To produce 1 kg of fertilizer A you need 300 g of potash (P), 400 g of phosphate (H), and 300 g of nitrogen (N). To produce 1 kg of fertilizer B you need 300 g of P, 300 g of H, and 400 g of N. To produce 1 kg of fertilizer C you need 500 g of P, 200 g of H, and 300 g of N. Finally, to produce 1 kg of fertilizer D you need 400 g of P, 400 g of H, and 200 g of N. Suppliers can provide 40 kg of P, 40 kg of H, and 30 kg of N per day. Net profit is $20, $40, $50, and $30 per kilogram of A, B, C, and D respectively. Formulate an LP problem for maximizing the profit of your company. Make sure to clearly describe all decision variables, their units, and the physical meaning of each constraint. No need to simplify constraints and/or objective. You can also get 2 bonus points (but no more than 100% for the whole exam) if you find the optimal value and all optimal solutions using Simplex Method!

3. Solve the LP problem provided in the Sage worksheet using the (\Regular") Simplex Method and, based on your solution, write down the following information. Entering and leaving variables on each step:

Step	1	2	3	4	5	6	7	8	9
Entering									
Leaving									

The optimal value (or explain why it does not exist):
An optimal solution (or explain why it does not exist):
All optimal solutions (or explain why there is only one or none):

2. CAS Use in Mid-Term Assessment

Dawn: I think these three questions really reflected our last three assignments. . . . That's what I was really surprised about because it made me feel a little bit better as I was going through the midterm because I was familiar with the format. . . . I think it's really fair. . . . I don't think it's fair to get a student kind of use to that format, where you can just do the assignments online on your computer, but then when it comes to your mid-term, you have to switch that mentality and go straight back to pen-and-paper. I don't think that really tests what you've learned.

Brittany: The computer component is important because no one, like engineers and analysts, does things by hand. Especially with 50 variables—how are you going to do that by hand? But you should still know the theory behind it . . . but the [software] aid is helpful for real-life applications.

2. CAS Use in Mid-Term Assessment

Ezra: I think it's necessary. If you depend on Sage heavily throughout the whole course, and you are not allowed to use it on the exam, that wouldn't be fair. . . . I think it's a natural outcome.

Felix: If you're going to do the Simplex Method by hand, you're probably going to do it wrong. . . . Yeah, look, I don't see any other way you could test this, other than not using technology and making it really simple . . . but the exam would be too easy. And if you just, straight up, give someone a Simplex Problem, or a couple of problems, and then just tell them to do them by hand, it would just take way too long, and people would make too many small mistakes. . . . So, yeah, I think it was fair. . . . the questions test your understanding with and without the technology.

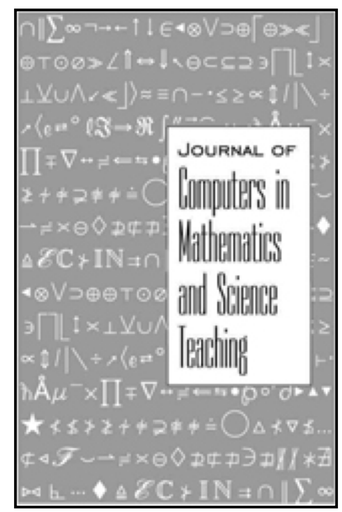
3. CAS Use in Final Examination Assessment

At the time of the site visit at the U Alberta, the final examination had not yet been written but was only a few weeks away.

Andrey: I'm trying to make it a 2-hour exam, but allow them three hours to write it, to give them some extra time so that they are not stressed. It will have problems that involve Sage, and so it will benefit from using computers. I will also try harder to make students show that they understand what the computer is actually doing. . . . And I really want students to demonstrate that they know how to set up the starting point for this situation, and then explain what steps they can do on it. If they want to, they can use computers to solve the problem and get the answer, but the task is not just to get the final answer, but to show how you can get to that certain point. . . . They will have seen the exam problems before, because I have taken them from the lecture notes, which they do have access to on eClass during the course. . . . I have not heard about other examples of using technology to this extent on exams in big courses. People have used it with maybe 10 or 20 students, but not for 60 or 90.

