Workshop on homogeneous plane continua

L. C. Hoehn (loganh@nipissingu.ca)

on joint work with L. G. Oversteegen

Nipissing University

March 9, 2016
50th Spring Topology and Dynamics Conference
Baylor University

Continua: basics 1

Continuum $\equiv$ compact connected metric space.

Continua: basics 1

Continuum $\equiv$ compact connected metric space.

- *Arc*: space homeomorphic to $[0, 1]$

Continua: basics 1

Continuum $\equiv$ compact connected metric space.

- *Arc*: space homeomorphic to $[0, 1]$
- *Graph*: union of finitely many arcs meeting only in endpoints

Continua: basics 1

Continuum $\equiv$ compact connected metric space.

- *Arc*: space homeomorphic to $[0, 1]$
- *Graph*: union of finitely many arcs meeting only in endpoints
- *Tree*: graph with no cycles

Continua: basics 1

Continuum $\equiv$ compact connected metric space.

- *Arc*: space homeomorphic to $[0,1]$
- *Graph*: union of finitely many arcs meeting only in endpoints
- *Tree*: graph with no cycles
- *Hilbert cube* $= [0,1]^{\mathbb{N}}$, with metric d

Continua: basics 1

Continuum $\equiv$ compact connected metric space.

- *Arc*: space homeomorphic to $[0, 1]$
- *Graph*: union of finitely many arcs meeting only in endpoints
- *Tree*: graph with no cycles
- *Hilbert cube* $= [0, 1]^{\mathbb{N}}$, with metric d

Every compact metric space can be embedded in $[0, 1]^{\mathbb{N}}$.

Continua: basics 1

Continuum $\equiv$ compact connected metric space.

- *Arc*: space homeomorphic to $[0,1]$
- *Graph*: union of finitely many arcs meeting only in endpoints
- *Tree*: graph with no cycles
- *Hilbert cube* $= [0,1]^{\mathbb{N}}$, with metric d

Every compact metric space can be embedded in $[0,1]^{\mathbb{N}}$.

Hausdorff distance $d_H(A,B) = \inf\{\varepsilon > 0 : N_\varepsilon(A) \supseteq B \text{ and } N_\varepsilon(B) \supseteq A\}$

Continua: basics 1

Continuum $\equiv$ compact connected metric space.

- *Arc*: space homeomorphic to $[0,1]$
- *Graph*: union of finitely many arcs meeting only in endpoints
- *Tree*: graph with no cycles
- *Hilbert cube* $= [0,1]^{\mathbb{N}}$, with metric d

Every compact metric space can be embedded in $[0,1]^{\mathbb{N}}$.

Hausdorff distance $d_H(A,B) = \inf\{\varepsilon > 0 : N_\varepsilon(A) \supseteq B \text{ and } N_\varepsilon(B) \supseteq A\}$

X is *tree-like* if for every $\varepsilon > 0$ there is an ε-map from X to a tree

Continua: basics 1

Continuum $\equiv$ compact connected metric space.

- *Arc*: space homeomorphic to $[0,1]$
- *Graph*: union of finitely many arcs meeting only in endpoints
- *Tree*: graph with no cycles
- *Hilbert cube* $= [0,1]^{\mathbb{N}}$, with metric d

Every compact metric space can be embedded in $[0,1]^{\mathbb{N}}$.

Hausdorff distance $d_H(A,B) = \inf\{\varepsilon > 0 : N_\varepsilon(A) \supseteq B \text{ and } N_\varepsilon(B) \supseteq A\}$

X is *tree-like* if for every $\varepsilon > 0$ there is an ε-map from X to a tree
$\Leftrightarrow$ X is an inverse limit of trees

Continua: basics 1

Continuum $\equiv$ compact connected metric space.

- *Arc*: space homeomorphic to $[0, 1]$
- *Graph*: union of finitely many arcs meeting only in endpoints
- *Tree*: graph with no cycles
- *Hilbert cube* $= [0, 1]^{\mathbb{N}}$, with metric d

Every compact metric space can be embedded in $[0, 1]^{\mathbb{N}}$.

Hausdorff distance $d_H(A, B) = \inf\{\varepsilon > 0 : N_\varepsilon(A) \supseteq B \text{ and } N_\varepsilon(B) \supseteq A\}$

X is *tree-like* if for every $\varepsilon > 0$ there is an ε-map from X to a tree
$\Leftrightarrow$ X is an inverse limit of trees
$\Leftrightarrow$ There is a sequence of trees $\langle T_n \rangle_{n=1}^{\infty}$ in $[0, 1]^{\mathbb{N}}$ and maps $f_n : X \to T_n$ such that $f_n \to \mathrm{id}_X$ (uniformly)

Continua: basics 2

X is *decomposable* if $X = A \cup B$ for some subcontinua $A, B \subsetneq X$

Continua: basics 2

X is *decomposable* if $X = A \cup B$ for some subcontinua $A, B \subsetneq X$

Indecomposable = not decomposable

Continua: basics 2

X is *decomposable* if $X = A \cup B$ for some subcontinua $A, B \subsetneq X$

Indecomposable = not decomposable

Hereditarily indecomposable = every subcontinuum is indecomposable

Continua: basics 2

X is *decomposable* if $X = A \cup B$ for some subcontinua $A, B \subsetneq X$

Indecomposable = not decomposable

Hereditarily indecomposable = every subcontinuum is indecomposable

Theorem (Bing 1952)

A continuum is homeomorphic to the pseudo-arc if and only if it is hereditarily indecomposable and arc-like.

Continua: basics 2

X is *decomposable* if $X = A \cup B$ for some subcontinua $A, B \subsetneq X$
Indecomposable = not decomposable
Hereditarily indecomposable = every subcontinuum is indecomposable

Theorem (Bing 1952)

A continuum is homeomorphic to the pseudo-arc if and only if it is hereditarily indecomposable and arc-like.

X has *span zero* if any continuum Z in $X \times X$ with $\pi_1(Z) \subseteq \pi_2(Z)$ meets $\Delta X = \{(x, x) : x \in X\}$

Continua: basics 2

X is *decomposable* if $X = A \cup B$ for some subcontinua $A, B \subsetneq X$

Indecomposable = not decomposable

Hereditarily indecomposable = every subcontinuum is indecomposable

Theorem (Bing 1952)

A continuum is homeomorphic to the pseudo-arc if and only if it is hereditarily indecomposable and arc-like.

X has *span zero* if any continuum Z in $X \times X$ with $\pi_1(Z) \subseteq \pi_2(Z)$ meets $\Delta X = \{(x, x) : x \in X\}$

- Arc-like $\Rightarrow$ span zero
- Span zero $\Rightarrow$ tree-like, but span zero $\not\Rightarrow$ arc-like in general

Continua: basics 2

X is *decomposable* if $X = A \cup B$ for some subcontinua $A, B \subsetneq X$

Indecomposable = not decomposable

Hereditarily indecomposable = every subcontinuum is indecomposable

Theorem (Bing 1952)

A continuum is homeomorphic to the pseudo-arc if and only if it is hereditarily indecomposable and arc-like.

X has *span zero* if any continuum Z in $X \times X$ with $\pi_1(Z) \subseteq \pi_2(Z)$ meets $\Delta X = \{(x, x) : x \in X\}$

- Arc-like $\Rightarrow$ span zero
- Span zero $\Rightarrow$ tree-like, but span zero $\not\Rightarrow$ arc-like in general

Theorem (Oversteegen-H 2015)

A continuum is homeomorphic to the pseudo-arc if and only if it is hereditarily indecomposable and has span zero.

Simple folds

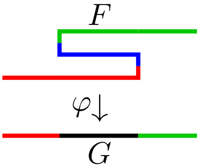
Simple fold on a graph G:

- Subgraphs $G_1, G_2, G_3 \subset G$ such that
  - $G_1 \cup G_3 = G$ and $G_1 \cap G_3 = G_2$;
  - $\overline{G_1 \setminus G_2} \cap \overline{G_3 \setminus G_2} = \emptyset$.
- Graph $F = F_1 \cup F_2 \cup F_3$ and map $\varphi : F \to G$ such that
  - $\varphi\restriction_{F_i} : F_i \to G_i$ is a homeomorphism for each $i = 1, 2, 3$;
  - $\partial G_1 = \varphi(F_1 \cap F_2)$, $\partial G_3 = \varphi(F_2 \cap F_3)$, and $F_1 \cap F_3 = \emptyset$.

Simple folds

Simple fold on a graph G:

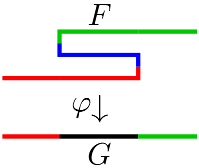
- Subgraphs $G_1, G_2, G_3 \subset G$ such that
  - $G_1 \cup G_3 = G$ and $G_1 \cap G_3 = G_2$;
  - $\overline{G_1 \setminus G_2} \cap \overline{G_3 \setminus G_2} = \emptyset$.
- Graph $F = F_1 \cup F_2 \cup F_3$ and map $\varphi : F \to G$ such that
  - $\varphi\restriction_{F_i} : F_i \to G_i$ is a homeomorphism for each $i = 1, 2, 3$;
  - $\partial G_1 = \varphi(F_1 \cap F_2)$, $\partial G_3 = \varphi(F_2 \cap F_3)$, and $F_1 \cap F_3 = \emptyset$.



Simple folds

Simple fold on a graph G:

- Subgraphs $G_1, G_2, G_3 \subset G$ such that
  - $G_1 \cup G_3 = G$ and $G_1 \cap G_3 = G_2$;
  - $\overline{G_1 \setminus G_2} \cap \overline{G_3 \setminus G_2} = \emptyset$.
- Graph $F = F_1 \cup F_2 \cup F_3$ and map $\varphi : F \to G$ such that
  - $\varphi\restriction_{F_i} : F_i \to G_i$ is a homeomorphism for each $i = 1, 2, 3$;
  - $\partial G_1 = \varphi(F_1 \cap F_2)$, $\partial G_3 = \varphi(F_2 \cap F_3)$, and $F_1 \cap F_3 = \emptyset$.



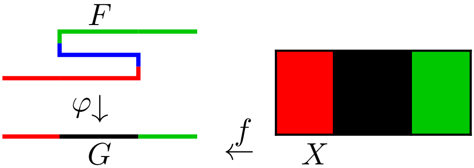
Theorem (cf. Krasinkiewicz-Minc 1977)

X is hereditarily indecomposable if and only if for any map $f : X \to G$ to a graph G, for any simple fold $\varphi : F \to G$, and for any $\varepsilon > 0$, there exists a map $g : X \to F$ such that $\varphi \circ g =_\varepsilon f$.

Simple folds

Simple fold on a graph G:

- Subgraphs $G_1, G_2, G_3 \subset G$ such that
  - $G_1 \cup G_3 = G$ and $G_1 \cap G_3 = G_2$;
  - $\overline{G_1 \setminus G_2} \cap \overline{G_3 \setminus G_2} = \emptyset$.
- Graph $F = F_1 \cup F_2 \cup F_3$ and map $\varphi : F \to G$ such that
  - $\varphi\restriction_{F_i} : F_i \to G_i$ is a homeomorphism for each $i = 1, 2, 3$;
  - $\partial G_1 = \varphi(F_1 \cap F_2)$, $\partial G_3 = \varphi(F_2 \cap F_3)$, and $F_1 \cap F_3 = \emptyset$.



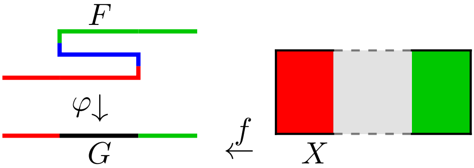
Theorem (cf. Krasinkiewicz-Minc 1977)

X is hereditarily indecomposable if and only if for any map $f : X \to G$ to a graph G, for any simple fold $\varphi : F \to G$, and for any $\varepsilon > 0$, there exists a map $g : X \to F$ such that $\varphi \circ g =_\varepsilon f$.