Jarvis, D. H., Buteau, C., Doran, C., & Novoseltsev, A. (2018). **Innovative CAS technology use in university mathematics teaching and assessment: Findings from a case study in Alberta, Canada.** *Journal of Computers in Mathematics and Science Teaching, 37*(4), 309-354. ISSN: 0731-9258

Abstract: In this paper, we report on a case study that focused on innovative uses of CAS technology in university mathematics teaching and assessment. The study involved a site visit to the University of Alberta campus (June 2015) during which: interviews were conducted with five mathematics faculty members and seven mathematics students; math lectures were attended; and artifacts were collected such as course outlines, software demonstrations, and assessment tools. Interviews were transcribed and the data entered into Atlas.ti qualitative research software for the purpose of thematic analysis. Findings center around the innovative use of the open source, CAS-based software both in the teaching (answer checking, interactive lecture demonstrations) and assessment (student-generated optimization problems, mid-terms, final exams) practices of one particular instructor who taught seven iterations of a Mathematical Programming and Optimization undergraduate course.



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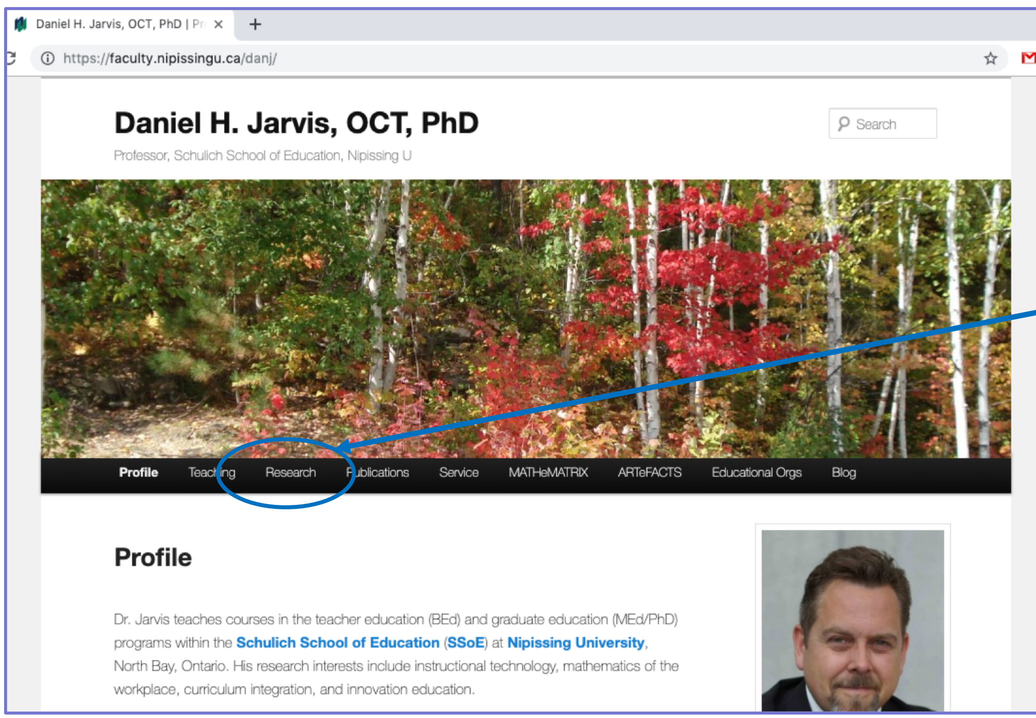
Thank you for your (Sat. am!) attendance and your attention.



Under *Research*, I have uploaded this ACA slideshow presentation.

https://faculty.nipissingu.ca/danj/