Simple folds

Simple fold on a graph G:

- Subgraphs $G_1, G_2, G_3 \subset G$ such that
  - $G_1 \cup G_3 = G$ and $G_1 \cap G_3 = G_2$;
  - $\overline{G_1 \setminus G_2} \cap \overline{G_3 \setminus G_2} = \emptyset$.
- Graph $F = F_1 \cup F_2 \cup F_3$ and map $\varphi : F \to G$ such that
  - $\varphi\restriction_{F_i} : F_i \to G_i$ is a homeomorphism for each $i = 1, 2, 3$;
  - $\partial G_1 = \varphi(F_1 \cap F_2)$, $\partial G_3 = \varphi(F_2 \cap F_3)$, and $F_1 \cap F_3 = \emptyset$.



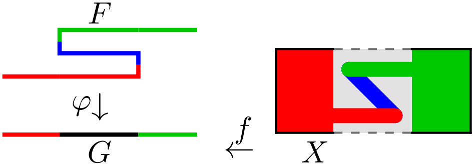
Theorem (cf. Krasinkiewicz-Minc 1977)

X is hereditarily indecomposable if and only if for any map $f : X \to G$ to a graph G, for any simple fold $\varphi : F \to G$, and for any $\varepsilon > 0$, there exists a map $g : X \to F$ such that $\varphi \circ g =_\varepsilon f$.

Simple folds

Simple fold on a graph G:

- Subgraphs $G_1, G_2, G_3 \subset G$ such that
  - $G_1 \cup G_3 = G$ and $G_1 \cap G_3 = G_2$;
  - $\overline{G_1 \setminus G_2} \cap \overline{G_3 \setminus G_2} = \emptyset$.
- Graph $F = F_1 \cup F_2 \cup F_3$ and map $\varphi : F \to G$ such that
  - $\varphi\restriction_{F_i} : F_i \to G_i$ is a homeomorphism for each $i = 1, 2, 3$;
  - $\partial G_1 = \varphi(F_1 \cap F_2)$, $\partial G_3 = \varphi(F_2 \cap F_3)$, and $F_1 \cap F_3 = \emptyset$.



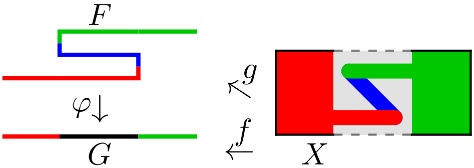
Theorem (cf. Krasinkiewicz-Minc 1977)

X is hereditarily indecomposable if and only if for any map $f : X \to G$ to a graph G, for any simple fold $\varphi : F \to G$, and for any $\varepsilon > 0$, there exists a map $g : X \to F$ such that $\varphi \circ g =_\varepsilon f$.

Simple folds

Simple fold on a graph G:

- Subgraphs $G_1, G_2, G_3 \subset G$ such that
  - $G_1 \cup G_3 = G$ and $G_1 \cap G_3 = G_2$;
  - $\overline{G_1 \setminus G_2} \cap \overline{G_3 \setminus G_2} = \emptyset$.
- Graph $F = F_1 \cup F_2 \cup F_3$ and map $\varphi : F \to G$ such that
  - $\varphi\restriction_{F_i} : F_i \to G_i$ is a homeomorphism for each $i = 1, 2, 3$;
  - $\partial G_1 = \varphi(F_1 \cap F_2)$, $\partial G_3 = \varphi(F_2 \cap F_3)$, and $F_1 \cap F_3 = \emptyset$.



Theorem (cf. Krasinkiewicz-Minc 1977)

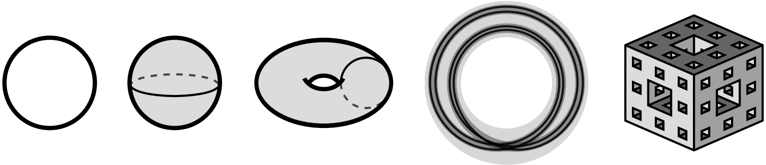
X is hereditarily indecomposable if and only if for any map $f : X \to G$ to a graph G, for any simple fold $\varphi : F \to G$, and for any $\varepsilon > 0$, there exists a map $g : X \to F$ such that $\varphi \circ g =_\varepsilon f$.

Homogeneous continua

X is *homogeneous* if $\forall\, x, y \in X \quad \exists\, h : X \overset{\approx}{\to} X \quad h(x) = y$

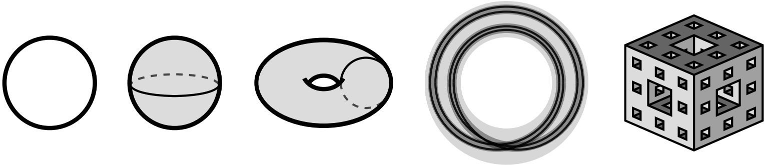
Homogeneous continua

X is *homogeneous* if $\forall x, y \in X \quad \exists h : X \stackrel{\approx}{\to} X \quad h(x) = y$



Homogeneous continua

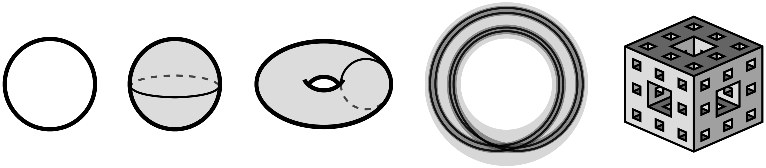
X is *homogeneous* if $\forall\, x, y \in X \quad \exists\, h : X \stackrel{\approx}{\to} X \quad h(x) = y$



Examples: Connected manifolds, topological groups

Homogeneous continua

X is *homogeneous* if $\forall x, y \in X \quad \exists\, h : X \overset{\approx}{\to} X \quad h(x) = y$



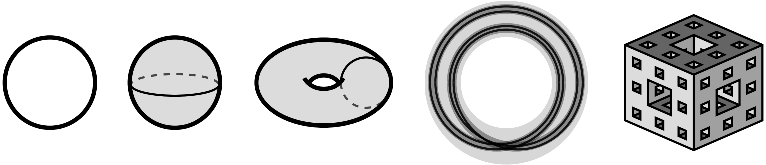
Examples: Connected manifolds, topological groups

Question (Knaster-Kuratowski 1920)

Is the circle the only non-degenerate homogeneous continuum in $\mathbb{R}^2$?

Homogeneous continua

X is *homogeneous* if $\forall\, x, y \in X \quad \exists\, h : X \overset{\approx}{\to} X \quad h(x) = y$



Examples: Connected manifolds, topological groups

Question (Knaster-Kuratowski 1920)

Is the circle the only non-degenerate homogeneous continuum in $\mathbb{R}^2$?

Answer: **No.** Known examples: circle, pseudo-arc, circle of pseudo-arcs

Homogeneous plane continua

Theorem (Jones 1955)

If $M \subset \mathbb{R}^2$ is decomposable and homogeneous, then M is a circle of X's, where X is indecomposable and homogeneous.

Homogeneous plane continua

Theorem (Jones 1955)

If $M \subset \mathbb{R}^2$ is decomposable and homogeneous, then M is a circle of X's, where X is indecomposable and homogeneous.

Theorem (Hagopian 1976)

If $X \subset \mathbb{R}^2$ is indecomposable and homogeneous, then X is hereditarily indecomposable.

Homogeneous plane continua

Theorem (Jones 1955)

If $M \subset \mathbb{R}^2$ is decomposable and homogeneous, then M is a circle of X's, where X is indecomposable and homogeneous.

Theorem (Hagopian 1976)

If $X \subset \mathbb{R}^2$ is indecomposable and homogeneous, then X is hereditarily indecomposable.

Theorem (Oversteegen-Tymchatyn 1982)

If $X \subset \mathbb{R}^2$ is indecomposable and homogeneous, then X has span zero.

Homogeneous plane continua

Theorem (Jones 1955)

If $M \subset \mathbb{R}^2$ is decomposable and homogeneous, then M is a circle of X's, where X is indecomposable and homogeneous.

Theorem (Hagopian 1976)

If $X \subset \mathbb{R}^2$ is indecomposable and homogeneous, then X is hereditarily indecomposable.

Theorem (Oversteegen-Tymchatyn 1982)

If $X \subset \mathbb{R}^2$ is indecomposable and homogeneous, then X has span zero.

Theorem (Oversteegen-H 2015)

A continuum is homeomorphic to the pseudo-arc if and only if it is hereditarily indecomposable and has span zero.

Span and separators

Suppose $X \subset [0,1]^{\mathbb{N}}$ has span zero

Span and separators

Suppose $X \subset [0,1]^{\mathbb{N}}$ has span zero $\quad \Rightarrow X$ is tree-like.

Span and separators

Suppose $X \subset [0,1]^{\mathbb{N}}$ has span zero $\quad \Rightarrow X$ is tree-like.

Theorem

$\forall\, \varepsilon > 0 \quad \exists\, \delta > 0$ such that if T is a tree and $I = [p,q]$ is an arc, with $d_H(T,X) < \delta$ and $d_H(I,X) < \delta$, then the set

$$M = \{(x,y) \in T \times I : d(x,y) < \tfrac{\varepsilon}{3}\}$$

separates $T \times \{p\}$ from $T \times \{q\}$.

Span and separators

Suppose $X \subset [0,1]^{\mathbb{N}}$ has span zero $\quad \Rightarrow X$ is tree-like.

Theorem

$\forall\, \varepsilon > 0 \quad \exists\, \delta > 0$ such that if T is a tree and $I = [p,q]$ is an arc, with $d_H(T,X) < \delta$ and $d_H(I,X) < \delta$, then the set

$$M = \{(x,y) \in T \times I : d(x,y) < \tfrac{\varepsilon}{3}\}$$

separates $T \times \{p\}$ from $T \times \{q\}$.

Fix $\varepsilon > 0$, take such $\delta \le \frac{\varepsilon}{3}$.

Span and separators

Suppose $X \subset [0,1]^{\mathbb{N}}$ has span zero $\quad \Rightarrow X$ is tree-like.

Theorem

$\forall\, \varepsilon > 0 \quad \exists\, \delta > 0$ such that if T is a tree and $I = [p,q]$ is an arc, with $d_H(T,X) < \delta$ and $d_H(I,X) < \delta$, then the set

$$M = \{(x,y) \in T \times I : d(x,y) < \tfrac{\varepsilon}{3}\}$$

separates $T \times \{p\}$ from $T \times \{q\}$.

Fix $\varepsilon > 0$, take such $\delta \leq \frac{\varepsilon}{3}$. Let $T \subset [0,1]^{\mathbb{N}}$ be a tree, $f : X \to T$ such that $f =_{\delta} \mathrm{id}_X$.

Span and separators

Suppose $X \subset [0,1]^{\mathbb{N}}$ has span zero $\Rightarrow X$ is tree-like.

Theorem

$\forall\, \varepsilon > 0 \quad \exists\, \delta > 0$ such that if T is a tree and $I = [p,q]$ is an arc, with $d_H(T,X) < \delta$ and $d_H(I,X) < \delta$, then the set

$$M = \{(x,y) \in T \times I : d(x,y) < \tfrac{\varepsilon}{3}\}$$

separates $T \times \{p\}$ from $T \times \{q\}$.

Fix $\varepsilon > 0$, take such $\delta \leq \frac{\varepsilon}{3}$. Let $T \subset [0,1]^{\mathbb{N}}$ be a tree, $f : X \to T$ such that $f =_\delta \mathrm{id}_X$. Let $I = [p,q] \subset [0,1]^{\mathbb{N}}$ be such that $d_H(I,X) < \delta$.

Span and separators

Suppose $X \subset [0,1]^{\mathbb{N}}$ has span zero $\Rightarrow X$ is tree-like.

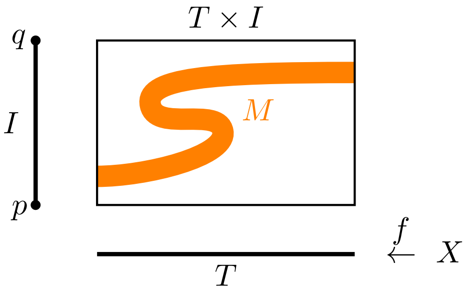
Theorem

$\forall\, \varepsilon > 0 \quad \exists\, \delta > 0$ such that if T is a tree and $I = [p,q]$ is an arc, with $d_H(T,X) < \delta$ and $d_H(I,X) < \delta$, then the set

$$M = \{(x,y) \in T \times I : d(x,y) < \tfrac{\varepsilon}{3}\}$$

separates $T \times \{p\}$ from $T \times \{q\}$.

Fix $\varepsilon > 0$, take such $\delta \leq \frac{\varepsilon}{3}$. Let $T \subset [0,1]^{\mathbb{N}}$ be a tree, $f : X \to T$ such that $f =_\delta \mathrm{id}_X$. Let $I = [p,q] \subset [0,1]^{\mathbb{N}}$ be such that $d_H(I,X) < \delta$.



Span and separators

Suppose $X \subset [0,1]^{\mathbb{N}}$ has span zero $\quad \Rightarrow X$ is tree-like.

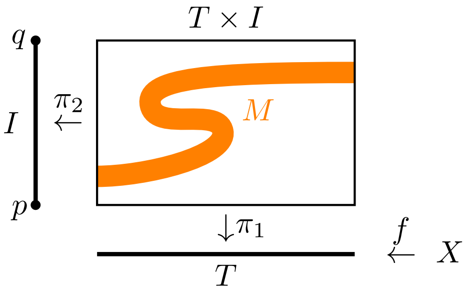
Theorem

$\forall\, \varepsilon > 0 \quad \exists\, \delta > 0$ such that if T is a tree and $I = [p,q]$ is an arc, with $d_H(T,X) < \delta$ and $d_H(I,X) < \delta$, then the set

$$M = \{(x,y) \in T \times I : d(x,y) < \tfrac{\varepsilon}{3}\}$$

separates $T \times \{p\}$ from $T \times \{q\}$.

Fix $\varepsilon > 0$, take such $\delta \leq \frac{\varepsilon}{3}$. Let $T \subset [0,1]^{\mathbb{N}}$ be a tree, $f : X \to T$ such that $f =_\delta \mathrm{id}_X$. Let $I = [p,q] \subset [0,1]^{\mathbb{N}}$ be such that $d_H(I,X) < \delta$.



Span and separators

Suppose $X \subset [0,1]^{\mathbb{N}}$ has span zero $\Rightarrow X$ is tree-like.

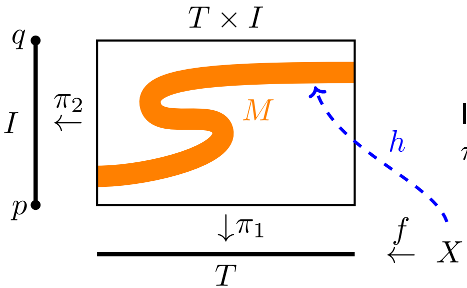
Theorem

$\forall\, \varepsilon > 0 \quad \exists\, \delta > 0$ such that if T is a tree and $I = [p,q]$ is an arc, with $d_H(T,X) < \delta$ and $d_H(I,X) < \delta$, then the set

$$M = \{(x,y) \in T \times I : d(x,y) < \tfrac{\varepsilon}{3}\}$$

separates $T \times \{p\}$ from $T \times \{q\}$.

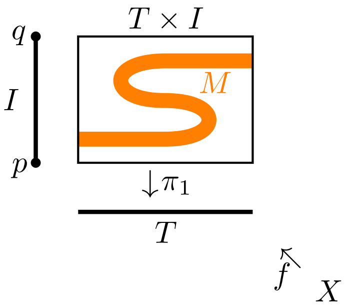
Fix $\varepsilon > 0$, take such $\delta \leq \frac{\varepsilon}{3}$. Let $T \subset [0,1]^{\mathbb{N}}$ be a tree, $f : X \to T$ such that $f =_\delta \mathrm{id}_X$. Let $I = [p,q] \subset [0,1]^{\mathbb{N}}$ be such that $d_H(I,X) < \delta$.



If $\exists\, h : X \to M$ such that $\pi_1 \circ h =_\delta f$, then $\pi_2 \circ h =_\varepsilon \mathrm{id}_X$.

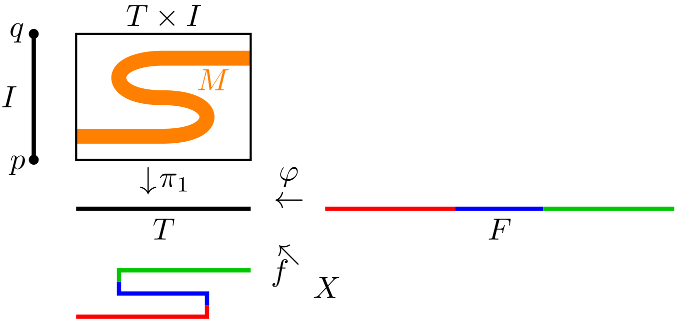
Unfolding separator: Example 1

If $\exists\, h : X \to M$ such that $\pi_1 \circ h =_\delta f$, then $\pi_2 \circ h =_\varepsilon \mathrm{id}_X$.



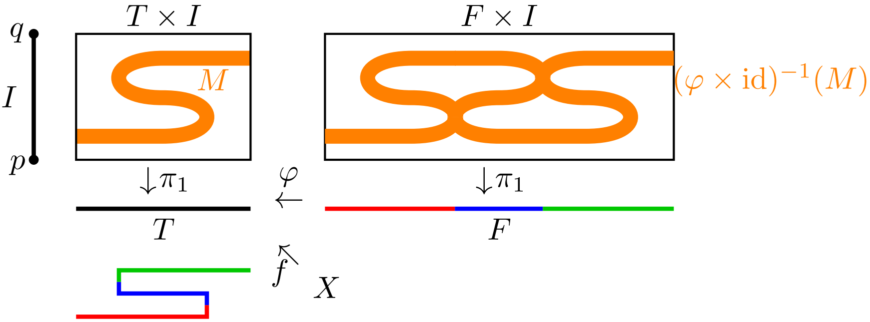
Unfolding separator: Example 1

If $\exists\ h : X \to M$ such that $\pi_1 \circ h =_\delta f$, then $\pi_2 \circ h =_\varepsilon \mathrm{id}_X$.



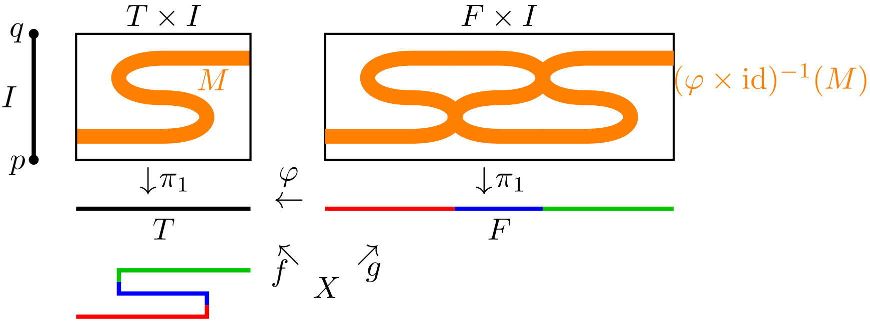
Unfolding separator: Example 1

If $\exists\, h : X \to M$ such that $\pi_1 \circ h =_\delta f$, then $\pi_2 \circ h =_\varepsilon \mathrm{id}_X$.



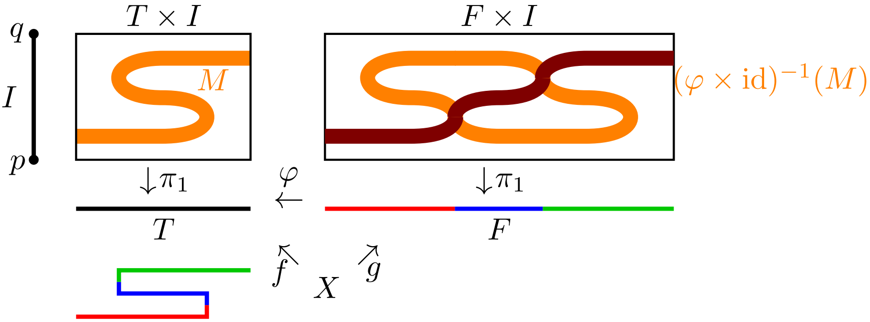
Unfolding separator: Example 1

If $\exists\, h : X \to M$ such that $\pi_1 \circ h =_\delta f$, then $\pi_2 \circ h =_\varepsilon \mathrm{id}_X$.



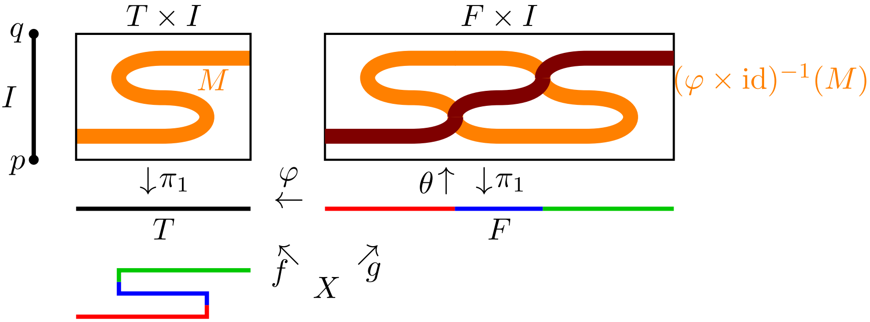
Unfolding separator: Example 1

If $\exists\, h : X \to M$ such that $\pi_1 \circ h =_\delta f$, then $\pi_2 \circ h =_\varepsilon \mathrm{id}_X$.



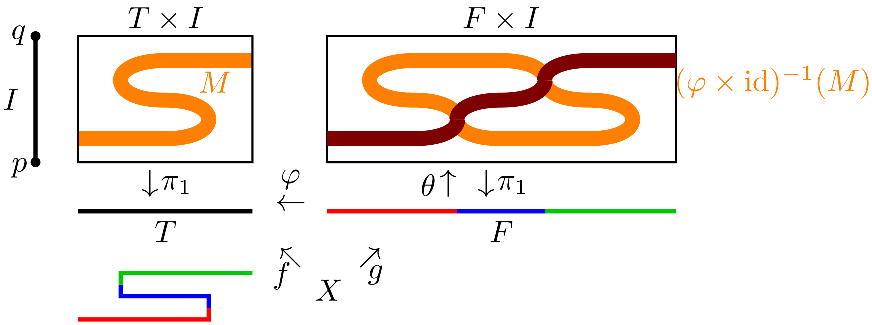
Unfolding separator: Example 1

If $\exists\, h : X \to M$ such that $\pi_1 \circ h =_\delta f$, then $\pi_2 \circ h =_\varepsilon \mathrm{id}_X$.



Unfolding separator: Example 1

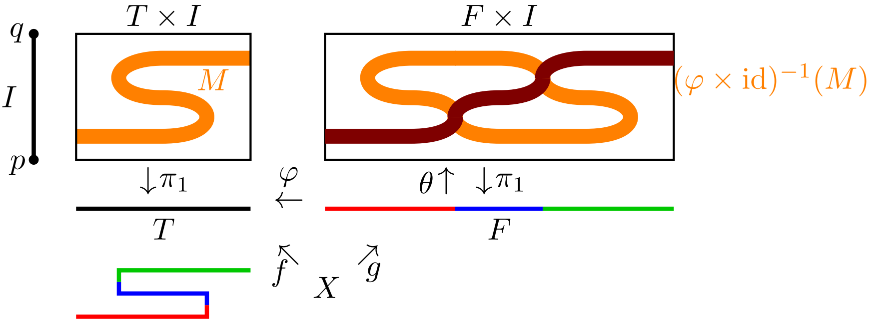
If $\exists\, h : X \to M$ such that $\pi_1 \circ h =_\delta f$, then $\pi_2 \circ h =_\varepsilon \mathrm{id}_X$.



Define $h : X \to M$ by $h = (\varphi \times \mathrm{id}) \circ \theta \circ g$.

Unfolding separator: Example 1

If $\exists\, h : X \to M$ such that $\pi_1 \circ h =_\delta f$, then $\pi_2 \circ h =_\varepsilon \mathrm{id}_X$.



Define $h : X \to M$ by $h = (\varphi \times \mathrm{id}) \circ \theta \circ g$.

Idea: Find sequence of simple folds $T \leftarrow F_1 \leftarrow F_2 \leftarrow \cdots \leftarrow F_n$ such that in F_n, separator has a subset S' such that π_1 maps S' one-to-one onto F_n.

Stairwell structures

(Informal definition)

Stairwell structures

(Informal definition)

A separator $S \subset G \times [0,1]$ has a *stairwell structure* if $S = S_1 \cup S_2 \cup \cdots \cup S_k$, where

Stairwell structures

(Informal definition)

A separator $S \subset G \times [0, 1]$ has a *stairwell structure* if $S = S_1 \cup S_2 \cup \cdots \cup S_k$, where

1. π_1 is one-to-one on each S_i

Stairwell structures

(Informal definition)

A separator $S \subset G \times [0,1]$ has a *stairwell structure* if $S = S_1 \cup S_2 \cup \cdots \cup S_k$, where

1. π_1 is one-to-one on each S_i
2. S_i connects only to S_{i-1} and S_{i+1} at its "ends"; S_1 has no "lower end" and S_k has no "upper end"

Stairwell structures

(Informal definition)

A separator $S \subset G \times [0,1]$ has a *stairwell structure* if $S = S_1 \cup S_2 \cup \cdots \cup S_k$, where

1. π_1 is one-to-one on each S_i
2. S_i connects only to S_{i-1} and S_{i+1} at its "ends"; S_1 has no "lower end" and S_k has no "upper end"
3. For a small neighborhood V of $S_i \cap S_{i+1}$, $\pi_1(S_i \cap V) = \pi_1(S_{i+1} \cap V)$

Stairwell structures

(Informal definition)

A separator $S \subset G \times [0,1]$ has a *stairwell structure* if $S = S_1 \cup S_2 \cup \cdots \cup S_k$, where

1. π_1 is one-to-one on each S_i
2. S_i connects only to S_{i-1} and S_{i+1} at its "ends"; S_1 has no "lower end" and S_k has no "upper end"
3. For a small neighborhood V of $S_i \cap S_{i+1}$, $\pi_1(S_i \cap V) = \pi_1(S_{i+1} \cap V)$
4. From any one component of the complement of $\pi_1(S_i)$ ("floor"), you never see S_i going both up and down

Stairwell structures

(Informal definition)

A separator $S \subset G \times [0,1]$ has a *stairwell structure* if $S = S_1 \cup S_2 \cup \cdots \cup S_k$, where

1. π_1 is one-to-one on each S_i
2. S_i connects only to S_{i-1} and S_{i+1} at its "ends"; S_1 has no "lower end" and S_k has no "upper end"
3. For a small neighborhood V of $S_i \cap S_{i+1}$, $\pi_1(S_i \cap V) = \pi_1(S_{i+1} \cap V)$
4. From any one component of the complement of $\pi_1(S_i)$ ("floor"), you never see S_i going both up and down
5. The projections of the ends of the S_i's are disjoint from each other and from the branch points of G

Stairwell structures

(Informal definition)

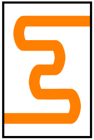
A separator $S \subset G \times [0,1]$ has a *stairwell structure* if $S = S_1 \cup S_2 \cup \cdots \cup S_k$, where

1. π_1 is one-to-one on each S_i
2. S_i connects only to S_{i-1} and S_{i+1} at its "ends"; S_1 has no "lower end" and S_k has no "upper end"
3. For a small neighborhood V of $S_i \cap S_{i+1}$, $\pi_1(S_i \cap V) = \pi_1(S_{i+1} \cap V)$
4. From any one component of the complement of $\pi_1(S_i)$ ("floor"), you never see S_i going both up and down
5. The projections of the ends of the S_i's are disjoint from each other and from the branch points of G

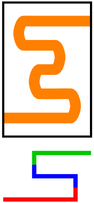
Theorem

Given any separator $M \subseteq G \times (0,1)$ and any open set $U \subseteq G \times (0,1)$ with $M \subseteq U$, there exists a separator $S \subset U$ with a stairwell structure of odd height.

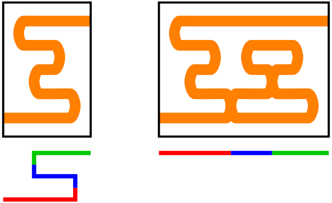
Unfolding separator: Example 2



Unfolding separator: Example 2

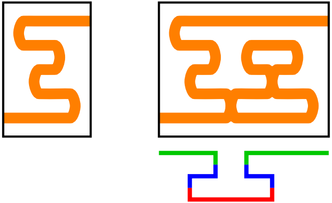


Unfolding separator: Example 2

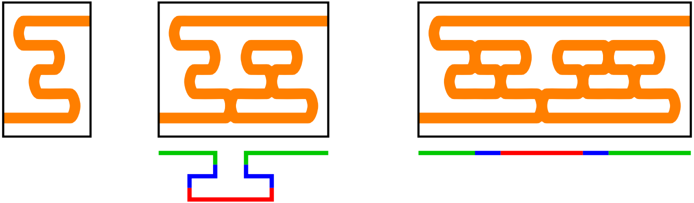


Unfolding separator: Example 2

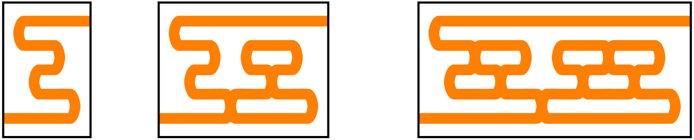
Unfolding separator: Example 2



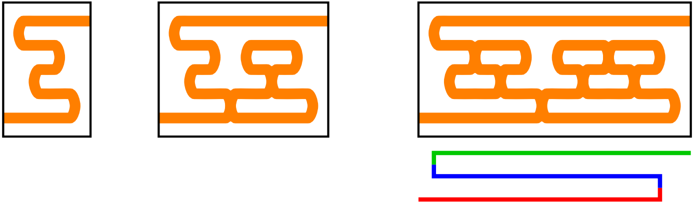
Unfolding separator: Example 2



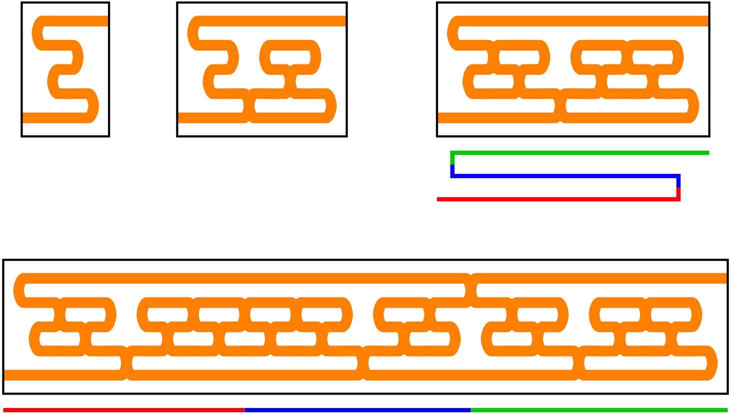
Unfolding separator: Example 2



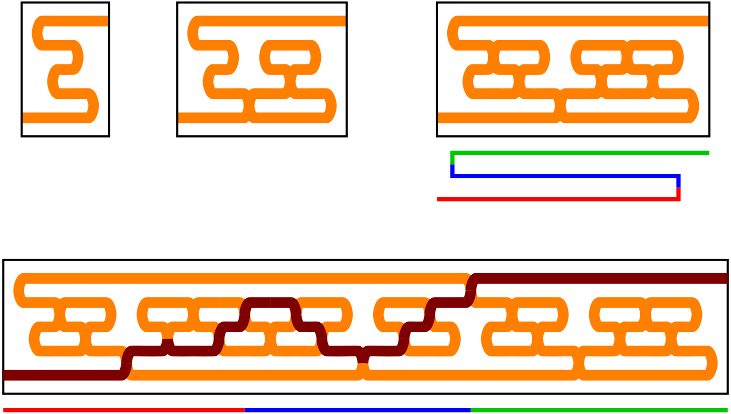
Unfolding separator: Example 2



Unfolding separator: Example 2



Unfolding separator: Example 2



Questions 1

X is *weakly chainable* if it is the continuous image of the pseudo-arc

Questions 1

X is *weakly chainable* if it is the continuous image of the pseudo-arc

- Arc-like $\Rightarrow$ span zero $\Rightarrow$ weakly chainable

Questions 1

X is *weakly chainable* if it is the continuous image of the pseudo-arc

- Arc-like ⇒ span zero ⇒ weakly chainable

Question

Is the pseudo-arc the only hereditarily indecomposable and weakly chainable continuum?

Questions 1

X is *weakly chainable* if it is the continuous image of the pseudo-arc

- Arc-like $\Rightarrow$ span zero $\Rightarrow$ weakly chainable

Question

Is the pseudo-arc the only hereditarily indecomposable and weakly chainable continuum?

A homogeneous continuum is tree-like if and only if it is hereditarily indecomposable (Rogers 1982, Krupski-Prajs 1990).

Questions 1

X is *weakly chainable* if it is the continuous image of the pseudo-arc

- Arc-like $\Rightarrow$ span zero $\Rightarrow$ weakly chainable

Question

Is the pseudo-arc the only hereditarily indecomposable and weakly chainable continuum?

A homogeneous continuum is tree-like if and only if it is hereditarily indecomposable (Rogers 1982, Krupski-Prajs 1990).

Question

Is the pseudo-arc the only homogeneous tree-like continuum?

Questions 2

X is *hereditarily equivalent* if it is homeomorphic to each of its non-degenerate subcontinua

Questions 2

X is *hereditarily equivalent* if it is homeomorphic to each of its non-degenerate subcontinua

- Known examples: arc and pseudo-arc

Questions 2

X is *hereditarily equivalent* if it is homeomorphic to each of its non-degenerate subcontinua

- Known examples: arc and pseudo-arc
- The arc is the only decomposable hereditarily equivalent continuum (Henderson 1960)

Questions 2

X is *hereditarily equivalent* if it is homeomorphic to each of its non-degenerate subcontinua

- Known examples: arc and pseudo-arc
- The arc is the only decomposable hereditarily equivalent continuum (Henderson 1960)
- An indecomposable hereditarily equivalent continuum is tree-like (Cook 1970)

Questions 2

X is *hereditarily equivalent* if it is homeomorphic to each of its non-degenerate subcontinua

- Known examples: arc and pseudo-arc
- The arc is the only decomposable hereditarily equivalent continuum (Henderson 1960)
- An indecomposable hereditarily equivalent continuum is tree-like (Cook 1970)

Our techniques can also prove that there are no other hereditarily equivalent continua in $\mathbb{R}^2$.

Questions 2

X is *hereditarily equivalent* if it is homeomorphic to each of its non-degenerate subcontinua

- Known examples: arc and pseudo-arc
- The arc is the only decomposable hereditarily equivalent continuum (Henderson 1960)
- An indecomposable hereditarily equivalent continuum is tree-like (Cook 1970)

Our techniques can also prove that there are no other hereditarily equivalent continua in $\mathbb{R}^2$.

Question

Are there any other hereditarily equivalent continua?

Questions 2

X is *hereditarily equivalent* if it is homeomorphic to each of its non-degenerate subcontinua

- Known examples: arc and pseudo-arc
- The arc is the only decomposable hereditarily equivalent continuum (Henderson 1960)
- An indecomposable hereditarily equivalent continuum is tree-like (Cook 1970)

Our techniques can also prove that there are no other hereditarily equivalent continua in $\mathbb{R}^2$.

Question

Are there any other hereditarily equivalent continua?

Question

Is every hereditarily equivalent continuum weakly chainable?